Phys 231 Fall 2002	Homework #1	1.3, 1.68, 2.3, 2.43, 2.62, 2.65, 2.72

1.13. The displacement of a particle moving under uniform acceleration is some function of the elapsed time and the acceleration. Suppose we write this displacement $s = ka^m t^n$, where k is a dimensionless constant. Show by dimensional analysis that this expression is satisfied if $m = 1$ and $n = 2$. Can this analysis give the value of k?

$$s = ka^m t^n$$
$$L = \left(\frac{L}{T^2}\right)^m T^n$$
$$L^1 = L^m T^{-2m} T^n$$
$$L^1 = L^m T^{n-2m}$$
$$\boxed{m = 1}$$
$$n - 2m = 0$$
$$n - 2\cdot 1 = 0$$
$$\boxed{n = 2}$$

Because k is dimensionless, there is no way of determining it using dimensional analysis.

1.68. One cubic centimeter of water has a mass of 1.00×10^{-3} kg. (a) Determine the mass of $1.00\ m^3$ of water. (b) Assuming biological substances are 98% water, estimate the mass of a cell that has a diameter of 1.0 um, a human kidney, and a fly. Assume that a kidney is roughly a sphere with a radius of 4.0 cm and that a fly is roughly a cylinder 4.0 mm long and 2.0 mm in diameter.

(a) $$\rho = \frac{m}{V} = \frac{1.00 \times 10^{-3}\ kg}{1.00\ cm^3} = 1.00 \times 10^{-3}\ \frac{kg}{cm^3}$$

$$m = \rho V = \left(1.00 \times 10^{-3}\ \frac{kg}{\cancel{cm^3}}\right)\left(1.00\ \cancel{m^3}\right) \times \left(\frac{100\ \cancel{cm}}{1\ \cancel{m}}\right)^3$$

$$\boxed{m = 1000\ kg}$$

(b) cell: $d = 1.0\ um$

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{d}{2}\right)^3 = \frac{4}{3}\pi\left(\frac{1.0\ um}{2}\right)^3 = 0.52\ um^3$$

$$m = \rho V = \left(1.00 \times 10^{-3}\ \frac{kg}{\cancel{cm^3}}\right)\left(0.52\ \cancel{um^3}\right) \times \left(\frac{100\ \cancel{cm}}{1\ \cancel{m}}\right)^3 \times \left(\frac{1\ \cancel{m}}{10^6\ \cancel{um}}\right)^3$$

$$m = (5.2 \times 10^{-16}\ kg)(0.98) = \boxed{5.1 \times 10^{-16}\ kg}$$

kidney: $r = 4.0\ cm$

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(4.0\ cm)^3 = 270\ cm^3$$

$$m = \rho V = \left(1.00 \times 10^{-3}\ \frac{kg}{\cancel{cm^3}}\right)(270\ \cancel{cm^3})$$

$$m = (0.27\ kg)(0.98) = \boxed{0.26\ kg}$$

fly: $L = 4.0\ mm$, $d = 2.0\ mm$

$$V = \pi r^2 L = \pi\left(\frac{d}{2}\right)^2 L = \pi\left(\frac{2.0\ mm}{2}\right)^2 (4.0\ mm) = 13\ mm^3$$

$$m = \rho V = \left(1.00 \times 10^{-3}\ \frac{kg}{\cancel{cm^3}}\right)\left(13\ \cancel{mm^3}\right) \times \left(\frac{100\ \cancel{cm}}{1\ m}\right)^3 \times \left(\frac{1\ m}{1000\ \cancel{mm}}\right)^3$$

$$m = (1.3 \times 10^{-5}\ kg)(0.98) = \boxed{1.2 \times 10^{-5}\ kg}$$

2.3. The displacement versus time for a certain particle moving along the x-axis is shown in Figure P2.3. Find the average velocity in the time intervals (a) 0 to 2 s, (b) 0 to 4 s, (c) 2 s to 4 s, (d) 4 s to 7 s, (e) 0 to 8 s.

(a) 0 to 2 s

$$\overline{V_x} \equiv \frac{\Delta x}{\Delta t} = \frac{10\ m}{2\ s} = 5\ m/s$$

(b) 0 to 4 s

$$\overline{V_x} \equiv \frac{\Delta x}{\Delta t} = \frac{5\ m}{4\ s} = 1.25\ m/s$$

(c) 2s to 4s

$$\overline{V_x} \equiv \frac{\Delta x}{\Delta t} = \frac{-5\ m}{2\ s} = -2.5\ m/s$$

(d) 4 s to 7 s

$$\overline{V_x} \equiv \frac{\Delta x}{\Delta t} = \frac{(-5-5)m}{3\ s} = -3.3\ m/s$$

(e) 0 to 8 s

$$\overline{V_x} \equiv \frac{\Delta x}{\Delta t} = \frac{0\ m}{8\ s} = 0\ m/s$$

2.43. A student throws a set of keys vertically upward to her sorority sister, who is in a window 4.00 m above. The keys are caught 1.5 s later by the sister's outstretched hand. (a) With what initial velocity were the keys thrown? (b) What was the velocity of the keys just before they were caught?

$y_i = 0\ m$ $\quad a_y = -9.80\ m/s^2$

$y_f = 4.00\ m$ $\quad V_{y_i} = ?$

$t = 1.5\ s$

(a) $y_f = y_i + V_{y_i} t + \frac{1}{2} a_y t^2$

$$V_{y_i} = \frac{y_f - y_i - \frac{1}{2} a_y t^2}{t} = \frac{4.00\ m - 0\ m - \frac{1}{2}(-9.80\ m/\cancel{s^2})(1.5\ \cancel{s})^2}{1.5\ s}$$

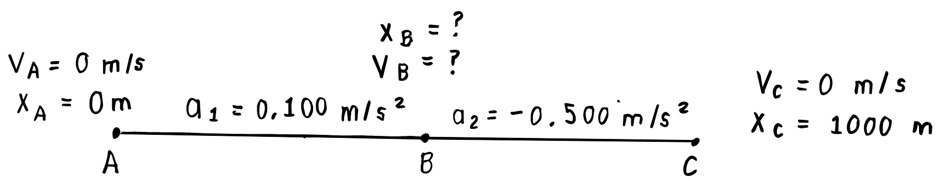
$$\boxed{V_{y_i} = 10\ m/s}$$

(b) $V_{y_f} = V_{y_i} + a_y t$

$V_{y_f} = 10\ m/s + (-9.80\ m/s^{\cancel{2}})(1.5\ \cancel{s})$

$$\boxed{V_{y_f} = -4.7\ m/s}$$

2.62. A commuter train travels between 2 downtown stations. Because the stations are only 1.00 km apart, the train never reaches its maximum possible cruising speed. The engineer minimizes the time t between the 2 stations by accelerating at a rate $a_1 = 0.100\ m/s^2$ for a time t_1 and then by breaking with acceleration $a_2 = -0.500\ m/s^2$ for a time t_2. Find the minimum time of travel and the time t_1.



$X_f = X_i + V_i t + \frac{1}{2} a_x t^2$
$X = 0 + 0 + \frac{1}{2}(0.100\ m/s^2)\, t_1^2$
$X = (0.05\ m/s^2)\, t_1^2$

$X_f = X_i + V_i t + \frac{1}{2} a_x t^2$
$1000\ m = X + v t_2 + \frac{1}{2}(-0.500\ m/s^2)\, t_2^2$
$1000\ m = X + v t_2 - (0.25\ m/s^2)\, t_2^2$
$X = 1000\ m - v t_2 + (0.25\ m/s^2)\, t_2^2$

53 $V_f = V_i + a t$
$V = 0 + (0.100\ m/s^2) t_1$
$V = (0.100\ m/s^2)\, t_1$

$V_f = V_i + a t$
$0 = V + (-0.500\ m/s^2)\, t_2$
$V = (0.500\ m/s^2)\, t_2$
$t_2 = (2.00\ s^2/m)\, v$
$t_2 = (2\ \cancel{s^2}/\cancel{m})(0.1\ \cancel{m}/\cancel{s^2})\, t_1$
$t_2 = 0.2\, t_1$

$X = X$
$(0.05\ m/s^2)\, t_1^2 = 1000\ m - v t_2 + (0.25\ m/s^2)\, t_2^2$
$(0.05\ m/s^2)\, t_1^2 = 1000\ m - (0.100\ m/s^2) t_1 (0.2\, t_1) + (0.25\ m/s^2)(0.2\, t_1)^2$
$(0.05\ m/s^2)\, t_1^2 = 1000\ m - (0.02\ m/s^2)\, t_1^2 + (0.01\ m/s^2)\, t_1^2$
$(0.06\ m/s^2)\, t_1^2 = 1000\ m$

$t_1^2 = \frac{1000\ \cancel{m}}{0.06\ \cancel{m}/s^2}$
$\boxed{t_1 = 129\ s}$

$t_2 = 0.2\, t_1 = 0.2(130\ s) = 26\ s$
$t = t_1 + t_2 = 129\ s + 26\ s = 156\ s$
$\boxed{t = 155\ s}$

2.65. A teenager has a car that speeds up at $3.00\ m/s^2$ and slows down at $-4.50\ m/s^2$. On a trip to the store, he accelerates from rest to $12.0\ m/s$, drives at a constant speed for $5.00\ s$, and then comes to a momentary stop at an intersection. He then accelerates to $18.0\ m/s$, drives at a constant speed for $20.0\ s$, slows down for $2.67\ s$, continues for $4.00\ s$ at this speed, and then comes to a stop.
(a) How long does the trip take? (b) How far has he traveled? (c) What is his average speed for the trip? (d) How long would it take to walk to the store and back if he walks at $1.50\ m/s$.?

(a) $V_f = V_0 + a t$

$$t = \frac{V_f - V_0}{a} = \frac{12.0\ \cancel{m}/s - 0\ \cancel{m}/s}{3.00\ \cancel{m}/s^2} = 4.00\ s$$

$$t = \frac{V_f - V_0}{a} = \frac{0\ m/s - 12.0\ m/s}{-4.50\ m/s^2} = 2.67\ s$$

$$t = \frac{V_f - V_0}{a} = \frac{18.0\ m/s - 0\ m/s}{3.00\ m/s^2} = 6.00\ s$$

$$V_f = V_0 + a t = 18.0\ m/s + (-4.50\ m/s^2)(2.67 s) = 6.00\ m/s$$

$$t = \frac{V_f - V_0}{a} = \frac{0\ m/s - 6.00\ m/s}{-4.50\ m/s^2} = 1.33\ s$$

$$t = 4.00 s + 5.00 s + 2.67 s + 6.00 s + 20.0 s + 2.67 s + 4.00 s + 1.33 s$$

$$t = 45.67\ s$$

(b) $X_f = X_i + v_i t + \frac{1}{2} a t^2$

$$X_f = 0 + 0 + \tfrac{1}{2}(3.00 m/s^2)(4.00 s)^2 = 24\ m$$

$$X_f = 0 + (12 m/s)(5.00 s) + 0 = 60\ m$$

$$X_f = 0 + (12 m/s)(2.67 s) + \tfrac{1}{2}(-4.50 m/s^2)(2.67 s)^2 = 16\ m$$

$$X_f = 0 + 0 + \tfrac{1}{2}(3.00 m/s^2)(6.00 s)^2 = 54\ m$$

$$X_f = 0 + (18 m/s)(20.0 s) + 0 = 360\ m$$

$$X_f = 0 + (18 m/s)(2.67 s) + \tfrac{1}{2}(-4.50 m/s^2)(2.67 s)^2 = 32.0\ m$$

$$X_f = 0 + (6.00 m/s)(4.00 s) + 0 = 24.0\ m$$

$$X_f = 0 + (6.00 m/s)(1.33 s) + \tfrac{1}{2}(-4.50 m/s^2)(1.33 s)^2 = 4.0\ m$$

$$X_f = 24\ m + 60\ m + 16\ m + 54\ m + 360\ m + 32.0\ m + 24.0\ m + 4.0\ m$$

$$X_f = 574\ m$$

(c) $$\frac{\text{total distance}}{\text{total time}} = \frac{574\ m}{45.7\ s} = 12.6 s$$

(d) $X_f = X_i + V_i t + \frac{1}{2} a t^2$

$$(574 m + 574 m) = (1.50\ m/s) t$$

$$t = 765\ s$$