14.23. Show that the spacecraft's distance from the Earth must be between 1.47×10^9 m and 1.48×10^9 m.

Let m represent the mass of the spacecraft, r_E the radius of the Earth's orbit, and x the distance from Earth to the spacecraft. The sun exerts a radial inward force of $F_S = \frac{G M_S m}{(r_E - x)^2}$ on the spacecraft while the Earth exerts a radial outward force of $F_E = \frac{G M_E m}{x^2}$ on it. The net force on the spacecraft must produce the correct centripetal acceleration for it to have an orbital period of 1.000 year. Thus,

$$F_S - F_E = \frac{G M_S m}{(r_E - x)^2} - \frac{G M_E m}{x^2} = \frac{m v^2}{(r_E - x)} = \frac{m}{(r_E - x)}\left[\frac{2\pi (r_E - x)}{T}\right]^2$$

$d = vT$

$2\pi (r_E - x) = vT$

$v = \frac{2\pi (r_E - x)}{T}$

$$\frac{G M_S m}{(r_E - x)^2} - \frac{G M_E m}{x^2} = \frac{4\pi^2 (r_E - x) m}{T^2} \quad (1)$$

We can test the assertion that x is between 1.47×10^9 m and 1.48×10^9 m by substituting both of these trial solutions, along with the following data: $M_S = 1.991 \times 10^{30}$ kg, $M_E = 5.983 \times 10^{24}$ kg, $r_E = 1.496 \times 10^{11}$ m, and $T = 1.000$ yr $= 3.156 \times 10^7$ s.

With $x = 1.47 \times 10^9$ m substituted into (1), we obtain

6.052×10^{-3} m/s^2 $- 1.85 \times 10^{-3}$ m/s^2 $\approx 5.871 \times 10^{-3}$ m/s^2

5.868×10^{-3} m/s^2 $\approx 5.871 \times 10^{-3}$ m/s^2

With $x = 1.48 \times 10^9$ m substitued into (1), we obtain

6.053×10^{-3} m/s^2 $- 1.82 \times 10^{-3}$ m/s^2 $\approx 5.8708 \times 10^{-3}$ m/s^2

5.8709×10^{-3} m/s^2 $\approx 5.8708 \times 10^{-3}$ m/s^2.

Since the first trial solution makes the left hand side of (1) slightly less than the right hand side, and the second trial solution does the opposite, the true solution is determined as between the trial values. To three digit precision, it is 1.48×10^9 m.

14.28. How much energy is required to move a 1,000 kg mass from the Earth's surface to an altitude twice the Earth's radius?

$$U = -G\frac{Mm}{r} \quad \text{and} \quad g = \frac{GM_E}{R_E^2}$$

$$\Delta U = -GMm\left(\frac{1}{3R_E} - \frac{1}{R_E}\right) = \frac{2}{3}mgR_E \quad \text{change in gravitational potential energy}$$

$$\Delta U = \frac{2}{3}(1000\text{ kg})(9.80\text{ m/s}^2)(6.37\times10^6\text{ m}) = 4.17\times10^{10}\text{ J}$$

14.31. A rigid ring of material rotates about a star. The rotational speed of the ring is 1.25×10^6 m/s, and its radius is 1.53×10^{11} m.

(a) Show that the centripetal acceleration of the inhabitants is 10.2 m/s².

$$a_r = \frac{v^2}{r} = \frac{(1.25\times10^6\text{ m/s})^2}{1.53\times10^{11}\text{ m}} = 10.2\text{ m/s}^2$$

(b) The inhabitants of this ring would experience a normal contact force $\vec{n}$. Acting alone, this normal force would produce an inward acceleration of 9.9 m/s². Additionally, the star at the center of the ring exerts a gravitational force on the ring and its inhabitants. The difference between the total acceleration and the acceleration provided by the normal force is due to the gravitational attraction of the central star. Show that the mass of the star is about 10^{32} kg.

$$\text{difference} = 10.2\frac{\text{m}}{\text{s}^2} - 9.9\frac{\text{m}}{\text{s}^2} = 0.312\frac{\text{m}}{\text{s}^2} = \frac{GM}{r^2}$$

$$M = \frac{(0.312\text{ m/s}^2)r^2}{G} = \frac{(0.312\text{ m/s}^2)(1.53\times10^{11}\text{ m})^2}{6.67\times10^{-11}\text{ N}\cdot\text{m/kg}^2} = 1.40\times10^{32}\text{ kg}$$

14.68. A rocket is given an initial speed vertically up of $v_i = 2\sqrt{Rg}$ at the surface of the Earth, which has radius R and surface free-fall acceleration g. The rocket motors are quickly cut off, and thereafter the rocket coasts under the action of gravitational forces only. Derive an expression for the subsequent speed v as a function of the distance r from the center of the Earth in terms of g, R, and r.

$$v_i = 2\sqrt{Rg} \qquad g = \frac{MG}{R^2}$$

Conservation of energy

$$\frac{mv^2}{2} - \frac{mGM}{r} = \frac{mv_i^2}{2} - \frac{mGM}{R}$$

$$\frac{\cancel{m}v^2}{2} = \frac{\cancel{m}v_i^2}{2} - \frac{\cancel{m}GM}{R} + \frac{\cancel{m}GM}{r}$$

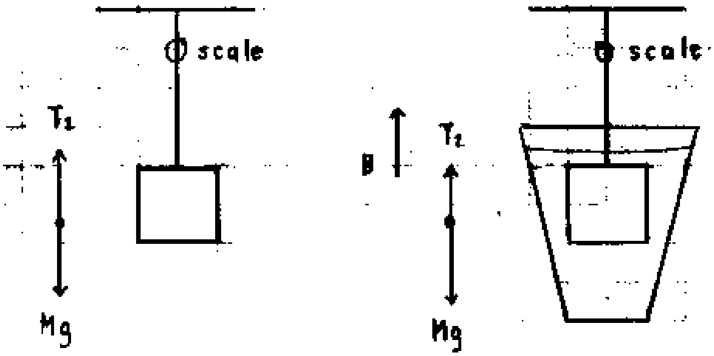
$$v^2 = v_i^2 - 2MG\left(\frac{1}{R} - \frac{1}{r}\right)$$

$$v = \sqrt{v_i^2 + 2MG\left(\frac{1}{r} - \frac{1}{R}\right)} = \sqrt{4Rg + 2MG\left(\frac{1}{r} - \frac{1}{R}\right)}$$

$$v = \sqrt{\frac{4MG}{R} + \frac{2MG}{r} - \frac{2MG}{R}} = \sqrt{\frac{2MG}{R} + \frac{2MG}{r}}$$

$$v = \sqrt{2MG\left(\frac{1}{R} + \frac{1}{r}\right)} = \sqrt{2R^2g\left(\frac{1}{R} + \frac{1}{r}\right)}$$

15.23. A piece of aluminum with mass 1.00 kg and density 2,700 kg/m³ is suspended from a string and then completely immersed in a container of water. Calculate the tension in the string (a) before and (b) after the metal is immersed.



(a) Before the metal is immersed:

$\Sigma F_y = T_1 - Mg = 0$ or

$T_1 = Mg = (1.00\ \text{kg})(9.80\ \text{m/s}^2) = 9.80\ \text{N}$

(b) After the metal is immersed:

$\Sigma F_y = T_2 + B - Mg = 0$ or

$T_2 = Mg - B = Mg - (\rho_w V)g$

$V = \dfrac{M}{\rho} = \dfrac{1.00\ \text{kg}}{2700\ \text{kg/m}^3}$

The magnitude of the buoyant force equals the weight of the fluid displaced by the object. The buoyant force acts upward.

$$T_2 = Mg - B = 9.80\ \text{N} - (1000\ \text{kg/m}^3)\left(\frac{1.00\ \text{kg}}{2700\ \text{kg/m}^3}\right)(9.80\ \text{m/s}^2) = 6.17\ N$$

15.43. A siphon is used to drain water from a tank. The siphon has a uniform diameter. Assume steady flow without friction. (a) If the distance h = 1.00 m, find the speed of outflow at the end of the siphon. (b) What is the limitation on the height of the top of the siphon above the water surface?

pressure, kinetic energy, gravitational potential energy

(a) $P + \frac{1}{2}\rho v^2 + \rho g y = \text{constant}$

$\cancel{P_0} + 0 + \cancel{\rho} g h = \cancel{P_0} + \frac{1}{2}\cancel{\rho} v_3^2 + 0$

$v_3 = \sqrt{2gh} = 4.43\ \text{m/s}$

(b) $P + \frac{1}{2}\cancel{\rho v_2^2} + \rho g(y+h) = P_0 + \frac{1}{2}\cancel{\rho v_3^2}$

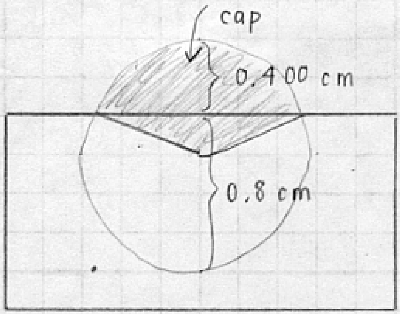
$A_3 v_3 = A_2 v_2$, $A_3 = A_2 \rightarrow v_3 = v_2$

$P = P_0 - \rho g(y+h) \geq 0$

Since $P \geq 0$,

$$(y+h) \leq \frac{P_0}{\rho g} = \frac{(1.013\times10^5\ \text{Pa})}{(10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)} = 10.3\ \text{m}$$

15.53. A wooden dowel has a diameter of 1.20 cm. It floats in water with 0.400 cm of its diameter above water level. Determine the density of the dowel.



$$\text{shaded} = \frac{\left(\pi - 2\sin^{-1}\frac{R-h}{R}\right)}{2\pi}\pi R^2$$ (including cap)

$$\text{cap} = \text{shaded} - (R-h)\sqrt{R^2-(R-h)^2}$$

$$\text{submerged area} = \pi R^2 - \text{cap}$$

$$\rho_{water} V_{under} \cancel{g} = \rho_{wood} V_{all} \cancel{g}$$

$$\rho_{wood} = \frac{\rho_{water} V_{under}}{V_{all}}$$