13.19. A 50.0 g mass connected to a spring with a force constant of 35.0 N/m oscillates on a horizontal, frictionless surface with an amplitude of 4.00 cm. Find (a) the total energy of the system and (b) the speed of the mass when the displacement is 1.00 cm. Find (c) the kinetic energy and (d) the potential energy when the displacement is 3.00 cm.

(a) $E = \frac{1}{2} k A^2 = \frac{1}{2} (35.0\ N/m)(4.00 \times 10^{-2}\ m)^2 = 28.0\ mJ$

(b) $\cancel{\frac{1}{2}} k A^2 = \cancel{\frac{1}{2}} m v^2 + \cancel{\frac{1}{2}} k x^2$

$$v = \sqrt{\frac{k}{m}(A^2 - x^2)} = \sqrt{\frac{35.0\ N/m}{50.0 \times 10^{-3}\ kg}\left[(4.00 \times 10^{-2}\ m)^2 - (1.00 \times 10^{-2} m)^2\right]}$$

$v = 1.02\ m/s$

(c) $\frac{1}{2} m v^2 = \frac{1}{2} k A^2 - \frac{1}{2} k x^2 = \frac{1}{2} k (A^2 - x^2)$

$= \frac{1}{2} (35.0\ N/m)\left[(4.00 \times 10^{-2}\ m)^2 - (3.00 \times 10^{-2} m)^2\right]$

$= 12.2\ mJ$

(d) $\frac{1}{2} k x^2 = E - \frac{1}{2} m v^2 = 28.0\ mJ - 12.2\ mJ = 15.8\ mJ$

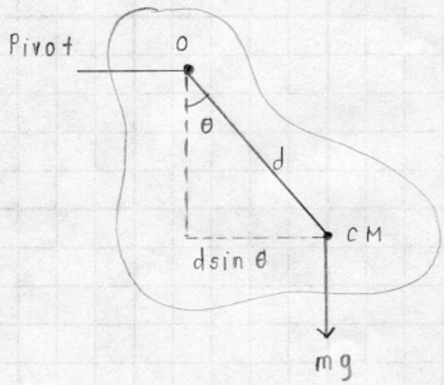
13.35. Consider the physical pendulum of Figure 13.13. (a) If I_{CM} is its moment of inertia about an axis that passes through its center of mass and is parallel to the axis that passes through its pivot point, show that the period is
$$T = 2\pi \sqrt{\frac{I_{CM} + md^2}{mgd}}$$
where d is the distance between the pivot point and the center of mass. (b) Show that the period has a minimum value when d satisfies $md^2 = I_{CM}$.

(a) The parallel axis theorem says

$I = I_{CM} + md^2$

$$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{I}{mgd}} \quad (13.28)$$

$$T = 2\pi \sqrt{\frac{I_{CM} + md^2}{mgd}}$$



(b) When d is very large

$$T \to 2\pi \sqrt{\frac{d}{g}}$$

When d is very small

$$T \to 2\pi \sqrt{\frac{I_{CM}}{mgd}}$$

So there must be a minimum, found by

$$\frac{dT}{dd} = 0 = \frac{d}{dd} 2\pi (I_{CM} + md^2)^{1/2} (mgd)^{-1/2}$$

$$= \cancel{2}\pi (I_{CM} + md^2)^{1/2} (-\cancel{\tfrac{1}{2}})(mgd)^{-3/2} mg + \cancel{2}\pi (mgd)^{-1/2} (\cancel{\tfrac{1}{2}})(I_{CM} + md^2)^{-1/2}\ 2md$$

$$\frac{-\pi (I_{CM} + md^2) mg}{(I_{CM} + md^2)^{1/2} (mgd)^{3/2}} + \frac{2\pi md\ (mgd)}{(I_{CM} + md^2)^{1/2}\ (mgd)^{3/2}} = 0$$

$- I_{CM} - md^2 + 2md^2 = 0$

$I_{CM} = md^2$

13.55. The mass of the deuterium molecule (D_2) is twice that of the hydrogen molecule (H_2). If the vibrational frequency of H_2 is 1.30×10^{14} Hz, what is the vibrational frequency of D_2? Assume that the "spring constant" of attracting forces is the same for the two molecules.

$$M_{D_2} = 2 M_{H_2}$$

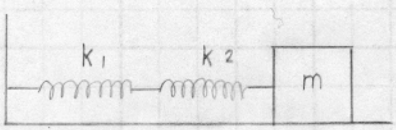
$$\frac{\omega_D}{\omega_H} = \frac{\sqrt{K/M_D}}{\sqrt{K/M_H}} = \sqrt{\frac{M_H}{M_D}} = \sqrt{\frac{1}{2}} \qquad \omega_D = \frac{1}{\sqrt{2}}\,\omega_H$$

$$\omega = 2\pi f$$

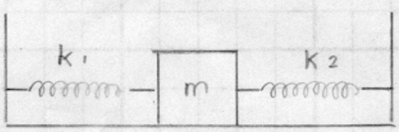
$$f_{D_2} = \frac{1}{\sqrt{2}} f_{H_2} = \frac{1}{\sqrt{2}}(1.30 \times 10^{14}\ \text{Hz}) = 0.919 \times 10^{14}\ \text{Hz}$$

13.71. A mass m is connected to two springs of force constants K_1 and K_2. In each case, the mass moves on a frictionless table and is displaced from equilibrium and then released. Show that in the two cases the mass exhibits simple harmonic motion with periods

(a) $T = 2\pi\sqrt{\dfrac{m(k_1+k_2)}{K_1 K_2}}$ (b) $T = 2\pi\sqrt{\dfrac{m}{K_1+K_2}}$



(a)



(b)

(a) When the mass is displaced a distance x from equilibrium, spring 1 is stretched a distance x_1 and spring 2 is stretched a distance x_2.

$$T = -k_1 x_1 = -k_2 x_2$$

$$\left.\begin{aligned} k_1 x_1 &= k_2 x_2 \\ x &= x_1 + x_2 \end{aligned}\right\}$$

$$\begin{aligned} k_1 x_1 &= k_2 x_2 \\ +\ k_1 x - k_1 x_1 &= k_1 x_2 \\ \hline k_1 x &= (k_1+k_2)x_2 \end{aligned} \qquad \begin{aligned} k_1 x_1 &= k_2 x_2 \\ -(k_2 x &= k_2 x_1 + k_2 x_2) \\ \hline (k_1+k_2)x_1 - k_2 x &= 0 \\ k_2 x &= (k_1+k_2)x_1 \end{aligned}$$

$$x_2 = \frac{k_1 x}{k_1+k_2} \qquad x_1 = \frac{k_2 x}{(k_1+k_2)}$$

The force on either spring is given by

$$F = -K_1 x_1 = -K_2 x_2 = -\frac{k_1 k_2}{(k_1+k_2)}x = -K_{eff}\,x$$

$$T = 2\pi\sqrt{\frac{m}{K_{eff}}} = 2\pi\sqrt{\frac{m(k_1+k_2)}{k_1 k_2}} \qquad \left(T = \frac{2\pi}{\omega}\right)$$

(b) Each spring is stretched by the distance x which the mass is displaced.

$F = -(k_1+k_2)x$ and $K_{eff} = (k_1+k_2)$

$$T = 2\pi\sqrt{\frac{m}{k_1+k_2}}$$

16.29. A sinusoidal wave train is described by the equation

$$y = (0.25\ m)\sin(0.30x - 40t)$$

where x and y are in meters and t is in seconds. Determine the (a) amplitude, (b) angular frequency, (c) angular wave number, (d) wavelength, (e) wave speed, and (f) direction of motion.

$y = A\sin(kx - \omega t)$

(a) $A = 0.250\ m$

(b) $\omega = 40.0\ rad/s$

(c) $k = 0.300\ rad/m$

(d) $\lambda = \frac{2\pi}{k} = \frac{2\pi}{0.300\ rad/m} = 20.9\ m$

(e) $v = f\lambda = \left(\frac{\omega}{2\pi}\right)\lambda = \left(\frac{40.0\ rad/s}{2\pi}\right)(20.9m) = 133\ m/s$

(f) moves to the right, in +x direction

16.42. It is found that a 6.00 m segment of a long string contains four complete waves and has a mass of 180 g. The string is vibrating sinusoidally with a frequency of 50.0 Hz and a peak-to-valley displacement of 15.0 cm. (a) Write the function that describes this wave travelling in the positive x-direction. (b) Determine the power being supplied to the string.

$\mu = 180\ g / 6.00\ m = 30.0\ g/m = 30.0\times10^{-3}\ kg/m$

$f = 50.0\ Hz$ $\qquad \lambda = 6.00\ m/4 = 1.50\ m$

$\omega = 2\pi f = 314\ /s$

$2A = 0.150\ m$

$A = 7.5\times10^{-2}\ m$

(a) $y = A\sin\left(\frac{2\pi}{\lambda}x - \omega t\right) = (7.50\times10^{-2} m)\sin(4.19x - 314t)$

(b) $P = \frac{1}{2}\mu\omega^2A^2v$ $\qquad v = f\lambda = (50.0\ Hz)(1.50\ m)$

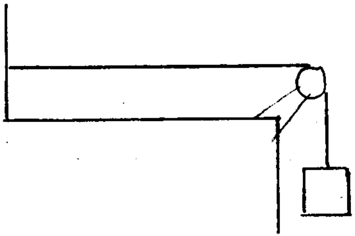
$P = \frac{1}{2}\left(30.0\times10^{-3}\ \frac{kg}{m}\right)(314/s)^2(7.50\times10^{-2}\ m)^2\left(75\ \frac{m}{s}\right)$

$P = 625\ W$

16.57. A sinusoidal wave in a rope is described by the wave function

$$y = (0.20\ \text{m}) \sin (0.75 \pi x + 18 \pi t)$$

The rope has a linear mass density of 0.25 kg/m. If the tension in the rope is provided by an arrangement like the one illustrated in Figure 16.12, what is the value of the suspended mass?



$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{mg}{\mu}} \longrightarrow m = \frac{\mu v^2}{g}$$

$$T = mg$$

$$v = \omega / k$$

$$m = \frac{\mu}{g}\left(\frac{\omega}{k}\right)^2 = \frac{0.25\ \text{kg/m}}{9.80\ \text{m/s}^2}\left(\frac{18\pi\ \text{s}^{-1}}{0.75\pi\ \text{m}^{-1}}\right)^2 = 14.5\ \text{kg}$$