Phys 231 HM # 2 | 3.31, 3.58, 4.7, 4.32, 4.39, 4.60, 4.63 | due 09/13/02

3.31. Consider two vectors $\vec{A} = 3\hat{\imath} - 2\hat{\jmath}$ and $\vec{B} = -\hat{\imath} - 4\hat{\jmath}$. Calculate the following.

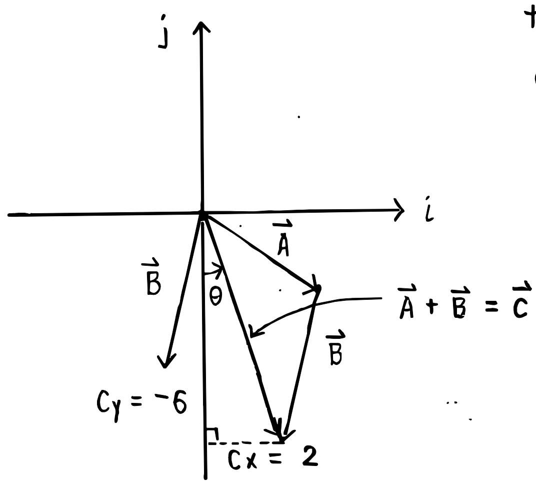
(a) $\vec{A} + \vec{B} = (3\hat{\imath} - 2\hat{\jmath}) + (-\hat{\imath} - 4\hat{\jmath}) = 2\hat{\imath} - 6\hat{\jmath}$

(b) $\vec{A} - \vec{B} = (3\hat{\imath} - 2\hat{\jmath}) - (-\hat{\imath} - 4\hat{\jmath}) = 4\hat{\imath} + 2\hat{\jmath}$

(c) $|\vec{A} + \vec{B}| = \sqrt{(2)^2 + (-6)^2} = 6.32$

(d) $|\vec{A} - \vec{B}| = \sqrt{(4)^2 + (2)^2} = 4.47$

(e) the direction of $\vec{A} + \vec{B}$

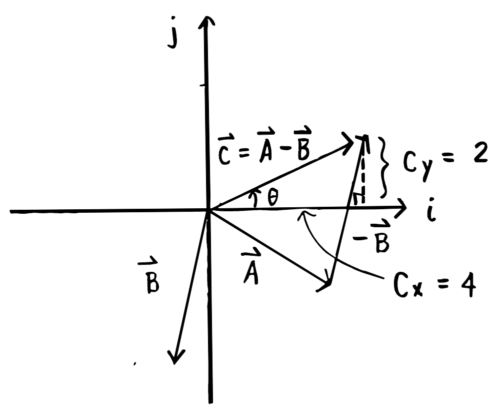


$\tan\theta = \frac{C_x}{C_y} = \frac{2}{-6} = -0.33$

$\theta = 18.4^\circ$ from $-y$ axis

$\theta = 270 + 18.4 = 288^\circ$ from $+x$ axis

the direction of $\vec{A} - \vec{B}$



$\tan\theta = \frac{C_y}{C_x} = \frac{2}{4} = \frac{1}{2}$

$\theta = \tan^{-1}(1/2) = 26.6^\circ$

3.58. In general, the instantaneous position of an object is specified by its position vector $\vec{P}$ leading from a fixed origin to the location of the object. Suppose that for a certain object the position vector is a function of time given by $\vec{P} = 4\hat{\imath} + 3\hat{\jmath} - 2t\hat{\jmath}$, where P is in meters and t in seconds. Evaluate $d\vec{P}/dt$. What does this derivative represent about the object?

$$\vec{P} = 4\hat{\imath} + 3\hat{\jmath} - 2t\hat{\jmath}$$

$$\frac{d\vec{P}}{dt} = \frac{d}{dt}(4\hat{\imath} + 3\hat{\jmath} - 2t\hat{\jmath}) = 0 + 0 - 2\hat{\jmath} = -(2.00\ m/s)\hat{\jmath}$$

This derivative represents the velocity of the object. The object is moving straight downward at 2.00 m/s.

4.7. A fish swimming in a horizontal plane has velocity $\vec{v}_i = (4.00\hat{\imath} + 1.00\hat{\jmath})$ m/s at a point in the ocean whose displacement from a certain rock is $\vec{r}_i = (10.0\hat{\imath} - 4.00\hat{\jmath})$ m. After the fish swims with constant acceleration for 20.0 s, its velocity is $\vec{v} = (20.0\hat{\imath} - 5.00\hat{\jmath})$ m/s.

(a) What are the components of the acceleration?

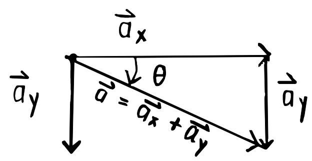
$$\vec{v}_i = (4.00\hat{\imath} + 1.00\hat{\jmath})\ m/s$$

$$\vec{v} = (20.0\hat{\imath} - 5.00\hat{\jmath})\ m/s$$

$$a_x = \frac{\Delta v_x}{\Delta t} = \frac{20.0 - 4.00}{20.0}\ m/s^2 = 0.800\ m/s^2$$

$$a_y = \frac{\Delta v_y}{\Delta t} = \frac{-5.00 - 1.00}{20.0}\ m/s^2 = -0.300\ m/s^2$$

(b) What is the direction of the acceleration with respect to the unit vector $\hat{\imath}$?



$$\tan\theta = \frac{a_y}{a_x}$$

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{-0.300}{0.800}\right)$$

$$\theta = -20.6^\circ$$

$$\theta = 360^\circ - 20.6^\circ = 339^\circ \text{ from } +x \text{ axis}$$

(c) Where is the fish at $t = 25.0$ s if it maintains its original acceleration and in what direction is it moving?

$$x_f = x_i + v_{ix}t + \tfrac{1}{2}a_x t^2$$

$$x_f = 10.0\ m + (4.00\ m/s)(25.0\ s) + \tfrac{1}{2}(0.800\ m/s^2)(25.0\ s)^2 = 360\ m$$

$$y_f = y_i + v_{iy}t + \tfrac{1}{2}a_y t^2$$

$$y_f = -4.00\ m + (1.00\ m/s)(25.0\ s) + \tfrac{1}{2}(-0.300\ m/s^2)(25.0\ s)^2 = -72.7\ m$$

$$v_{fx} = v_{ix} + a_x t$$

$$v_{fx} = (4.00\ m/s) + (0.800\ m/s^2)(25.0\ s) = 24\ m/s$$

$$v_{fy} = v_{iy} + a_y t$$

$$v_{fy} = (1.00\ m/s) + (-0.300\ m/s^2)(25.0\ s) = -6.5\ m/s$$

$$\theta = \tan^{-1}\left(\frac{v_{fy}}{v_{fx}}\right) = \tan^{-1}\left(\frac{-6.5}{24}\right) = -15.2^\circ$$

4.32. An automobile whose speed is increasing at a rate of $0.600\ m/s^2$ travels along a circular road of radius 20.0 m. When the instantaneous speed of the automobile is 4.00 m/s, find

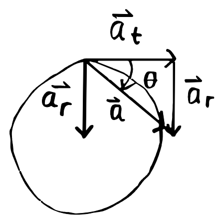
(a) the tangential acceleration component

$a_t = 0.600\ m/s^2$

(b) the radial acceleration component

$$a_r = \frac{v^2}{r} = \frac{(4.00\ m/s)^2}{20.0\ m} = 0.800\ m/s^2$$

(c) the magnitude and direction of the total acceleration.



$$\vec{a} = \vec{a}_t + \vec{a}_r$$

$$a = \sqrt{(a_t)^2 + (a_r)^2}$$

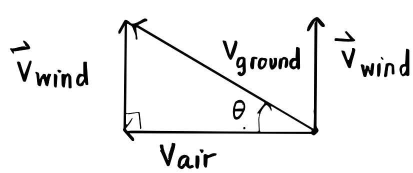
$$a = \sqrt{(0.600\ m/s^2)^2 + (0.800\ m/s^2)}$$

$$a = 1.00\ m/s^2$$

$$\theta = \tan^{-1}\left(\frac{a_r}{a_t}\right) = \tan^{-1}\left(\frac{0.800}{0.600}\right)$$

$\theta = 53.1°$ inward from path

4.39. The pilot of an airplane notes that the compass indicates a heading due west. The airplane's speed relative to the air is 150 km/h. If there is a wind of 30.0 km/h toward the north, find the velocity of the airplane relative to the ground.



$$\vec{V}_{ground} = \vec{V}_{air} + \vec{V}_{wind}$$

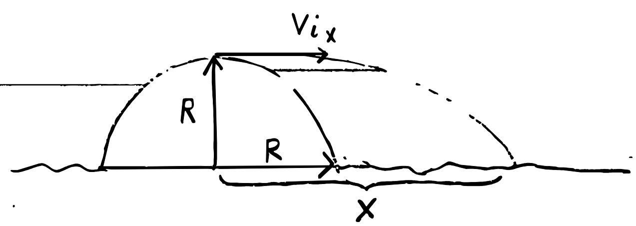
$$V_{ground} = \sqrt{(V_{air})^2 + (V_{wind})^2} = \sqrt{(150\ km/h)^2 + (30\ km/h)^2}$$

$$V_{ground} = 153\ km/h$$

$\theta = \tan^{-1}\left(\frac{V_{wind}}{V_{air}}\right) = \tan^{-1}\left(\frac{30.0}{150}\right) = 11.3°$ north of west

4.60. A person standing at the top of a hemispherical rock of radius R kicks a ball (initially at rest on the top of the rock) to give it horizontal velocity $\vec{v}_i$ as in Figure P4.60.

(a) What must be its minimum initial speed if the ball is never to hit the rock after it is kicked?



$x_i = 0$ m
$y_i = R$
$a_x = 0$
$a_y = -g$
$v_{iy} = 0$
$v_{ix} = v_i$

$$y_f = y_i + v_{iy} + \frac{1}{2} a_y t^2$$
$$y_f = R + 0 + \frac{1}{2}(-g)t^2$$
$$y_f = R - \frac{1}{2} g t^2$$

$$x_f = x_i + v_{ix} t + \frac{1}{2} a_x t^2$$
$$x_f = 0 + v_i t + 0$$
$$x = v_i t$$

For points on the rock : $y_r^2 + x^2 = R^2$
For all other points : $y_b^2 + x^2 > R^2$

$$y_b^2 + x^2 > R^2$$
$$\left(R - \frac{1}{2} g t^2\right)^2 + x^2 > R^2$$

substitute $t = x / v_i$

$$\left(R - \frac{1}{2} g \cdot \frac{x^2}{v_i^2}\right)^2 + x^2 > R^2$$
$$\cancel{R^2} - \frac{g x^2 R}{v_i^2} + \frac{g^2 x^4}{4 v_i^4} + x^2 > \cancel{R^2}$$
$$-\frac{gR}{v_i^2} + \frac{g^2 x^2}{4 v_i^4} + 1 > 0$$
$$-4gR v_i^2 + g^2 x^2 + 4 v_i^4 > 0$$
$$4 v_i^4 - 4gR v_i^2 + g^2 x^2 > 0$$
$$v_i^2 > \frac{4gR \pm \sqrt{16 g^2 R^2 - 16 g^2 x^2}}{8} > \frac{gR \pm g\sqrt{R^2 - x^2}}{2}$$

$x = 0$

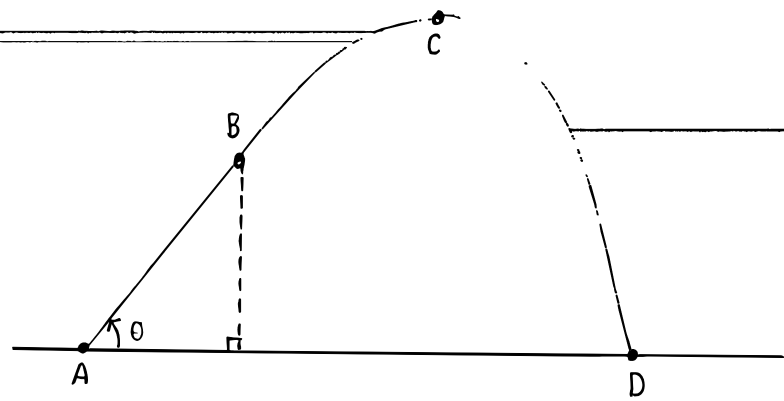
$v_i^2 = (gR \pm gR)/2$ $\qquad v_i = \sqrt{gR}$

(b) With this initial speed, how far from the base of the rock does the ball hit the ground?

$$y_f = y_i + v_{iy} t + \frac{1}{2} a_y t^2$$
$$0 = R + 0 + \frac{1}{2}(-g)t^2$$
$$R = \frac{1}{2} g t^2$$
$$t = \sqrt{\frac{2R}{g}}$$

$$v_i = \sqrt{gR}$$
$$x = v_i t = \sqrt{\cancel{g} R} \cdot \sqrt{\frac{2R}{\cancel{g}}} = \sqrt{2}\, R$$
$$x - R = \sqrt{2}\, R - R$$
$$x - R = R(\sqrt{2} - 1)$$

4.63.



Path AB

$\theta = 53°$
$V_A = 100$ m/s
$t = 3.00$ s
$a = 30.0$ m/s^2
$X_A = 0$ m
$X_B = ?$
$y_A = 0$ m
$y_B = ?$
$V_{fB} = ?$

$a_x = a\cos\theta = (30.0\,m/s^2)\cos 53 = 18.1\,m/s^2$
$a_y = a\sin\theta = (30.0\,m/s^2)\sin 53 = 24.0\,m/s^2$
$V_{Ax} = V_A\cos\theta = (100\,m/s)\cos 53 = 60.2\,m/s$
$V_{Ay} = V_A\sin\theta = (100\,m/s)\sin 53 = 79.9\,m/s$

$V_{fy} = V_{iy} + a_y t$
$V_{By} = V_{Ay} + a_y t$
$V_{By} = 79.9\,m/s + (24.0\,m/s^2)(3.00\,s) = 152\,m/s$

$V_{fx} = V_{ix} + a_x t$
$V_{Bx} = V_{Ax} + a_x t$
$V_{Bx} = 60.2\,m/s + (18.1\,m/s^2)(3.00\,s) = 114\,m/s$

$y_f = y_i + V_{iy}t + \frac{1}{2}a_y t^2$
$y_B = y_A + V_{Ay}t + \frac{1}{2}a_y t^2$
$y_B = 0 + (79.9\,m/s)(3.00\,s) + \frac{1}{2}(24.0\,m/s^2)(3.00\,s)^2 = 347\,m$

$X_f = X_i + V_{ix}t + \frac{1}{2}a_x t^2$
$X_B = X_A + V_{Ax}t + \frac{1}{2}a_x t^2$
$X_B = 0 + (60.2\,m/s)(3.00\,s) + \frac{1}{2}(18.1\,m/s^2)(3.00\,s)^2 = 262\,m$

Path BC

$V_{Bx} = 114$ m/s
$V_{By} = 152$ m/s
$y_B = 347$ m
$X_B = 262$ m
$a_x = 0$ m/s^2
$a_y = -9.8$ m/s^2
$V_{cy} = 0$ m/s

$V_{cx} = ?$
$X_c = ?$
$y_c = ?$
$t = ?$

$V_{fy} = V_{iy} + a_y t$
$V_{cy} = V_{By} + a_y t$
$0 = 152\,m/s + (-9.8\,m/s^2)t$
$\therefore t = 15.5\,s$

$X_f = X_i + V_{ix}t + \frac{1}{2}a_x t^2$
$X_c = X_B + V_{Bx}t + 0$
$X_c = 262\,m + (114\,m/s)(15.5\,s) = 2029\,m$

$y_f = y_i + V_{iy}t + \frac{1}{2}a_y t^2$
$y_c = y_B + V_{By}t + \frac{1}{2}a_y t^2$
$y_c = 347\,m + (152\,m/s)(15.5\,s) + \frac{1}{2}(-9.8\,m/s^2)(15.5\,s)^2$
$y_c = 1525.775\,m$

$V_{fx} = V_{ix} + a_x t$
$V_{c} = V_{B} + 0 = 114\,m/s$

Path CD

$V_{Cx} = 114\ m/s$
$V_{Cy} = 0\ m/s$
$y_C = 1525.775\ m$
$x_C = 2029\ m$
$a_x = 0\ m/s^2$
$a_y = -9.8\ m/s^2$
$t = ?$
$x_D = ?$
$y_D = 0$
$V_{Dx} = ?$
$V_{Dy} = ?$

$V_{fy}^2 = V_{iy}^2 + 2a_y(y_f - y_i)$
$V_{Dy}^2 = V_{Cy}^2 + 2a_y(y_D - y_C)$
$V_{Dy}^2 = 0 + 2(-9.8\ m/s^2)(0\ m - 1525.775\ m)$
$V_{Dy} = -173\ m/s$

$V_{fx} = V_{ix} + a_x t$
$V_{Dx} = V_{Cx} + a_x t$
$V_{Dx} = 114\ m/s + 0 = 114\ m/s$

$V_{fy} = V_{iy} + a_y t$
$V_{Dy} = V_{Cy} + a_y t$
$-173\ m/s = 0 + (-9.8\ m/s^2)t$
$t = 17.6\ s$

$x_f = x_i + V_{ix} t + \frac{1}{2} a_x t^2$
$x_D = x_C + V_{Cx} t + \frac{1}{2} a_x t^2$
$x_D = 2029\ m + (114\ m/s)(17.6\ s) + 0$
$x_D = 4035.4\ m$

(a) Find the maximum altitude reached by the rocket,
$y_C = 1525.775\ m$

(b) Find its total time of flight.
$t = t_{AB} + t_{BC} + t_{CD}$
$t = (3.00 s + 15.5 s + 17.6 s) = 36.1\ s$

(c) Find its horizontal range.
$x_D = 4035\ m$