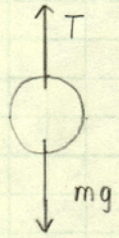
Phys 231 HM # 3	5: 27, 35, 48, 58, 64, 67, 69	due 9/20/2007

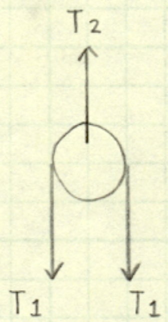
5.27. The systems shown in Figure P5.27 are in equilibrium. If the spring scales are calibrated in newtons, what do they read? (Neglect the masses of the pulleys and strings, and assume the incline is frictionless.

(a)



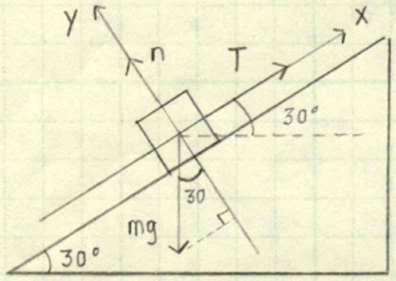
$$F = T - mg$$
$$F = ma = 0$$
$$T - mg = 0$$
$$T = mg = (5.00\ kg)(9.80\ m/s^2)$$
$$T = 49.0\ N$$

(b)



$$F = T_2 - 2T_1$$
$$F = ma = 0$$
$$T_2 - 2T_1 = 0$$
$$T_2 = 2T_1 = 2mg$$
$$T_2 = 2(5.00\ kg)(9.80\ m/s^2)$$
$$T_2 = 98.0\ N$$

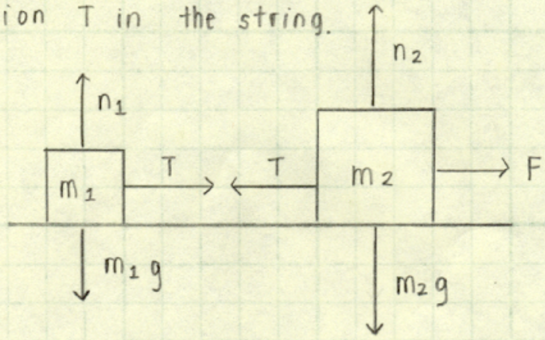
(c)



<u>x - component</u>

$$T - mg \sin 30 = 0$$
$$T = mg \sin 30 = (5.00\ kg)(9.80\ m/s^2) \sin 30$$
$$T = 24.5\ N$$

5.35. Two masses m_1 and m_2 situated on a frictionless, horizontal surface are connected by a light string. A force $\vec{F}$ is exerted on one of the masses to the right (Fig. P5.35). Determine the acceleration of the system and the tension T in the string.



<u>x - component</u>

For m_2: $F - T = m_2 a$ (1)

For m_1: $T = m_1 a$ (2)

$T = F - m_2 a$ (1)

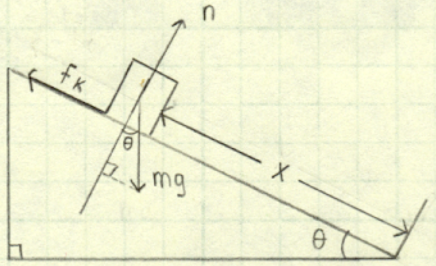
$T = m_1 a$ (2)

$$F - m_2 a = m_1 a$$
$$F = (m_1 + m_2) a$$
$$a = \frac{F}{m_1 + m_2}$$

$T = m_1 a$ (2)

$$T = \frac{F m_1}{m_1 + m_2}$$

5.48. Determine the stopping distance for a skier moving down a slope with friction with an initial speed of 20.0 m/s (Fig. P5.48). Assume $\mu_K = 0.180$ and $\theta = 5.00°$.



y-component

$n - mg\cos\theta = 0$

$n = mg\cos\theta$

x-component

$mg\sin\theta - f_K = ma_x$

$f_K = \mu_K n = \mu_K mg\cos\theta$

$\cancel{m}g\sin\theta - \mu_K \cancel{m}g\cos\theta = \cancel{m}a_x$

$a_x = g(\sin\theta - \mu_K\cos\theta)$

$a_x = 9.80\,m/s^2(\sin 5 - 0.180\cos 5)$

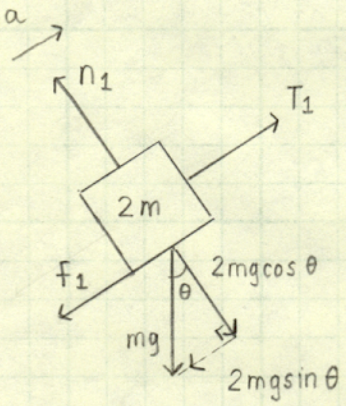
$a_x = -0.903\,m/s^2$

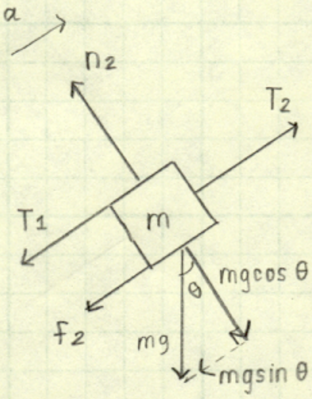
$V_{fx}^2 - V_{ix}^2 = 2a_x(X_f - X_i)$

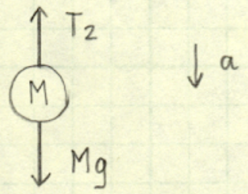
$0 - (20\,m/s)^2 = 2(-0.903\,m/s^2)(X_f - 0\,m)$

$X_f = 221\ m$

5.58. Consider the three connected objects shown in Figure P5.58. If the inclined plane is frictionless and the system is in equilibrium, find (in terms of m, g, and θ) (a) the mass and (b) the tensions T_1 and T_2. If the value of M is double the value found in part (a), find (c) the acceleration of each object, and (d) the tensions T_1 and T_2. If the coefficient of static friction between m and 2m and the inclined plane is μ_s, and the system is in equilibrium, find (e) the minimum value of M and (f) the maximum value of M, (g) Compare the values of T_2 when M has its minimum and maximum values.







y-component

$n_1 - 2mg\cos\theta = 0$

$n_1 = 2mg\cos\theta$

x-component

(1) $T_1 - f_1 - 2mg\sin\theta = 2ma$

y-component

$n_2 - mg\cos\theta = 0$

$n_2 = mg\cos\theta$

x-component

$T_2 - T_1 - f_2 - mg\sin\theta = ma$ (2)

y-component

$T_2 - Mg = -Ma$

$T_2 = M(g - a)$ (3)

Parts (a) and (b) : Equilibrium $(a = 0)$
Frictionless incline $(f_1 = f_2 = 0)$

(1) $\boxed{T_1 = 2mg\sin\theta}$
(2) $T_2 - T_1 = mg\sin\theta$
(3) $\boxed{T_2 = Mg}$

$$T_2 - T_1 = mg\sin\theta$$
$$M\cancel{g} - 2m\cancel{g}\sin\theta = m\cancel{g}\sin\theta$$
$$\boxed{M = 3m\sin\theta}$$

Parts (c) and (d) : $M = 6m\sin\theta$
$f_1 = f_2 = 0$

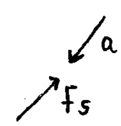
(1) $T_1 = 2m(g\sin\theta + a)$
(2) $T_2 - T_1 = m(g\sin\theta + a)$
(3) $T_2 = 6m\sin\theta\,(g - a)$

$$T_2 - T_1 = m(g\sin\theta + a)$$
$$6\cancel{m}\sin\theta(g-a) - 2\cancel{m}(g\sin\theta + a) = \cancel{m}(g\sin\theta + a)$$
$$6g\sin\theta - 6a\sin\theta - 2g\sin\theta - 2a = g\sin\theta + a$$
$$-6a\sin\theta - 2a - a = g\sin\theta + 2g\sin\theta - 6g\sin\theta$$
$$+6a\sin\theta + 3a = +3g\sin\theta$$
$$a(6\sin\theta + 3) = 3g\sin\theta$$
$$\boxed{a = \frac{g\sin\theta}{2\sin\theta + 1}}$$

$$T_1 = 2m\left(g\sin\theta + \frac{g\sin\theta}{2\sin\theta + 1}\right) = 2mg\sin\theta\left(1 + \frac{1}{2\sin\theta + 1}\right)$$
$$T_1 = 2mg\sin\theta\left(\frac{2\sin\theta + 2}{2\sin\theta + 1}\right)$$
$$\boxed{T_1 = 4mg\sin\theta\left(\frac{1 + \sin\theta}{1 + 2\sin\theta}\right)}$$

Part (e) : Equilibrium $(a = 0)$, How small can M be before the system goes down the incline ?

$f_1 = \mu_s n_1 = 2\mu_s mg\cos\theta$
$f_2 = \mu_s n_2 = \mu_s mg\cos\theta$
} both directed up the incline
$M = M_{min}$

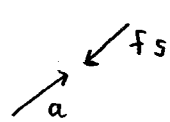


(1) $T_1 = 2mg(\sin\theta - \mu_s\cos\theta)$
(2) $T_2 - T_1 = mg(\sin\theta - \mu_s\cos\theta)$
(3) $T_2 = M_{min}\,g$

$$M_{min} = \frac{T_2}{g} = \frac{mg(\sin\theta - \mu_s\cos\theta) + T_1}{g} = \frac{mg(\sin\theta - \mu_s\cos\theta) + 2mg(\sin\theta - \mu_s\cos\theta)}{g}$$
$$\boxed{M_{min} = 3m(\sin\theta - \mu_s\cos\theta)}$$

Part (f) : Equilibrium $(a = 0)$, How big can M be before the system goes up the incline ?

$f_1 = \mu_s n_1 = 2\mu_s mg\cos\theta$
$f_2 = \mu_s n_2 = \mu_s mg\cos\theta$
} both directed down the incline



(1) $T_1 = 2mg(\sin\theta + \mu_s\cos\theta)$
(2) $T_2 - T_1 = mg(\sin\theta + \mu_s\cos\theta)$
(3) $T_2 = M_{max}\,g$

$$\boxed{M_{max} = 3m(\sin\theta + \mu_s\cos\theta)}$$

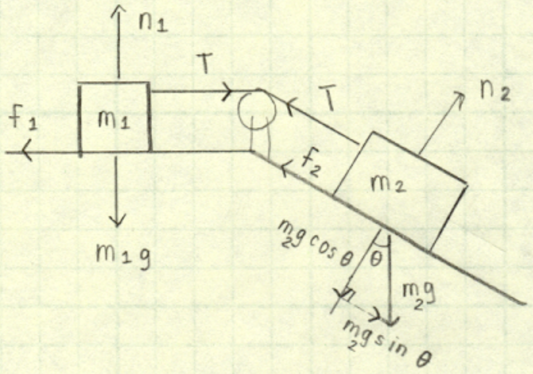
Part (g):

$T_{2,max} = g M_{max}$

$T_{2,min} = g M_{min}$

$T_{2,max} - T_{2,min} = g M_{max} - g M_{min} = 6 m g \mu_s \cos\theta$

5.64. A 2.00-kg aluminum block and a 6.00-kg copper block are connected by a light string over a frictionless pulley. They sit on a steel surface, as shown in Figure P5.64, and $\theta = 30.0°$. Do they start to move once any holding mechanism is released? If so, determine (a) their acceleration and (b) the tension in the string. If not, determine the sum of the magnitudes of the forces of friction acting on the blocks.



For the system to start to move, the force tending to move m_2 down the incline, $m_2 g \sin\theta$, must exceed the maximum friction force.

$f_{max} = f_{1,max} + f_{2,max} = \mu_{s,1} n_1 + \mu_{s,2} n_2 = \mu_{s,1} m_1 g + \mu_{s,2} m_2 g \cos\theta$

$\mu_{s,1} = 0.610$ (aluminum on steel)

$\mu_{s,2} = 0.530$ (copper on steel)

$m_1 = 2.00$ kg, $m_2 = 6.00$ kg, $\theta = 30°$

$f_{max} = (0.610)(2.00 \text{ kg})(9.80 \text{ m/s}^2) + (0.530)(6.00 \text{ kg}) \cos 30$

$f_{max} = 38.9$ N

This exceeds the force tending to cause the system to move:

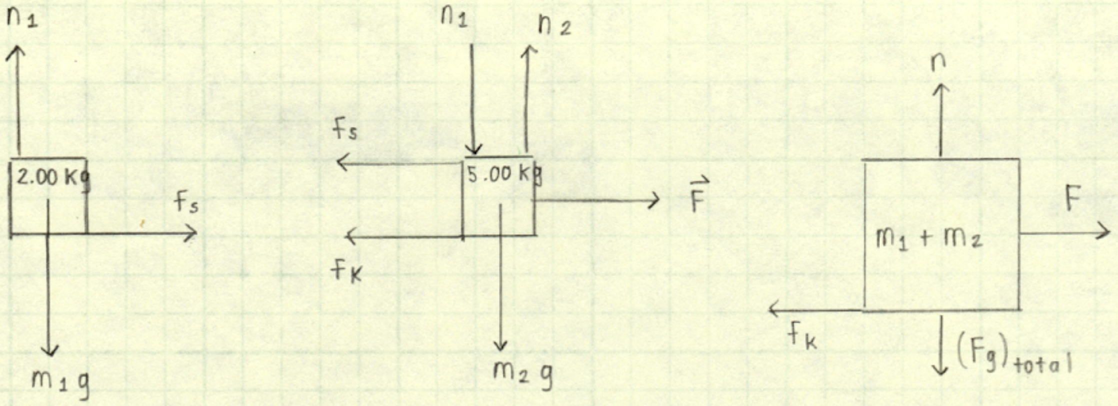
$m_2 g \sin\theta = (6.00 \text{ kg})(9.80 \text{ m/s}^2) \sin 30 = 29.4$ N

So the system will not start to move.

The friction forces increase in magnitudes until the total friction force retarding the motion, $f = f_1 + f_2$, equals the force tending to set the system in motion. That is until $f = m_2 g \sin\theta = 29.4$ N

5.67. A 2.00-kg block is placed on top of a 5.00-kg as in Figure P5.67. The coefficient of kinetic friction between the 5.00-kg block and the surface is 0.200. A horizontal force $\vec{F}$ is applied to the 5.00-kg block. (a) Draw a free-body diagram for each block. What force accelerates the 2.00-kg block? (b) Calculate the magnitude of the force necessary to pull both blocks to the right with an acceleration of 3.00 m/s^2. (c) Find the minimum coefficient of static friction between the blocks such that the 2.00 kg block does not slip under an acceleration of 3.00 m/s^2.

(a)



The force of static friction between the blocks accelerates the 2.00 kg block.

(b) For both blocks together:

$$F - f_K = ma$$
$$F - \mu_K (m_1 + m_2) g = (m_1 + m_2) a$$
$$F = (m_1 + m_2) a + \mu_K (m_1 + m_2) g$$
$$F = (m_1 + m_2)(a + \mu_K g)$$
$$F = (2.00\,\text{kg} + 5.00\,\text{kg})[3.00\,\text{m/s}^2 + 0.200\,(9.80\,\text{m/s}^2)]$$
$$F = 34.7\ \text{N}$$

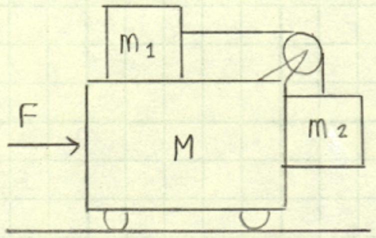
(c)

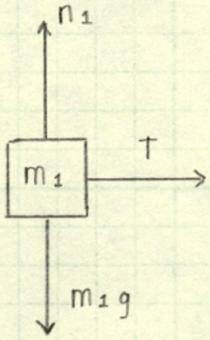
$$f_s = \mu_s n_1 = m_1 a$$
$$n_1 = m_1 g$$
$$\mu_s (\cancel{m_1} g) = \cancel{m_1} a$$
$$\mu_s = \frac{a}{g} = \frac{3.00\,\text{m/s}^2}{9.80\,\text{m/s}^2} = 0.306$$

OR

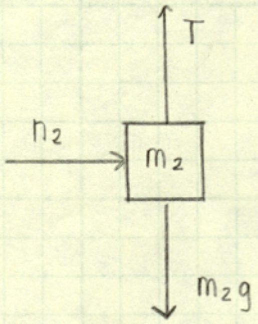
$$f_s = \mu_s n_2 = m_2 a$$
$$n_2 = m_2 g$$
$$\mu_s (\cancel{m_2} g) = \cancel{m_2} a$$
$$\mu_s = \frac{a}{g}$$

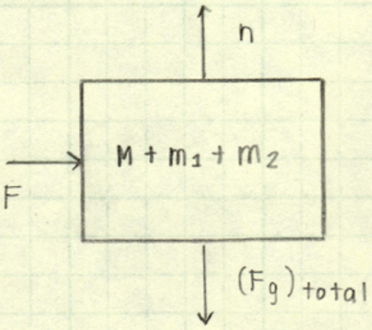
5.69. What horizontal force must be applied to the cart shown in Figure P5.69 so that the blocks remain stationary relative to the cart? Assume all surfaces, wheels, and pulley are frictionless. (Hint: Note that the force exerted by the string accelerates m_1.)





ropes always pull on bodies in the direction of the rope





normal forces are always $\perp$ to the contacting surfaces

For m_1: $T = m_1 a$

$$a = \frac{T}{m_1} = \frac{m_2 g}{m_1}$$

For m_2: $T - m_2 g = 0$

$$T = m_2 g$$

For all 3 blocks: $F = (M + m_1 + m_2)a$

$$F = (M + m_1 + m_2)\frac{m_2 g}{m_1}$$