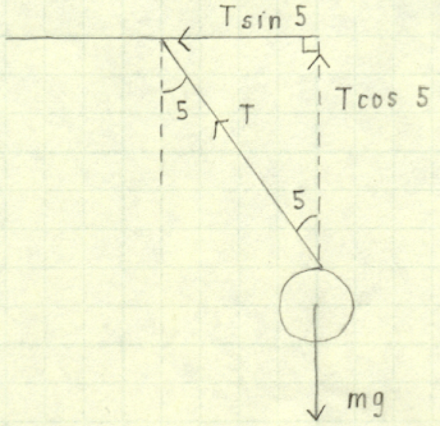
6.13. Consider a <u>conical</u> pendulum with an 80.0-kg bob on a 10.0-m wire making an angle of $\theta = 5.00°$ with the vertical (Figure P6.13). Determine (a) the horizontal and vertical components of the force exerted by the wire on the pendulum and (b) the radial acceleration of the bob.



(a) y-direction

$$T\cos 5° = mg$$

$$T = \frac{mg}{\cos 5} = \frac{(80.0\,kg)(9.80\,m/s^2)}{\cos 5.00°}$$

$$T = 787\ N$$

$$T_x = T\sin 5 = (787\ N)\sin 5° = 68.6\ N$$

$$T_y = T\cos 5 = (787\ N)\cos 5° = 784\ N$$

$$T = (68.6\ N)\hat{\imath} + (784\ N)\hat{\jmath}$$

(b) x-direction

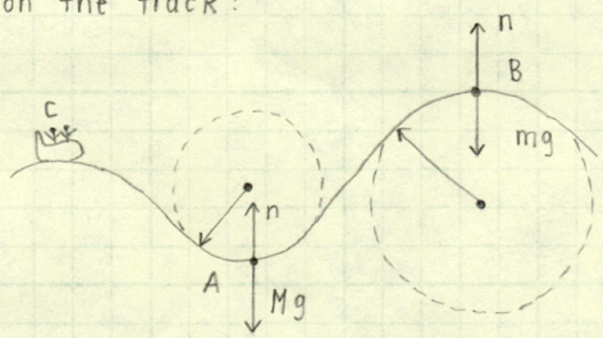
$$T\sin 5° = ma_r$$

$$a_r = \frac{T\sin 5°}{m} = \frac{(787\ N)\sin 5°}{80.0\ kg}$$

$$a_r = 0.857\ m/s^2$$

The force providing the centripedal acceleration is $T\sin\theta$.

6.21. A roller coaster car has a mass of 500 kg when fully loaded with passengers (Fig. P6.21). (a) If the car has a speed of 20.0 m/s at point A, what is the force exerted by the track on the car at this point? (b) What is the maximum speed the car can have at B and still remain on the track?



(a)

$$\sum F = \frac{Mv^2}{R} = n - Mg$$

radial acceleration points to the radius

$$n = Mg + \frac{Mv^2}{R} = M\left(g + \frac{v^2}{R}\right) = (500\ kg)\left(9.80\frac{m}{s^2} + \frac{(20\ m/s)^2}{10.0\ m}\right)$$

$$n = 2.49 \times 10^4\ N$$

(b)

$$n - Mg = -\frac{Mv^2}{R}$$

The max speed at B corresponds to $n = 0$.

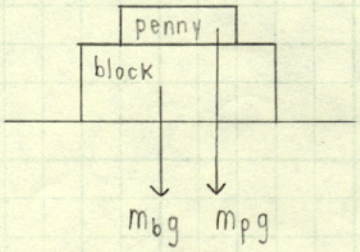
$$-\cancel{M}g = -\frac{\cancel{M}v_{max}^2}{R}$$

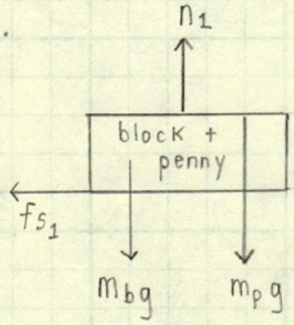
$$v_{max} = \sqrt{Rg} = \sqrt{(15.0\ m)(9.80\ m/s^2)}$$

$$v_{max} = 12.1\ m/s$$

6.58. A penny of mass 3.10 g rests on a small 20.0 g block supported by a spinning disk (Fig. P6.58). If the coefficient of friction between block and disk are 0.750 (static) and 0.640 (kinetic) while those for the penny and block are 0.450 (kinetic) and 0.520 (static), what is the maximum rate of rotation (in revolutions per minute) that the disk can have before either the block or penny starts to slip.

First, let's consider the block.





y-direction

$n_1 - (m_b + m_p)g = 0$

$n_1 = (m_b + m_p)g$

x-direction

At the point of slipping, the required centripetal force equals the maximum friction force:

$$(m_p + m_b)\frac{v_{max}^2}{r} = f_{s_1 max}$$

$$f_{s1} \leq \mu_{s1} n_1 = \mu_{s1}(m_b + m_p)g$$

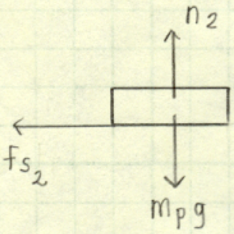
$$f_{s1 max} = \mu_{s1}(m_b + m_p)g$$

$$\cancel{(m_p + m_b)}\frac{v_{max}^2}{r} = \mu_{s1}\cancel{(m_b + m_p)}g$$

$$v_{max} = \sqrt{\mu_{s1} r g} = \sqrt{(0.750)(0.120)(9.80)}$$

$$v_{max} = 0.939 \text{ m/s}$$

Now, let's consider just the penny.



y-direction

$n_2 - m_p g = 0$

$n_2 = m_p g$

x-direction

When the penny is about to slip on the block, the required centripedal force equals the max frictional force:

$$m_p \frac{v_{max}^2}{r} = f_{s_2} max$$

$$f_{s_2} \leq \mu_{s2} n_2 = \mu_{s2}(m_p g)$$

$$f_{s2} max = \mu_{s2} m_p g$$

$$\cancel{m_p}\frac{v_{max}^2}{r} = \mu_{s2}\cancel{m_p}g$$

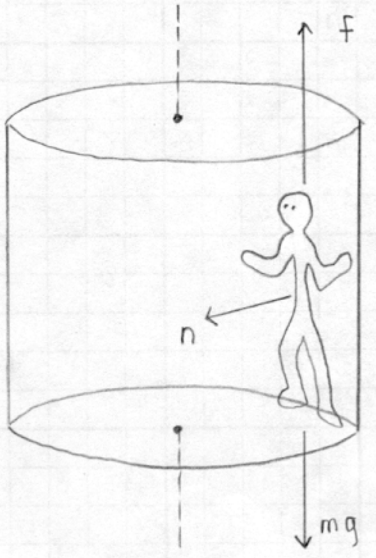
$$v_{max} = \sqrt{\mu_{s2} r g} = \sqrt{(0.520)(0.120)(9.80)}$$

$$v_{max} = 0.782 \text{ m/s}$$

This is less than the max speed for the block, so the penny slips before the block.

$$\text{Max rpm} = \frac{v_{max}}{2\pi r} = (0.782 \text{ m}/\cancel{s})\left[\frac{1 \text{ rev}}{2\pi(0.120 \text{ m})}\right]\left(\frac{60 \cancel{s}}{1 \text{ min}}\right) = 62.2 \frac{\text{rev}}{\text{min}}$$

6.63. An amusement park ride consists of a large vertical cylinder that spins about its axis fast enough that any person inside is held up against the wall when the floor drops away (Fig. P6.63). The coefficient of static friction between person and wall is μ_s, and the radius of the cylinder is R.
(a) Show that the maximum period from falling is $T = (4\pi^2 R \mu_s / g)^{1/2}$.
(b) Obtain a numerical value for T if $R = 4.00$ m and $\mu_s = 0.400$. How many revolutions per minute does the cylinder make?



(a) x-direction

$$n = \frac{mv^2}{R}$$

y-direction

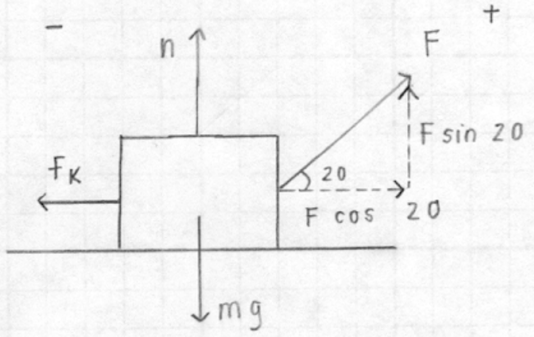
$$f - mg = 0$$
$$f = \mu_s n$$
$$f = \mu_s \frac{mv^2}{R}$$
$$\mu_s \frac{\cancel{m}v^2}{R} = \cancel{m}g$$
$$\mu_s v^2 = Rg$$

$$d = vT$$
$$v = \frac{d}{T} = \frac{2\pi R}{T}$$
$$\mu_s v^2 = Rg$$
$$\mu_s \frac{4\pi^2 R^2}{T^2} = Rg$$
$$T = \sqrt{\frac{4\pi^2 R \mu_s}{g}}$$

(b)
$$T = \sqrt{\frac{4\pi^2 (4.00\text{ m})(0.400)}{9.8\text{ m/s}^2}} = 2.54\text{ s}$$

$$\#\frac{\text{rev}}{\text{min}} = \frac{1\text{ rev}}{2.54\,\cancel{\text{s}}}\left(\frac{60\,\cancel{\text{s}}}{\text{min}}\right) = 23.6\text{ rev/min}$$

7.6. A 15.0 kg block is dragged over a rough, horizontal surface by a 70 N force acting at 20° above the horizontal. The block is displaced 5.00 m, and the coefficient of kinetic friction is 0.300. Find the work done by (a) the 70 N force, (b) the normal force, and (c) the force of gravity. (d) What is the energy loss due to friction? (e) Find the total change in the block's kinetic energy.



y-direction

$$n - mg + F\sin 20 = 0$$
$$n = mg - F\sin 20$$
$$n = (15\text{ kg})(9.8\text{ m/s}^2) - (70\text{ N})\sin 20$$
$$n = 123\text{ N}$$

$$f_K = \mu_K n = 0.300(123\text{ N}) = 36.9\text{ N}$$

(a) $W = \vec{F} \cdot \vec{d} = Fd\cos\theta = (70\text{ N})(5.00\text{ m})\cos 20 = 329\text{ J}$
(b) $W = Fd\cos\theta = (123\text{ N})(5.00\text{ m})\cos 90 = 0\text{ J}$
(c) $W = Fd\cos\theta = mgd\cos\theta = (15\text{ kg})(9.8\text{ m/s}^2)(5.00\text{ m})\cos 90 = 0\text{ J}$
(d) $W = Fd\cos\theta = (36.9\text{ N})(5.00\text{ m})\cos 180 = -185\text{ J}$
(e) $\Delta K = K_f - K_i = \Sigma W = 329\text{ J} - 185\text{ J} = 144\text{ J}$

7.13. Using the definition of the scalar product, find the angles between (a) $\vec{A} = 3\hat{\imath} - 2\hat{\jmath}$ and $\vec{B} = 4\hat{\imath} - 4\hat{\jmath}$; (b) $\vec{A} = -2\hat{\imath} + 4\hat{\jmath}$ and $\vec{B} = 3\hat{\imath} - 4\hat{\jmath} + 2\hat{k}$; (c) $\vec{A} = \hat{\imath} - 2\hat{\jmath} + 2\hat{k}$ and $\vec{B} = 3\hat{\jmath} + 4\hat{k}$.

$\vec{A} \cdot \vec{B} = AB\cos\theta$

$\theta = \cos^{-1} \dfrac{\vec{A} \cdot \vec{B}}{AB}$

(a) $\vec{A} = 3\hat{\imath} - 2\hat{\jmath}$ $\quad \vec{B} = 4\hat{\imath} - 4\hat{\jmath}$

$\theta = \cos^{-1} \dfrac{12.0 + 8.00}{\sqrt{(13)(32)}} = 11.3°$

(b) $\vec{A} = -2\hat{\imath} + 4\hat{\jmath}$ $\quad \vec{B} = 3\hat{\imath} - 4\hat{\jmath} + 2\hat{k}$

$\theta = \cos^{-1} \dfrac{-6 - 16}{\sqrt{(20)(29)}} = 156°$

(c) $\vec{A} = \hat{\imath} - 2\hat{\jmath} + 2\hat{k}$ $\quad \vec{B} = 3\hat{\jmath} + 4\hat{k}$

$\theta = \cos^{-1} \dfrac{-6 + 8}{\sqrt{(9)(25)}} = 82.3°$

7.18. A force $\vec{F} = (4x\hat{\imath} + 3y\hat{\jmath})$ N acts on an object as it moves in the x direction from the origin to $x = 5.00$ m. Find the work $W = \int \vec{F} \cdot d\vec{r}$ done on the object by the force.

$$W = \int_i^f \vec{W} \cdot d\vec{s} = \int_0^5 (4x\hat{\imath} + 3y\hat{\jmath})\text{ N} \cdot dx\,\hat{\imath}$$

$$= \int_0^5 \left(4\frac{\text{N}}{\text{m}}\right) x\,dx = 4\frac{\text{N}}{\text{m}} \cdot \frac{1}{2}x^2 \Big|_0^5 = 2\frac{\text{N}}{\text{m}}(25 - 0)\text{ m}^2$$

$W = 50.0\text{ J}$