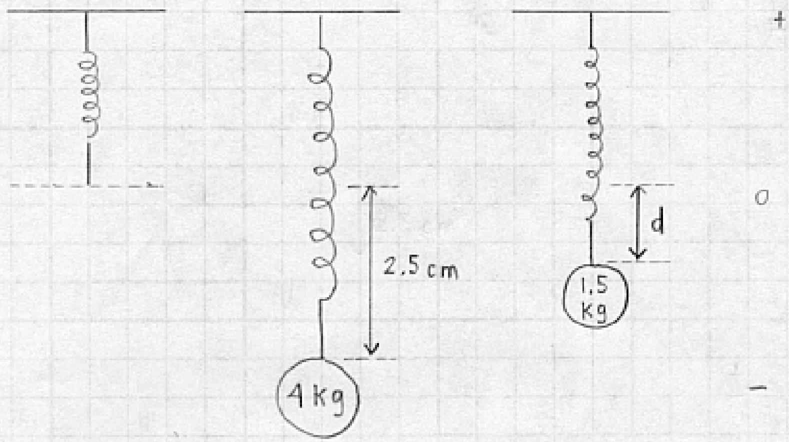
7.19. When a 4 kg mass is hung vertically on a certain light spring that obeys Hooke's law, the spring stretches 2.5 cm. If the 4 kg mass is removed, (a) how far will the spring stretch if a 1.5 kg mass is hung on it and (b) how much must an external agent do to stretch the same spring 4 cm from its unstretched position?

(a)



$$F = -ky$$

$$k = \frac{-F}{y} = \frac{-mg}{y} = \frac{-(4\ \text{kg})(9.8\ \text{m/s}^2)}{2.5\ \text{cm}} = 1.57\times10^3\ \text{N/m}$$

$$y = \frac{-F}{k} = \frac{-mg}{k} = \frac{-(1.5\ \text{kg})(-9.8\ \text{m/s}^2)}{1.57\times10^3\ \text{N/m}} = 0.938\ \text{cm}$$

(b)
$$W = \int \vec{F}\cdot d\vec{r} = \int -ky\,\hat{y}\cdot(dx\,\hat{x} + dy\,\hat{y} + dz\,\hat{z}) = \int_{-4}^{0} -ky\,dy = \int_{0}^{-4} ky\,dy$$

$$W = \frac{1}{2}ky^2\Big|_0^{-4} = \frac{1}{2}\left(1.57\times10^3\ \frac{\text{N}}{\text{m}}\right)\left(4\times10^{-2}\ \text{m}\right)^2 = 1.25\ \text{J}$$

7.44. If a certain horse can maintain 1.00 hp of output for 2.00 h, how many 70.0 kg bundles of shingles can the horse hoist (using some pulley arrangement) to the roof of a house 8.00 m tall, assuming 70 % efficiency.

$$\text{efficiency} = e = \frac{\text{useful energy output}}{\text{total energy output}} = \frac{(nmg)(h)\cos 0^\circ}{Pt}$$

$$n = \frac{ePt}{mgh} = \frac{(0.7)(1\ \cancel{\text{hp}})(2\ \cancel{\text{h}})}{(70\ \text{kg})(9.8\ \text{m/s}^2)(8\ \text{m})} \times \frac{3600\ \text{s}}{1\ \cancel{\text{hr}}} \times \frac{0.746\ \cancel{\text{kW}}}{1\ \cancel{\text{hp}}} \times \frac{1000\ \cancel{\text{W}}}{1\ \cancel{\text{kW}}} \times \frac{1\ \text{J/s}}{1\ \cancel{\text{W}}}$$

$n =$ 685 bundles

7.59. A 4 kg particle moves along the x axis. Its position varies with time according to $x = t + 2.0t^3$, where x is in meters and t is in seconds. Find (a) the kinetic energy at any time t, (b) the acceleration of the particle and the force acting on it at time t, (c) the power being delivered to the particle at time t, and (d) the work done on the particle in the interval t = 0 to t = 2.00 s.

(a) $v = \dfrac{dx}{dt} = 1 + 6t^2$

$K = \frac{1}{2}mv^2 = \frac{1}{2}(4\ \text{Kg})(1 + 6.00t^2)^2 = (2.00 + 24.0t^2 + 72.0t^4)\ \text{J}$

(b) $a = \dfrac{dv}{dt} = (12.0t)\ \text{m/s}^2$

$F = ma = 4.00(12.0t) = (48.0t)\ \text{N}$

(c) $P = \dfrac{dW}{dt} = \vec{F} \cdot \dfrac{d\vec{s}}{dt} = \vec{F} \cdot \vec{v} = Fv = (48.0t)(1 + 6t^2) = (48.0t + 288t^3)\ \text{W}$

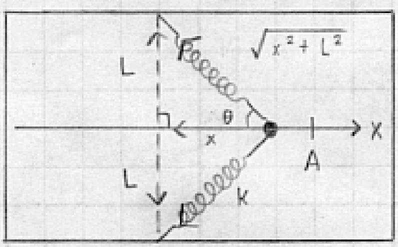
$dW = \vec{F} \cdot d\vec{s}$

(d) $W = \int \vec{F} \cdot d\vec{s} = \int P\,dt = \int_0^{2.00} (48.0t + 288t^3)\,dt = 1250\ \text{J}$

7.66. A particle is attached between 2 identical springs on a horizontal frictionless table. Both springs have spring constant K and are initially unstressed. (a) If the particle is pulled a distance x along a direction perpendicular to the initial configuration of the springs, as in Figure P 7.66, show that the force exerted on the particle by the springs is

$$\vec{F} = -2kx\left(1 - \frac{1}{\sqrt{x^2 + L^2}}\right)\hat{\imath}$$

(b) Determine the amount of work done by this force in moving the particle from x = A to x = 0.



(a) The new length of each spring is $\sqrt{x^2 + L^2}$, so its extension is $\sqrt{x^2 + L^2} - L$ and the force it exerts is $F = K(\sqrt{x^2 + L^2} - L)$ toward its fixed end. The y components of the 2 spring forces add to zero. Their x-components add to

$F_x = F\cos\theta$

$F_x = F \cdot \dfrac{x}{\sqrt{x^2 + L^2}}$

$$\vec{F} = -2k\hat{\imath}\,\frac{(\sqrt{x^2 + L^2} - L)x}{\sqrt{x^2 + L^2}} = -2Kx\hat{\imath}\left(1 - \frac{L}{\sqrt{x^2 + L^2}}\right)$$

(b) $W = \int_i^f F_x\,dx = \int_A^0 -2kx\left(1 - \frac{L}{\sqrt{x^2+L^2}}\right)dx$

$W = -2K\int_A^0 x\,dx + 2KL\int_A^0 x(x^2+L^2)^{-1/2}\,dx$

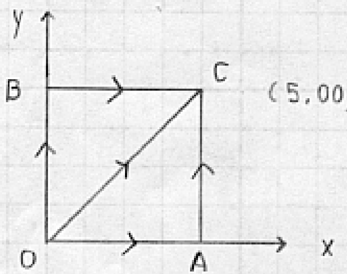
$W = -\cancel{2}K\frac{x^2}{\cancel{2}}\Big|_A^0 + KL\int u^{-1/2}\,du$

$u = x^2 + L^2$
$du = 2x\,dx$
$W = KA^2 + KL \cdot 2u^{1/2}\Big|_{A^2+L^2}^{L^2}$

$W = kA^2 + 2kL\left[L - (A^2+L^2)^{1/2}\right]$

$W = 2kL^2 + kA^2 - 2kL(A^2+L^2)^{1/2}$

8.3, A 4 kg particle moves from the origin to position C, which has coordinates $x = 5.00$ m and $y = 5.00$ m. One force is gravity. Using Equation 7.2, calculate the work done by gravity as the particle moves from O to C along (a) OAC, (b) OBC, and (c) OC. Your results should all be identical. Why?



(5.00, 5.00) m

$F_g = mg = (4.00\text{ kg})(9.8\text{ m/s}^2) = 39.2\text{ N}$

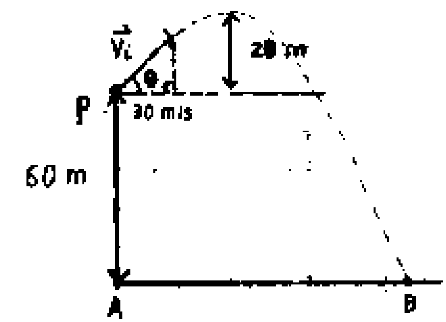
(a) Work along OAC = work along OA + work along AC
$W = F_g(OA)\cos(90) + F_g(AC)\cos 180°$
$W = 0 + (39.\text{N})(5.00\text{ m})(-1) = -196\text{ J}$

(b) Work along OBC = Work along OB + Work along BC
$W = F_g(OB)\cos(180) + F_g(BC)\cos(90)$
$W = (39.2\text{ N})(5.00\text{ m})(-1) + 0 = -196\text{ J}$

(c) $W = F_g(OC)\cos 135$
$W = (39.2\text{ N})(\sqrt{5^2+5^2})\cos 135 = -196\text{ J}$

Gravitational forces are conservative.

8.10. A particle of mass 0.5 kg is shot from P. The particle has an initial velocity $\vec{v}_i$ with a horizontal component of 30.0 m/s. The particle rises to a max height of 20m above P. Using the law of conservation of energy, determine (a) the vertical component of $\vec{v}_i$, (b) the work done by the gravitational force on the particle during its motion from P to B, and (c) the horizontal and vertical components of the velocity vector when the particle reaches B.



(a) $E_i = E_f$

$$\frac{1}{2} m v_{ix}^2 + \frac{1}{2} m v_{iy}^2 = \cancel{\frac{1}{2} m v_{ix}^2} + mgh$$

$$\frac{1}{2} \cancel{m} v_{iy}^2 = \cancel{m} gh$$

$$v_{iy} = \sqrt{2gh} = \sqrt{(2)(9.8)(20)} = 19.8 \text{ m/s}$$

(b) $\Delta KE + \Delta PE = 0$ energy is conserved

$\Delta KE = -\Delta PE$

$W_g = -\Delta PE = -(mg y_f + mg y_i) = 0 + (0.5 \text{ kg})(9.8 \text{ m/s}^2)(60 \text{ m})$

$W_g = 294 \text{ J}$

(c) $v_{ix} = v_{fx} = 30 \text{ m/s}$

$\Delta KE = -\Delta PE = W_g$

$\frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = 294 \text{ J}$

$v_i = \sqrt{v_{ix}^2 + v_{iy}^2} = \sqrt{(30 \text{ m/s})^2 + (19.8 \text{ m/s})^2}$

$v_i = 36 \text{ m/s}$

$$v_f = \sqrt{\frac{(294 \text{ J} + \frac{1}{2} m v_i^2)\, 2}{m}} = \sqrt{\frac{2[294 \text{ J} + (0.5)(0.5 \text{ kg})(36 \text{ m/s})^2]}{0.5 \text{ kg}}}$$

$v_f = 50 \text{ m/s}$

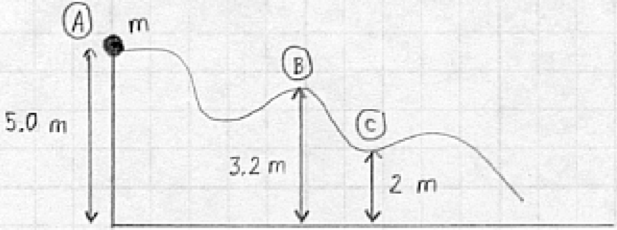
$v_f^2 = v_{fx}^2 + v_{fy}^2$

$v_{fy} = \pm\sqrt{v_f^2 - v_{fx}^2} = -\sqrt{(50 \text{ m/s})^2 - (30 \text{ m/s})^2}$

$v_{fy} = -39.6 \text{ m/s}$

$v_B = (30 \text{ m/s})\hat{i} - (39.6 \text{ m/s})\hat{j}$

8.13. A particle of mass $m = 5$ kg is released from point Ⓐ and slides on the frictionless track shown in Figure P8.13. Determine (a) the particle's speed at points Ⓑ and Ⓒ and (b) the net work done by the force of gravity in moving the particle from Ⓐ to Ⓒ.



$E_A = E_B$

$\cancel{m}g(5.0\,m) = \cancel{m}g(3.2\,m) + \frac{1}{2}\cancel{m}v_B^2$

$v_B^2 = 2(5.0\,m - 3.2\,m)(9.8\,m/s^2)$

$v_B = 5.94\ m/s$

$E_B = E_C$

$\cancel{m}g(3.2\,m) + \frac{1}{2}\cancel{m}(5.94\,m/s)^2 = \cancel{m}g(2\,m) + \frac{1}{2}\cancel{m}v_C^2$

$v_C^2 = 2\left[g(3.2\,m - 2\,m) + \frac{1}{2}(5.94\,m/s)^2\right]$

$v_C^2 = 2\left[(9.8\,m/s^2)(3.2\,m - 2\,m) + \frac{1}{2}(5.94\,m/s)^2\right]$

$v_C = 7.67\ m/s$

$W = \vec{F}\cdot\vec{d} = (mg)(d)\cos 0$

$W = (5\,kg)(9.8\,m/s^2)(3.0\,m)(1)$

$W = 147\ J$