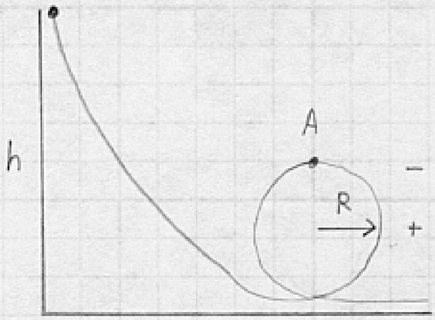
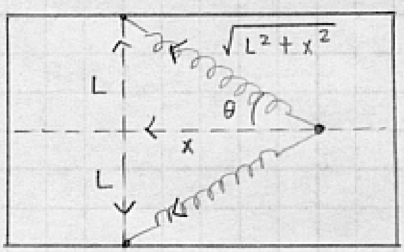
8.15. A bead slides without friction around a loop-the-loop. If the bead is released from a height $h = 3.50\,R$, what is its speed at point A? How great is the normal force on it if its mass is 5.00 g.



$$E_i = E_f$$
$$U_i + K_i = U_f + K_f$$
$$mgh + 0 = mg(2R) + \tfrac{1}{2}mv^2$$
$$\tfrac{1}{2}\cancel{m}v^2 = \cancel{m}g(3.50\,R) - \cancel{m}g(2R)$$
$$v = \sqrt{3gR}$$

$$n + mg = ma$$
$$n + mg = m\frac{v^2}{R}$$
$$n = m\left(\frac{v^2}{R} - g\right) = m\left(\frac{3g\cancel{R}}{\cancel{R}} - g\right)$$
$$n = 2\,mg = 2(5\times10^{-3}\,kg)(9.80\,m/s^2)$$
$$n = 0.0980\ N\ \text{downward}.$$

8.47. A particle of mass m is attached between two identical springs on a horizontal frictionless table. The springs have spring constant K, and each is initially unstressed. (a) If the mass is pulled a distance x along a direction perpendicular to the initial configuration of the springs, show that the potential energy of the system is $U(x) = Kx^2 + 2KL(L - \sqrt{x^2 + L^2})$.



When the mass moves distance x, the length of each spring changes from L to $\sqrt{x^2+L^2}$, so each exerts force $F = K(\sqrt{x^2+L^2} - L)$ toward its fixed end. The y-components cancel out and the x-components add to:

$$F_x = F\cos\theta$$
$$\cos\theta = x / \sqrt{x^2+L^2}$$
$$F_x = -2K\left(\sqrt{x^2+L^2} - L\right)\left(\frac{x}{\sqrt{x^2+L^2}}\right) = -2Kx + \frac{2KLx}{\sqrt{x^2+L^2}}$$

Choose $U = 0$ at $x = 0$. Then at any point

$$F_x = -\frac{dU}{dx}$$
$$dU = -F_x\,dx$$
$$U = \int_0^x \left(2Kx - \frac{2KLx}{\sqrt{x^2+L^2}}\right)dx = 2K\int_0^x x\,dx - 2KL\int_0^x \frac{x}{\sqrt{x^2+L^2}}\,dx$$

$u = x^2 + L^2$
$du = 2x\,dx$

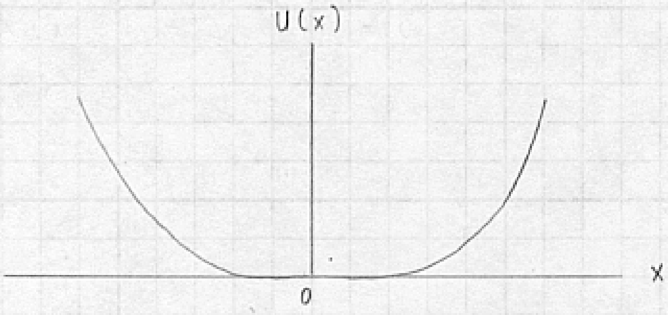
$$U = \cancel{2}K\cdot\frac{1}{\cancel{2}}x^2\Big|_0^x - KL\int_{L^2}^{x^2+L^2} u^{-1/2}\,du$$
$$U = Kx^2 - KL\cdot 2u^{1/2}\Big|_{L^2}^{x^2+L^2} = Kx^2 - 2KL\left(\sqrt{x^2+L^2} - L\right)$$
$$U = Kx^2 + 2KL\left(L - \sqrt{x^2+L^2}\right)$$

(b) Make a plot of $U(x)$ versus x and identify all equilibrium points. Assume that $L = 1.20$ m and $k = 40.0$ N/m.

$U(x) = 40.0x^2 + 2(40.0)(1.20)(1.20 - \sqrt{x^2 + (1.20)^2})$
$U(x) = 40.0x^2 + 96(1.20 - \sqrt{x^2 + 1.44})$

x (m)	0	0.2	0.4	0.6	0.8	1	1.5	2	2.5
U(x) (J)	0	0.011	0.168	0.802	2.35	5.24	20.8	51.3	99.0

For negative x, $U(x)$ has the same value as for positive x.
The only equilibrium point (where $F_x = 0$) is $x = 0$.

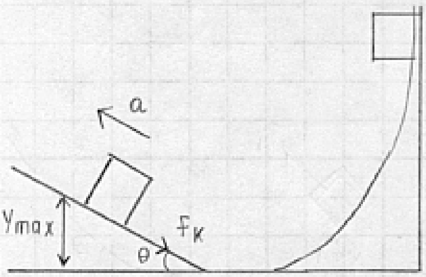


(c) If the mass is pulled 0.5 m to the right and then released, what is its speed when it reaches the equilibrium point $x = 0$?

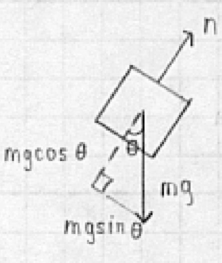
$E_i = E_f$
$K_i + U_i = K_f + U_f$
$0 + 0.400\text{ J} = \frac{1}{2}mv^2$

$$v = \sqrt{\frac{0.800\text{ J}}{m}}$$

8.50. A block slides down a curved frictionless track and then up an inclined plane. The coefficient of kinetic friction between the block and the incline is μ_K. Use energy methods to show that the maximum height reached by the block is
$$y_{max} = \frac{h}{1 + \mu_K \cot\theta}$$

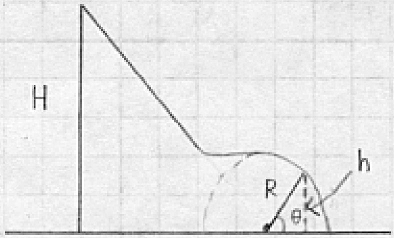


$E_i + W_f = E_f$
$W_f = \vec{F}_K \cdot \vec{d} = -\mu_K mgd\cos\theta$
$F_K = \mu_K n = \mu_K mg\cos\theta$
$\cancel{m}gh - \mu_K \cancel{m}gd\cos\theta = \cancel{mg}\, y_{max}$
$\sin\theta = y_{max}/d$
$d = y_{max}/\sin\theta$
$h - \mu_K y_{max}\cot\theta = y_{max}$
$y_{max} + \mu_K \cot\theta\, y_{max} = h$
$$y_{max} = \frac{h}{(1 + \mu_K\cot\theta)}$$



$n = mg\cos\theta$

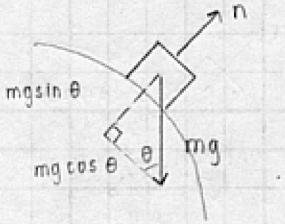
8.72. A child starts from rest and slides down the frictionless slide. In terms of R and H, at what height h will he lose contact with the section of radius R?



$$E_i = E_f$$
$$U_i + K_i = U_f + K_f$$
$$\cancel{m}gH + 0 = \cancel{m}gh + \frac{1}{2}\cancel{m}v^2$$
$$v = \sqrt{2g(H-h)}$$

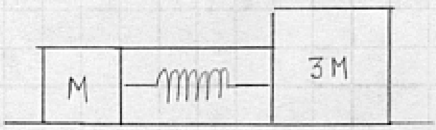
(remember the normal force will be zero):

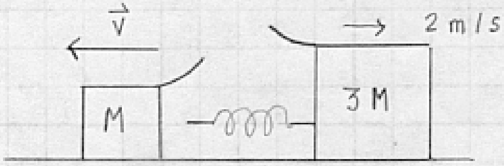
$$\Sigma F_r = ma_r = \frac{mv^2}{R}$$



$$mg\sin\theta - n = \frac{mv^2}{R}$$
$$mg\sin\theta - 0 = \frac{m}{R}2g(H-h)$$
$$\sin\theta = h/R$$
$$\cancel{mg}h/\cancel{R} = 2\cancel{mg}(H-h)/\cancel{R}$$
$$h = 2H - 2h$$
$$3h = 2H$$
$$h = (2/3)H$$

9.6. Two blocks of masses M and 3M are placed on a horizontal, frictionless surface. A light spring is attached to one of them, and the blocks are pushed together with the spring between them. A cord initially holding the blocks together is burned; after this, the block of mass 3M moves to the right with a speed of 2 m/s. (a) What is the speed of the the block of mass M? (b) Find the original elastic energy in the spring if M = 0.350 kg.



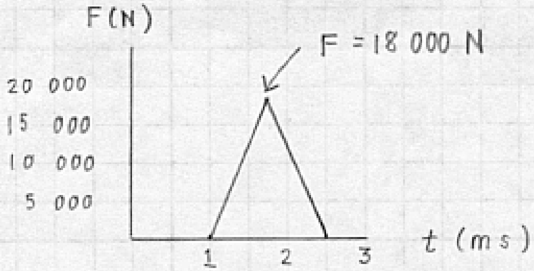


(a) For the system of two blocks

$$p_i = p_f$$
$$0 = \cancel{M}v + 3\cancel{M}(2\ m/s)$$
$$v = -6.00\ m/s \text{ (to the left)}$$

(b) $$\frac{1}{2}kx^2 = \frac{1}{2}Mv_M^2 + \frac{1}{2}(3M)v_{3M}^2$$
$$= \frac{1}{2}(0.35\ kg)(-6\ m/s)^2 + \frac{1}{2}(1.05\ kg)(2\ m/s)^2$$
$$= 8.4\ J$$

9.9. An estimated force – time curve for a baseball struck by a bat is shown. From this curve, determine (a) the impulse delivered to the ball, (b) the average force exerted on the ball, and (c) the peak force exerted on the ball.



$F = ma$

$F = m\,dv/dt$

$F\,dt = m\,dv$

$F\,dt = dp$

(a) $I = \int F\,dt =$ area under curve

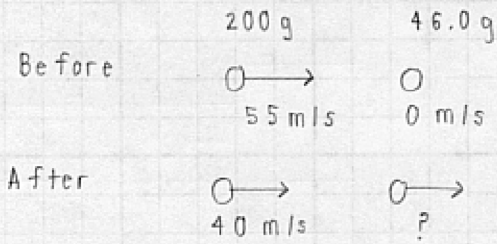
$= \frac{1}{2}(1.5 \times 10^{-3}\ s)(18\,000\ N) = 13.5\ N \cdot s$

(b) $I = \overline{F}\Delta t$ → the constant force that would give same impulse as time varying force

$\overline{F} = \frac{I}{\Delta t} = \frac{13.5\ N \cdot s}{1.5 \times 10^{-3}\ s} = 9.00\ kN$

(c) $F_{max} = 18.0\ kN$

9.15. High-speed stroboscopic photographs show that the head of a golf club of mass 200 g is traveling at 55.0 m/s just before it strikes a 46.0 g golf ball at rest on a tree. After the collision, the club head travels (in the same direction) at 40.0 m/s. Find the speed of the golf ball just after impact.



$p_i = p_f$

$(200\ g)(55\ m/s) = (200\ g)(40\ m/s) + (46.0\ g)\ v$

$v = 65.2\ m/s$