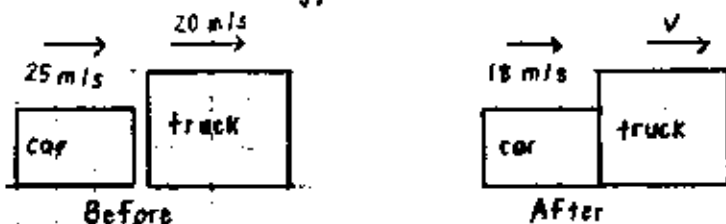


9.21. A 1,200 kg car traveling initially with a speed of 25.0 m/s in an easterly direction crashes into the rear end of a 9,000 kg truck moving in the same direction at 20 m/s. The velocity of the car right after the collision is 18 m/s to the east. (a) What is the velocity of the truck right after the collision? (b) How much mechanical energy is lost in the collision? Account for this loss in energy.



(a) $p_i = p_f$

$$m_c v_{ic} + m_t v_{it} = m_c v_{fc} + m_t v_{ft}$$

$$v_{ft} = \frac{m_c (v_{ic} - v_{fc}) + m_t v_{it}}{m_t}$$

$$v_{ft} = \frac{(1,200 \text{ kg})(25 \text{ m/s} - 18 \text{ m/s}) + (9,000 \text{ kg})(20 \text{ m/s})}{9,000 \text{ kg}} = 20.9 \text{ m/s east}$$

(b) $K_{\text{lost}} = K_i - K_f$ inelastic collision \rightarrow energy not conserved

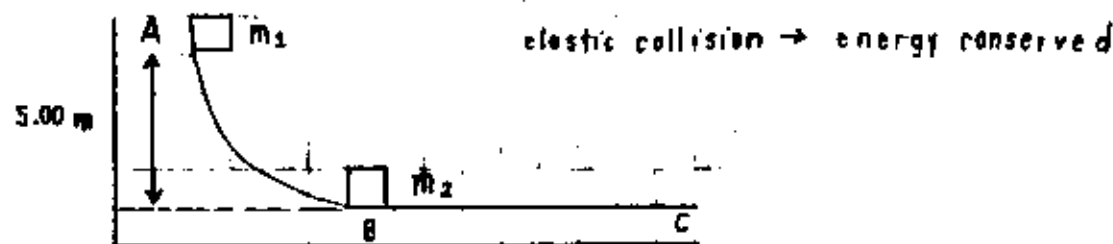
$$= \left(\frac{1}{2} m_c v_{ic}^2 + \frac{1}{2} m_t v_{it}^2 \right) - \left(\frac{1}{2} m_c v_{fc}^2 + \frac{1}{2} m_t v_{ft}^2 \right)$$

$$= \frac{1}{2} \left[m_c (v_{ic}^2 - v_{fc}^2) + m_t (v_{it}^2 - v_{ft}^2) \right]$$

$$= \frac{1}{2} \left[1,200 \text{ kg} \left\{ (25 \text{ m/s})^2 - (18 \text{ m/s})^2 \right\} + 9,000 \text{ kg} \left\{ (20 \text{ m/s})^2 - (20.9333 \text{ m/s})^2 \right\} \right]$$

$$= 8.68 \text{ kJ becomes internal energy}$$

9.26. Consider a frictionless track ABC. A block of mass $m_1 = 5 \text{ kg}$ is released from A. It makes a head-on elastic collision at B with a block of mass $m_2 = 10 \text{ kg}$ that is initially at rest. Calculate the max height to which m_1 rises after the collision.



First, let's find the velocity of m_1 at B:

$$m_1 g h = \frac{1}{2} m_1 v_{1i}^2$$

$$v_{1i} = \sqrt{2gh} = \sqrt{2(9.8 \text{ m/s}^2)(5.00 \text{ m})} = 9.90 \text{ m/s}$$

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2} \right) v_{1i} + \left(\frac{2m_2}{m_1 + m_2} \right) v_{2i} \rightarrow 0$$

$$v_{1f} = \left(\frac{5 \text{ kg} - 10 \text{ kg}}{5 \text{ kg} + 10 \text{ kg}} \right) 9.90 \text{ m/s} = -3.30 \text{ m/s}$$

At highest point (after collision)

$$m_1 g h_{\text{max}} = \frac{1}{2} m_1 v_{1f}^2$$

$$h_{\text{max}} = \frac{v_{1f}^2}{2g} = \frac{(-3.30 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = 0.556 \text{ m}$$

9.31. Two automobiles of equal mass approach an intersection. One vehicle is traveling with velocity 13 m/s toward the east and the other is traveling north with a speed of V_{2i} . Neither driver sees the other. The vehicles collide in the intersection and stick together, leaving parallel skid marks at an angle of 55° north of east. The speed limit for both roads is 35 mi/h , and the driver of the northward moving vehicle claims he was within the speed limit when the collision occurred. Is he telling the truth?

inelastic \rightarrow energy not conserved

We use conservation of momentum for both northward and eastward components:

eastern direction

$$M(13 \text{ m/s}) = 2M(V_f \cos 55)$$

northern direction

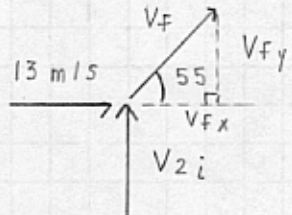
$$M(V_{2i}) = 2M(V_f \sin 55)$$

Divide the northern equation by the eastern direction:

$$\frac{V_{2i}}{13 \text{ m/s}} = \tan 55$$

$$V_{2i} = (13 \text{ m/s}) \tan 55 = 18.6 \text{ m/s} = 41.5 \text{ mi/hr}$$

Untruthful!



$$\cos 55 = V_{fx} / V_f$$

$$\sin 55 = V_{fy} / V_f$$

9.45. A rod of length 30.0 cm has linear density (mass/length) given by $\lambda = 50.0 \text{ g/m} + 20.0x \text{ g/m}^2$ where x is the distance from one end, measured in meters. (a) What is the mass of the rod? (b) How far from the $x=0$ end is its center of mass?

$$(a) M = \int dm = \int_0^{0.3 \text{ m}} \lambda dx = \int_0^{0.3 \text{ m}} (50.0 \text{ g/m} + 20.0x \text{ g/m}^2) dx$$

$$\lambda = \frac{dm}{dx} \quad M = \left[50x \text{ g/m} + 10.0x^2 \text{ g/m}^2 \right]_0^{0.3 \text{ m}} = 15.9 \text{ g}$$

$$(b) X_{cm} = \frac{\int_{\text{all mass}} x dm}{M} = \frac{1}{M} \int_0^{0.3} \lambda x dx$$

$$= \frac{1}{M} \int_0^{0.3} (50.0x \text{ g/m} + 20.0x^2 \text{ g/m}^2) dx$$

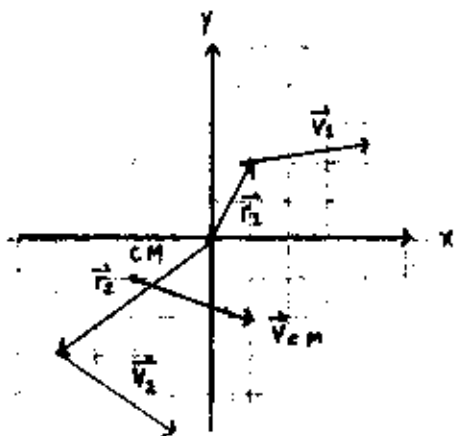
$$= \frac{1}{M} \left[25.0x^2 \text{ g/m} + \frac{20}{3}x^3 \text{ g/m}^2 \right]_0^{0.3}$$

$$= \frac{1}{15.9 \text{ g}} \left[25.0(0.3 \text{ m})^2 \text{ g/m} + \frac{20}{3}(0.3 \text{ m})^3 \text{ g/m}^2 \right]$$

$$= 0.153 \text{ m}$$

9.46. Consider a system of two particles in the xy plane: $m_1 = 2.00 \text{ kg}$ is at $\vec{r}_1 = (1.00\hat{i} + 2.00\hat{j}) \text{ m}$ and has velocity $(3.00\hat{i} + 0.500\hat{j}) \text{ m/s}$; $m_2 = 3.00 \text{ kg}$ is at $\vec{r}_2 = (-4.00\hat{i} - 3.00\hat{j}) \text{ m}$ and has velocity $(3.00\hat{i} - 2.00\hat{j}) \text{ m/s}$. (a) Plot these particles on a grid. Draw their position vectors and show their velocities. (b) Find the position of the center of mass of the system and mark it on the grid. (c) Determine the velocity of the center of mass and also show it on the diagram. (d) What is the total linear momentum of the system?

(a)

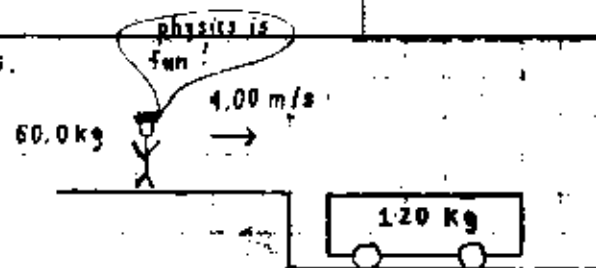


$$\begin{aligned} \text{(b)} \quad \vec{r}_{CM} &= \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2} = \frac{2.00 \text{ kg} \times (1.00 \text{ m}, 2.00 \text{ m}) + 3.00 \text{ kg} \times (-4.00 \text{ m}, -3.00 \text{ m})}{2.00 \text{ kg} + 3.00 \text{ kg}} \\ &= \frac{(2.00 \text{ kg} \cdot \text{m}, 4.00 \text{ kg} \cdot \text{m}) + (-12.0 \text{ kg} \cdot \text{m}, -9.00 \text{ kg} \cdot \text{m})}{5.00 \text{ kg}} \\ &= (-2.00\hat{i} - 1.00\hat{j}) \text{ m} \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad \vec{v}_{CM} &= \frac{\vec{p}}{M} = \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2}{m_1 + m_2} \\ &= \frac{2.00 \text{ kg} \times (3.00 \text{ m/s}, 0.50 \text{ m/s}) + 3.00 \text{ kg} \times (3.00 \text{ m/s}, -2.00 \text{ m/s})}{2.00 \text{ kg} + 3.00 \text{ kg}} \\ &= \frac{(6.00 \text{ kg} \cdot \text{m/s}, 1.0 \text{ kg} \cdot \text{m/s}) + (9.00 \text{ kg} \cdot \text{m/s}, -6.00 \text{ kg} \cdot \text{m/s})}{5.00 \text{ kg}} \\ &= (3.00\hat{i} - 1.00\hat{j}) \text{ m/s} \end{aligned}$$

$$\begin{aligned} \text{(d)} \quad \vec{p} &= M \vec{v}_{CM} = (m_1 + m_2) \vec{v}_{CM} = (2.00 \text{ kg} + 3.00 \text{ kg})(3.00\hat{i} - 1.00\hat{j}) \text{ m/s} \\ &= (15.0\hat{i} - 5.00\hat{j}) \text{ m/s} \end{aligned}$$

$$\begin{aligned} \vec{p} &= m_1 \vec{v}_1 + m_2 \vec{v}_2 = (2.00 \text{ kg})(3.00\hat{i} + 0.500\hat{j}) + (3.00 \text{ kg})(3.00\hat{i} - 2.00\hat{j}) \\ &= (15.0\hat{i} - 5.00\hat{j}) \text{ m/s} \end{aligned}$$



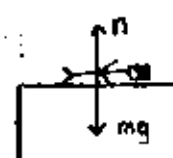
(a) Find the velocity of the person and cart relative to the ground.

$$p_i = p_f$$

$$m_p v_{pi} + m_c v_{ci} = m_p v_{pf} + m_c v_{cf}$$

$$v_f = \frac{m_p v_{pi}}{m_p + m_c} = \frac{(60 \text{ kg})(4.0 \text{ m/s})}{60 \text{ kg} + 120 \text{ kg}} = 1.33 \text{ m/s}$$

(b) Find the frictional force acting on the person while he is sliding across the top surface of the cart.



y-direction

$$n - mg = 0$$

$$n = mg$$

$$f_k = \mu_k n$$

$$f_k = \mu_k (mg)$$

$$f_k = 0.400 (60 \text{ kg})(9.80 \text{ m/s}^2)$$

$$f_k = 235 \text{ N}$$

(c) How long does the frictional force act on the person?

$$\Delta p = p_f - p_i = I \rightarrow p_i + I = p_f$$

$$mv_i + Ft = mv_f$$

$$(60 \text{ kg})(4 \text{ m/s}) - (235 \text{ N})t = (60 \text{ kg})(1.33 \text{ m/s})$$

$$t = 0.680 \text{ s}$$

(d) Find the change in momentum of the person and the change in momentum of the cart?

$$\text{person: } mv_f - mv_i = 60.0 \text{ kg}(1.33 \text{ m/s} - 4.00 \text{ m/s}) = -160 \text{ N}\cdot\text{s} \hat{i}$$

$$\text{cart: } mv_f - mv_i = 120 \text{ kg}(1.33 \text{ m/s}) = 160 \text{ N}\cdot\text{s} \hat{i}$$

(e) Determine the displacement of the person relative to the ground while he is sliding on the cart.

$$a = \frac{v_f - v_i}{t_f - t_i} = \frac{v_f - v_i}{t}$$

$$v_f = v_i + v_i t + \frac{1}{2} a t^2$$

$$x_f = v_i t + \frac{1}{2} \left(\frac{v_f - v_i}{t} \right) t^2 = v_i t + \left(\frac{v_f - v_i}{2} \right) t$$

$$x_f = \frac{1}{2} (v_i + v_f) t = \frac{1}{2} (4.00 + 1.33) \text{ m/s} (0.680 \text{ s}) = 1.81 \text{ m}$$

(f) Determine the displacement of the cart relative to the ground while the person is sliding.

$$x_f = \frac{1}{2} (v_i + v_f) t = \frac{1}{2} (0 + 1.333 \text{ m/s})(0.680 \text{ s}) = 0.454 \text{ m}$$

(g) Find the change in kinetic energy of the person

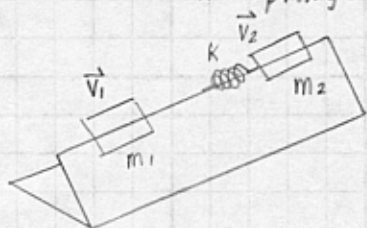
$$\frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = \frac{1}{2} (60.0 \text{ kg})(1.33 \text{ m/s})^2 - \frac{1}{2} (60.0 \text{ kg})(4.00 \text{ m/s})^2 = -427 \text{ J}$$

(h) Find the change in KE of the cart.

$$\frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = \frac{1}{2} (120 \text{ kg})(1.33 \text{ m/s})^2 = 107 \text{ J}$$

(i) Equal frictional forces act through different distances on person and cart to do different amounts of work.

9.66. Two gliders are set in motion on an air track. A spring of force constant k is attached to the rear side of one glider. The first glider of mass m_1 has a velocity of \vec{v}_1 , and the second glider of mass m_2 has a velocity of \vec{v}_2 ($v_1 > v_2$). When m_1 collides with the spring attached to m_2 and compresses the spring to its maximum compression x_m , the velocity of the gliders is \vec{v} . In terms of \vec{v}_1 , \vec{v}_2 , m_1 , m_2 , and k , find (a) the velocity \vec{v} at max compression, (b) the max compression x_m , and (c) the velocities of each glider after m_1 has lost contact with the spring.



(a) When the spring is fully compressed, each cart moves with the same velocity v . Apply conservation of momentum.

$$p_i = p_f$$

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v$$

$$v = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

(b) Only conservative forces act, therefore $\Delta E = 0$

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} (m_1 + m_2) v^2 + \frac{1}{2} k x_m^2$$

Substitute for v from (a) and solve for x_m .

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} (m_1 + m_2) \frac{(m_1 v_1 + m_2 v_2)^2}{(m_1 + m_2)^2} + \frac{1}{2} k x_m^2$$

$$m_1 v_1^2 + m_2 v_2^2 = \frac{(m_1 v_1)^2 + (m_2 v_2)^2 + 2 m_1 m_2 v_1 v_2}{(m_1 + m_2)} + k x_m^2$$

$$x_m^2 = \frac{(m_1 + m_2) m_1 v_1^2 + (m_1 + m_2) m_2 v_2^2 - (m_1 v_1)^2 - (m_2 v_2)^2 - 2 m_1 m_2 v_1 v_2}{k (m_1 + m_2)}$$

$$x_m = \sqrt{\frac{m_1 m_2 (v_1^2 + v_2^2 - 2 v_1 v_2)}{k (m_1 + m_2)}} = (v_1 - v_2) \sqrt{\frac{m_1 m_2}{k (m_1 + m_2)}}$$

(c) conservation of momentum

$$m_1 v_1 + m_2 v_2 = m_1 v_{1f} + m_2 v_{2f}$$

$$m_1 (v_1 - v_{1f}) = m_2 (v_{2f} - v_2) \quad (1)$$

conservation of energy

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

$$m_1 (v_1^2 - v_{1f}^2) = m_2 (v_{2f}^2 - v_2^2)$$

$$m_1 (v_1 - v_{1f})(v_1 + v_{1f}) = m_2 (v_{2f} - v_2)(v_{2f} + v_2) \quad (2)$$

combining (1) and (2)

$$m_2 (v_{2f} - v_2)(v_1 + v_{1f}) = m_2 (v_{2f} - v_2)(v_{2f} + v_2)$$

$$(v_1 + v_{1f}) = (v_{2f} + v_2) \quad \text{or} \quad v_{1f} = v_{2f} + v_2 - v_1 \quad (3)$$

Substituting (3) into (1)

$$m_1 (v_1 - v_{2f} - v_2 + v_1) = m_2 (v_{2f} - v_2)$$

$$m_1 v_1 - m_1 v_2 + m_1 v_1 + m_2 v_2 = m_2 v_{2f} + m_1 v_{2f}$$

$$v_{2f} = \frac{2 m_1 v_1 + (m_2 - m_1) v_2}{m_1 + m_2} = \left(\frac{2 m_1}{m_1 + m_2} \right) v_1 + \left(\frac{m_2 - m_1}{m_1 + m_2} \right) v_2$$

Substituting this into (3) gives

$$v_{1f} = \left(\frac{2 m_1}{m_1 + m_2} \right) v_1 - \left(\frac{m_1 + m_2}{m_1 + m_2} \right) v_1 + \left(\frac{m_2 - m_1}{m_1 + m_2} \right) v_2 + \left(\frac{m_1 + m_2}{m_1 + m_2} \right) v_2$$

$$V_{1f} = \left(\frac{2m_2}{m_1 + m_2} \right) V_2 + \left(\frac{m_1 - m_2}{m_1 + m_2} \right) V_1$$