7. The angular position of a swinging door is described by $\theta = 5.00 + 10.0t + 2.00t^2$ rad. Determine the angular position, angular speed, and angular acceleration of the door (a) at $t = 0$ and (b) at $t = 3.00$ s.

(a) $\theta|_{t=0} = 5.00$

$$\omega\Big|_{t=0} = \frac{d\theta}{dt}\Big|_{t=0} = 10.0 + 4.00t = 10.0 \text{ rad/s}$$

$$\alpha_{t=0} = \frac{d\omega}{dt}\Big|_{t=0} = 4.00 \text{ rad/s}^2$$

(b) $\theta|_{t=3.00s} = 5.00 + 10.0(3.00) + 2.00(3.00s)^2 = 53.0$ rad

$$\omega\Big|_{t=3.00s} = \frac{d\theta}{dt}\Big|_{t=0} = 10.0 + 4.00t = 10.0 + 4.00(3.00) = 22.0 \text{ rad/s}$$

$$\alpha_{t=3.00s} = \frac{d\omega}{dt}\Big|_{t=3.00s} = 4.00 \text{ rad/s}^2$$

17. A car accelerates uniformly from rest and reaches and reaches a speed of 22.0 m/s in 9.00 s. If the diameter of a tire is 58.0 cm, find (a) the number of revolutions the tire makes during this motion, assuming that no slipping occurs. (b) What is the final rotational speed of a tire in revolutions per second?

(a) $s = \bar{v}t = (11.0 \text{ m/s})(9.00 \text{ s}) = 99.0 \text{ m}$

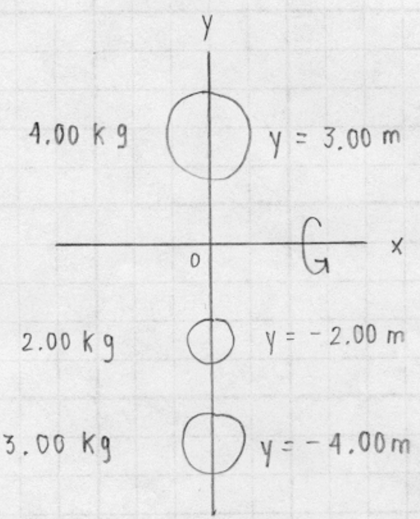
$s = r\theta$

$$\theta = \frac{s}{r} = \frac{99.0 \text{ m}}{0.290 \text{ m}} = 341 \cancel{\text{rad}} \times \frac{1 \text{ rev}}{2\pi \cancel{\text{rad}}} = 54.3 \text{ rev}$$

(b) $v = \omega r$

$$\omega = \frac{v}{r} = \frac{22.0 \text{ m/s}}{0.290 \text{ m}} = 75.9 \frac{\cancel{\text{rad}}}{\text{s}} \times \frac{1 \text{ rev}}{2\pi \text{ rad}} = 12.1 \text{ rev/s}$$

23. Three particles are connected by rigid rods of negligible mass lying along the y axis. If the system rotates about the x axis with an angular speed of 2.00 rad/s, find (a) the moment of inertia about the x axis and the total rotational KE evaluated from $\frac{1}{2}I\omega^2$ and (b) the linear speed of each particle and the total KE evaluated from $\sum \frac{1}{2} m_i v_i^2$.



$m_1 = 4.00\ kg,\quad r_1 = y_1 = 3.00\,m$

$m_2 = 2.00\ kg,\quad r_2 = |y_2| = 2.00\ m$

$m_3 = 3.00\ kg,\quad r_3 = |y_3| = 4.00\ m$

$\omega = 2.00\ rad/s$

(a) $I_x = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2$

$I_x = (4.00)(3.00)^2 + (2.00)(2.00)^2 + (3.00)(4.00)^2 = 92.0\ kg \cdot m^2$

$K_R = \frac{1}{2} I_x \omega^2 = \frac{1}{2}(92.0)(2.00)^2 = 184\ J$

(b) $V_1 = r_1 \omega = (3.00)(2.00) = 6.00\ m/s$

$V_2 = r_2 \omega = (2.00)(2.00) = 4.00\ m/s$

$V_3 = r_3 \omega = (4.00)(2.00) = 8.00\ m/s$

$K_1 = \frac{1}{2} m_1 V_1^2 = \frac{1}{2}(4.00)(6.00)^2 = 72.0\ J$

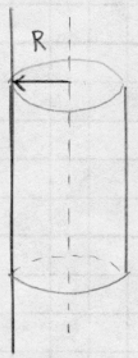
$K_2 = \frac{1}{2} m_2 V_2^2 = \frac{1}{2}(2.00)(4.00)^2 = 16.0\ J$

$K_3 = \frac{1}{2} m_3 V_3^2 = \frac{1}{2}(3.00)(8.00)^2 = 96.0\ J$

$K = K_1 + K_2 + K_3 = 72.0 + 16.0 + 96.0 = 184\ J$

30. Use the parallel-axis theorem and table 10.2 to find the moment of inertia of (a) a solid cylinder about an axis parallel to the center of mass axis and passing through the edge of the cylinder and (b) a solid sphere about an axis tangent to its surface.

(a)

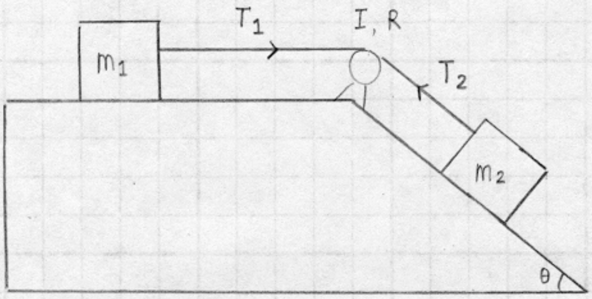


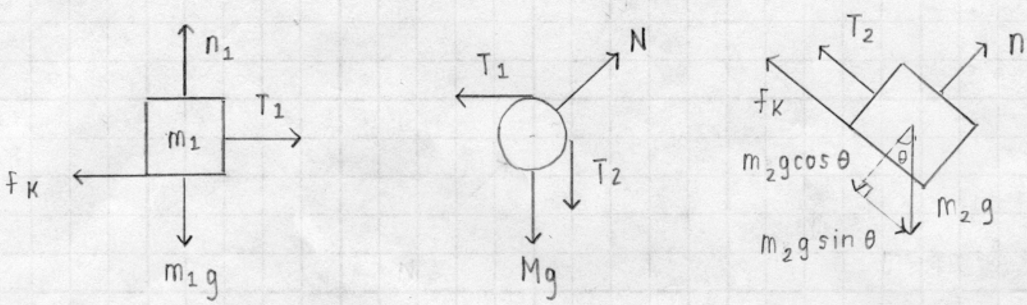
$I = I_{CM} + MD^2 = \frac{1}{2} MR^2 + MR^2 = \frac{3}{2} MR^2$

(b)

$I = I_{CM} + MD^2 = \frac{2}{5} MR^2 + MR^2 = \frac{7}{5} MR^2$

39. A block of mass $m_1 = 2.00$ kg and a block of mass $m_2 = 6.00$ kg are connected by a massless string over a pulley in the shape of a disk having radius $R = 0.250$ m and mass $M = 10.0$ kg. These blocks are allowed to move on fixed block - wedge of angle $\theta = 30.0°$. The coefficient of kinetic friction for both blocks is 0.360. Draw free body diagrams of both blocks and of the pulley. Determine (a) the acceleration of the 2 blocks and (b) the tensions in the string on both sides of the pulley.





For m_1

y-direction: $n_1 - m_1 g = 0$

$n_1 = m_1 g$

x-direction: $T_1 - f_K = m_1 a_x$

$f_K = \mu_K n_1 = \mu_K m_1 g$

$T_1 - \mu_K m_1 g = m_1 a_x$ (1)

For pulley

$\sum \tau = I\alpha$

$-T_1 \cancel{R} + T_2 \cancel{R} = \frac{1}{2} M R^{\cancel{2}} (a/R)$

$a_t = r\alpha$

$-T_1 + T_2 = \frac{1}{2} M a$ (2)

For m_2

$n_2 - m_2 g \cos\theta = 0$

$n_2 = m_2 g \cos\theta$

$-T_2 - f_K + m_2 g \sin\theta = m_2 a$

$f_K = \mu_K n_2 = \mu_K m_2 g \cos\theta$

$-T_2 - \mu_K m_2 g \cos\theta + m_2 g \sin\theta = m_2 a$ (3)

(a) add equations (1) (2) and (3)

$\cancel{T_1} - \mu_K m_1 g - \cancel{T_1} + \cancel{T_2} - \cancel{T_2} - \mu_K m_2 g \cos\theta + m_2 g \sin\theta = m_1 a + \frac{1}{2} M a + m_2 a$

$$a = \frac{-\mu_K g (m_1 + m_2 \cos\theta) + m_2 g \sin\theta}{m_1 + \frac{1}{2} M + m_2} = \frac{-(0.36)(9.8)(2.00 + 6.00 \cos 30) + (6.00)(9.8) \sin 30}{2.00 + 5.00 + 6.00}$$

$a = 0.309 \text{ m/s}^2$

(b) $T_1 = m_1 a + \mu_k m_1 g$

$T_1 = m_1(a + \mu_k g) = 2.00[0.309 + 0.36(9.80)]$

$T_1 = 7.67 \text{ N}$

$T_2 = \frac{1}{2} Ma + T_1 = \frac{1}{2}(10)(0.309) + 7.67 = 9.22 \text{ N}$

64. (a) What is the rotational energy of the Earth about its spin axis? The radius of the earth is 6,370 km, and its mass is 5.98×10^{24} kg. Treat the earth as a sphere of moment of inertia $\frac{2}{5}MR^2$. (b) The rotational energy of the earth is decreasing steadily because of tidal friction. Estimate the change in 1 day, given that the rotational period increases by about 10 μs each year.

(a) $E = \frac{1}{2} I \omega^2 = \frac{1}{2}\left(\frac{2}{5} MR^2\right)\omega^2$

$E = \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{5}(5.98 \times 10^{24} \text{ kg})(6.37 \times 10^6 \text{ m})^2 \left(\frac{2\pi}{24 \text{ h}} \times \frac{1 \text{ hr}}{60 \text{ m}} \times \frac{1 \text{ m}}{60 \text{ s}}\right)^2$

$E = 2.57 \times 10^{29} \text{ J}$

$v = \omega r$

$d = vt$

$\omega = \frac{v}{r} = \frac{(2\pi r / T)}{r} = \frac{2\pi}{T}$

(b) $\frac{dE}{dt} = \frac{d}{dt}\left[\frac{1}{\cancel{2}}\left(\frac{\cancel{2}}{5} MR^2\right)\left(\frac{2\pi}{T}\right)^2\right]$

$= \frac{1}{5} MR^2 (2\pi)^2 (-2T^{-3}) \frac{dT}{dt} = \underbrace{\frac{1}{5} MR^2 \left(\frac{2\pi}{T}\right)^2}_{E} \left(\frac{-2}{T}\right) \frac{dT}{dt}$

$= (2.57 \times 10^{29} \text{ J})\left(\frac{-2}{86400 \cancel{s}}\right)\left(\frac{10 \times 10^{-6} \cancel{s}}{3.16 \times 10^7 \cancel{s}}\right)\left(\frac{86400 \cancel{s}}{\text{day}}\right)$

$= -1.63 \times 10^{17} \text{ J/day}$

Δ seconds / year

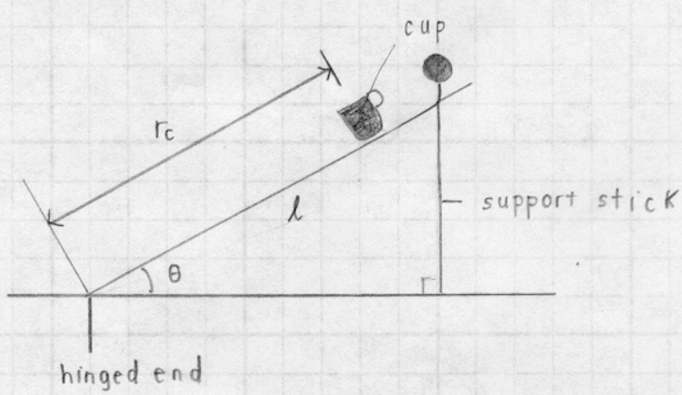
seconds / year

conversion

72. A ball rests at one end of a uniform board of length ℓ, hinged at the other end, and elevated at an angle θ. A light cup is attached to the board at r_c so that it will catch the ball when the support stick is suddenly removed. (a) Show that the ball will lag behind the falling board when θ is less than 35.3°; and that (b) the ball will fall into the cup when the board is supported at this limiting angle and the cup is placed at

$r_c = \frac{2\ell}{3\cos\theta}$

(c) If a ball is at the end of a 1.00 m stick at this critical angle, show that the cup must be 18.4 cm from the moving end.



For the board just starting to move,

$\sum \tau = I \alpha$

$\vec{\tau} = \vec{r} \times \vec{F}$

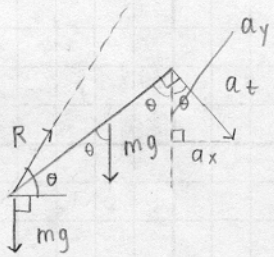
$\vec{\tau} = (\ell/2) mg \sin(\theta + 90)$

$\sin(u + v) = \sin u \cos v + \cos u \sin v$

$\vec{\tau} = mg(\sin\theta \cos 90 + \cos\theta \sin 90)\, \ell/2$

$\vec{\tau} = mg(\ell/2) \cos\theta$

$I = (1/3) m \ell^2$



$$\cancel{m} g \frac{\cancel{\ell}}{2} \cos\theta = \frac{1}{3} \cancel{m} \ell^{\cancel{2}} \alpha$$

$$\alpha = \frac{3}{2} \frac{g}{\ell} \cos\theta$$

The tangential acceleration of the end is

$$a_t = \ell \alpha = \frac{3}{2} g \cos\theta$$

$\cos\theta = a_y / a_t$

$a_y = a_t \cos\theta = \frac{3}{2} g \cos^2\theta$

If this is greater than g, the board will pull ahead of the ball in falling.

(a) $\frac{3}{2} g \cos^2\theta \geq g$

$\cos^2\theta \geq 2/3$

$\cos\theta \geq \sqrt{\frac{2}{3}}$ $\qquad \theta \leq 35.3°$

(b) When $\theta = 35.3°$, the cup will land underneath the release point of the ball if

$$r_c = \ell \cos\theta = \frac{\ell \cos^2\theta}{\cos\theta} = \frac{2\ell}{3\cos\theta}$$

$\cos\theta = r_c / \ell$

(c) $\ell = 1.00\text{ m}$ and $\theta = 35.3°$

$$r_c = \frac{2(1.00\text{ m})}{3\sqrt{2/3}} = 0.816\text{ m}$$

$1.00\text{ m} - 0.816\text{ m} = 0.184\text{ m}$