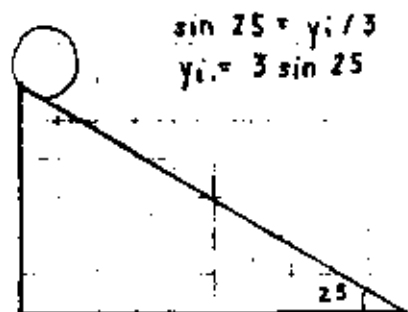


11.7. A metal can containing condensed mushroom soup has a mass of 215 g, a height of 10.8 cm, and a diameter of 6.38 cm. It is placed at rest at the top of a 3.00-m-long incline that is at an angle of 25.0° to the horizontal and is then released to roll straight down. Assuming energy conservation, calculate the moment of inertia of the can if it takes 1.50 s to reach the bottom of the incline. Which pieces of data, if any, are unnecessary for calculating the solution.



$$\sin 25 = y_i / 3$$

$$y_i = 3 \sin 25$$

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{3.00 \text{ m}}{1.50 \text{ s}} = 2.00 \text{ m/s}$$

$$\bar{v} = \frac{(v_f + v_i)}{2}$$

$$v_f = 2\bar{v} - v_i = 2(2.00 \text{ m/s}) - 0 = 4.00 \text{ m/s}$$

$$v = \omega r$$

$$\omega_f = \frac{v_f}{r} = \frac{4.00 \text{ m/s}}{(6.38 \times 10^{-2} \text{ m})/2} = 125.39 \text{ /s}$$

$$\Delta K_{\text{trans}} + \Delta K_{\text{rot}} + \Delta U_g = 0$$

$$(K_{\text{trans}} + K_{\text{rot}} + U_g)_i = (K_{\text{trans}} + K_{\text{rot}} + U_g)_f$$

$$0 + 0 + mgy_i = \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2 + 0$$

$$\frac{1}{2}I\omega_f^2 = mgy_i - \frac{1}{2}mv_f^2$$

$$I = \frac{2m}{\omega_f^2} \left(gy_i - \frac{1}{2}v_f^2 \right)$$

$$I = \frac{2(0.215 \text{ kg})}{(125.39 \text{ /s})^2} \left[(9.8 \text{ m/s}^2)(3 \text{ m})\sin 25 - \frac{1}{2}(4.00 \text{ m/s})^2 \right]$$

$$I = 1.21 \times 10^{-4} \text{ kg} \cdot \text{m}^2$$

The height of the can is unnecessary.

11.15. A force of $\vec{F} = 2.00\hat{i} + 3.00\hat{j}$ N is applied to an object that is pivoted about a fixed axis aligned along the z coordinate axis. If the force is applied at the point $\vec{r} = (4.00\hat{i} + 5.00\hat{j} + 0\hat{k})$ m, find (a) the magnitude of the net torque about the z axis and (b) the direction of the torque vector $\vec{\tau}$.

$$(a) \quad \vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4.00 & 5.00 & 0 \\ 2.00 & 3.00 & 0 \end{vmatrix} = \hat{k}(12.0 - 10.0) = 2.00\hat{k}$$

$$\tau = 2.00 \text{ N} \cdot \text{m}$$

(b) The torque vector is in the direction of the unit vector \hat{k} , or in the z direction.

11.19. A student claims that she has found a vector \vec{A} such that $(2\hat{i} - 3\hat{j} + 4\hat{k}) \times \vec{A} = (4\hat{i} + 3\hat{j} - \hat{k})$. Do you believe this claim? Explain.

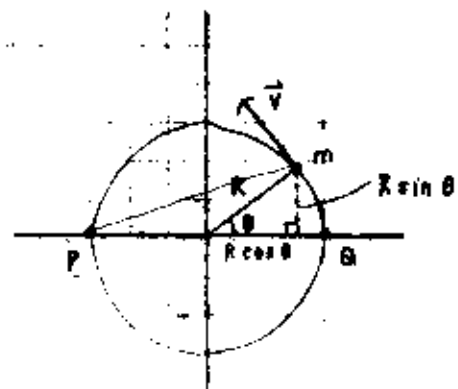
The cross product vector must be perpendicular to both of the factors, so its dot product with either factor must be zero.

Does $(2\hat{i} - 3\hat{j} + 4\hat{k}) \cdot (4\hat{i} + 3\hat{j} - \hat{k}) = 0$?

$$8 - 9 - 4 = -5 \neq 0$$

No, the cross product could not work out that way.

11.23. A particle of mass m moves in a circle of radius R at a constant speed v . If the motion begins at point Q , determine the angular momentum of the particle about point P as a function of time.



The vector from point P to the mass is

$$\vec{r} = R\hat{i} + R\cos\theta\hat{i} + R\sin\theta\hat{j}$$

$$\vec{r} = R[(1 + \cos\theta)\hat{i} + \sin\theta\hat{j}]$$

$$\theta = \omega t = s/R = vt/R$$

$$s = R\theta$$

$$s = vt$$

$$\vec{r} = R[(1 + \cos(vt/R))\hat{i} + \sin(vt/R)\hat{j}]$$

$$\vec{v} = d\vec{r}/dt = -v\sin(vt/R)\hat{i} + v\cos(vt/R)\hat{j}$$

$$\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times m\vec{v}$$

$$\vec{L} = mvR \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 + \cos(vt/R) & \sin(vt/R) & 0 \\ -\sin(vt/R) & \cos(vt/R) & 0 \end{vmatrix}$$

$$\vec{L} = \hat{k} \left(\cos\frac{vt}{R} + \cos^2\frac{vt}{R} + \sin^2\frac{vt}{R} \right) mvR$$

$$\vec{L} = mvR\hat{k} [1 + \cos(vt/R)]$$

11.36. A uniform rod with a mass of 100g and a length of 50.0 cm rotates in a horizontal plane about a fixed, vertical, frictionless pin passing through its center. Two small beads, each having a mass 30g are mounted on the rod so that they are able to slide without friction along its length. Initially, the beads are held by catches at positions 10 cm on each side of center; at this time, the system rotates at an angular speed of 20 rad/s. Suddenly, the catches are released, and the small beads slide outward along the rod. Find (a) the angular speed of the system at the instant the beads reach the ends of the rod and (b) the angular speed of the rod after the beads fly off the rod's end.

(a) M = mass of rod, m = mass of each bead

$$I_i \omega_i = I_f \omega_f \quad \text{2 beads}$$

$$\left[\frac{1}{12} M L^2 + 2m r_1^2 \right] \omega_i = \left[\frac{1}{12} M L^2 + 2m r_2^2 \right] \omega_f$$

$$\left[\frac{1}{12} (0.1 \text{ kg})(0.5 \text{ m})^2 + 2(0.03 \text{ kg})(0.1 \text{ m})^2 \right] (20 \frac{\text{rad}}{\text{s}}) =$$

$$\left[\frac{1}{12} (0.1 \text{ kg})(0.5 \text{ m})^2 + 2(0.03 \text{ kg})(0.25 \text{ m})^2 \right] \omega_f$$

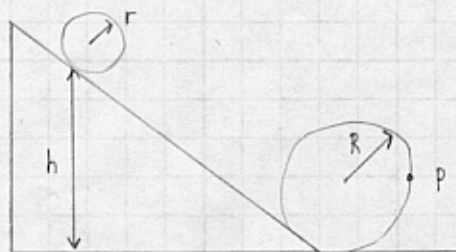
$$\omega_f = 9.20 \text{ /s}$$

(b) Since there is no external torque on the rod, $L = \text{constant}$ and ω is unchanged.

$$\sum \tau_{\text{ext}} = \frac{d\vec{L}}{dt} = 0$$

$$\vec{L} = \text{constant}$$

11.51. A solid sphere of mass m and radius r rolls without slipping along the track. The sphere starts from rest with its lowest point at height h above the bottom of a loop of radius R , which is much larger than r . (a) What is the minimum value that h can have (in terms of R) if the sphere is to complete the loop? (b) What are the force components on the sphere at point P if $h = 3R$.



Initially, the center of mass of the sphere is $h+r$ above the bottom of the loop. As the mass reaches the top of the loop, this distance above the reference level is $2R-r$.

$$\Delta K_{\text{trans}} + \Delta K_{\text{rot}} + \Delta U_g = 0$$

$$(K_{\text{trans}} + K_{\text{rot}} + U_g)_i = (K_{\text{trans}} + K_{\text{rot}} + U_g)_f$$

$$0 + 0 + mg(h+r) = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + mg(2R-r)$$

$$I = \left(\frac{2}{5}\right)mr^2$$

$$mg(h+r) = \frac{1}{2}m(r\omega)^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\omega^2 + mg(2R-r)$$

$$gh + gr = \frac{r^2\omega^2}{2} + \frac{r^2\omega^2}{5} + 2gR - gr$$

$$gh = \frac{7}{10}r^2\omega^2 + 2gR - 2gr$$

$$gh = \frac{7}{10}v^2 + 2gR - 2gr$$

$h = h_{\text{min}}$ when the speed of the sphere at the top of the loop satisfies the condition

$$\sum F = mg + n = m \frac{v^2}{(R-r)}$$

$$v^2 = g(R-r)$$

$$v^2 = \frac{(mg+n)(R-r)}{m}$$

When $n=0$, v^2 is smaller.

When v^2 is smaller, $h = h_{\text{min}}$.

$$h_{\text{min}} = \frac{7g(R-r)}{10g} + 2R - 2r$$

$$h_{\text{min}} = 2(R-r) + 0.700(R-r) = 2.70(R-r) \approx 2.70R$$

(b) When the sphere is initially at $h = 3R$ and finally at point P , conservation of energy gives

$$mg(3R+r) = mgR + \frac{1}{2}mv^2 + \frac{1}{5}mv^2$$

$$v^2 = \frac{10}{7}(2R+r)g$$

At point P,

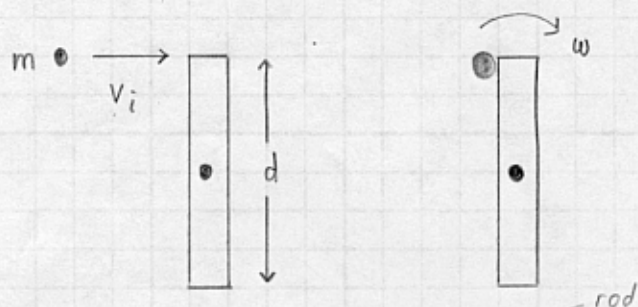
$$F - mg = -m\alpha r$$

$$Fr = (2/5)mr^2\alpha$$

Eliminating f by substitution yields $\alpha = 5g/7r$ so $\Sigma F_y = -(5/7)mg$

$$\Sigma F_x = -n = -\frac{mv^2}{R-r} = -\frac{(10/7)(2R+r)mg}{R-r} = \frac{-20mg}{7} \quad (\text{since } R \gg r)$$

11.54. A projectile of mass m moves to the right with speed v_i . The projectile strikes and sticks to the end of a stationary rod of mass M and length d that is pivoted about a frictionless axle through its center. (a) Find the angular speed of the system right after the collision. (b) Determine the fractional loss in mechanical energy due to the collision.



(a) angular momentum is conserved

$$\vec{L} = \vec{r} \times \vec{p} = r m v \sin \theta = I \omega$$

$$\frac{d}{2} m v_i \sin 90 = \left[\frac{1}{12} M d^2 + m \left(\frac{d}{2} \right)^2 \right] \omega$$

$$6 d m v_i = (M d^2 + 3 m d^2) \omega$$

$$\omega = \frac{6 m v_i}{M d + 3 m d}$$

(b) The original energy is $\frac{1}{2} m v_i^2$.

The final energy is

$$\frac{1}{2} I \omega^2 = \frac{1}{2} \left(\frac{1}{12} M d^2 + m \left(\frac{d}{2} \right)^2 \right) \frac{36 m^2 v_i^2}{(M d + 3 m d)^2}$$

$$= \frac{d}{24} (M d + 3 m d) \frac{36 m^2 v_i^2}{(M d + 3 m d)^2}$$

$$= \frac{3 m^2 v_i^2 d}{2 (M d + 3 m d)}$$

The loss of energy is

$$\frac{1}{2} m v_i^2 - \frac{3 m^2 v_i^2 d}{2 (M d + 3 m d)} = \frac{(M d + 3 m d) m v_i^2 - 3 m^2 v_i^2 d}{2 (M d + 3 m d)} = \frac{m M v_i^2 d}{2 (M d + 3 m d)}$$

The fractional loss of energy is

$$\frac{m M v_i^2 d}{2 (M d + 3 m d)} \cdot \frac{1}{\frac{1}{2} m v_i^2} = \frac{M}{M + 3 m}$$