1) A force $F_x = -9\,x^2\,N$, where x is in meters, acts on a 2.0 kg body as it moves along the positive x-axis. The velocity of the body at $x = 1m$ is $9.0\,m/s$. (a) What is the velocity of the body at $x = 2m$? (5 pts) (b) At what value of x will the body have a velocity of $6.0\,m/s$? (5 pts)

a) $W = \displaystyle\int_{1m}^{2m} (-9x^2)\,dx = \frac{1}{2}m v_f^2 - \underbrace{\frac{1}{2}m v_i^2}_{81\,J}$ $\qquad v_i = 9.0\,m/s \ \ at \ x = 1m$

$-3x^3 \ \Big|_{1m}^{2m} \overbrace{(N\text{-}m)}^{J} = \frac{1}{2}m v_f^2 - 81\,J \ \Rightarrow \ v_f = \sqrt{81 - 21} \qquad m/s$

$\simeq 7.7\,m/s$

b) $-3x^3 \Big|_{1m}^{d} \overset{to\ find\ this}{=} 36\,J - 81\,J$

$\frac{1}{2}m v_f^2 = 36\,J$

$\frac{1}{2}m v_i^2 = 81\,J$

$-3d^3 = -48 \ \Rightarrow \ d \simeq 2.5\,m$

2) In the Figure below, a 12 kg block is released *from rest* on a 30^0 frictionless incline. Below the block is a spring that can be compressed 2.0 *cm* by a force of 270 *N*. The block *momentarily* stops when it compresses the spring by 5.5 *cm*. (a) How far does the block move down the incline from its rest position to this stopping point? (5 pts) (b) What is the speed of the block just as it touches the spring? (5 pts)
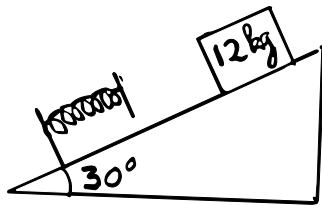
Spring constant:

$$k = \frac{270\,N}{0.02\,m} = 1.35 \times 10^4 \ \frac{N}{m}$$

a) Energy conservation: $5.5\,cm = 5.5 \times 10^{-2}\,m$

$$\frac{1}{2}kx^2 = mgh \qquad \Rightarrow \quad h = \frac{1}{2} \times \frac{1.35 \times (5.5)^2}{12 \times 9.8} \quad m$$

$$= 0.17 \ m$$

Distance that the block moves:

$$\ell = \frac{h}{\sin 30^0} = 2h \approx 0.35\,m$$

b) Just as the block touches the spring, the difference in potential is $mgh - mgh'$

$h' = (0.055m)\sin 30^0 = 0.0275m$
$h = \ell \sin 30^\circ$

$$mg(0.17m - 0.0275m) = \frac{1}{2}mv^2 \quad \Rightarrow \quad v = 1.70\,m/s$$

3

3) Two hard spheres approach each other head-on with the same speed and collide elastically. After collision, *one* of the spheres, whose mass is 300 g, remains at rest. (a) What is the mass of the other sphere? (5 pts) (b) What is the speed of the two-sphere Center of Mass if the initial speed of each sphere is $2.0\,m/s$? (5 pts).

a) This is the case of elastic collision with $v_{1i} = v$ and $v_{2i} = -v$. Let m_2 be the mass at rest after collision i.e. $v_{2f} = 0$ and $m_2 = 300g$.

From the formulae of p. 1

$$0 = v \left(\frac{2m_1}{m_1+m_2} - \frac{(m_2-m_1)}{m_1+m_2} \right)$$

$\Rightarrow 2m_1 = m_2 - m_1 \quad \Rightarrow \quad 3m_1 = m_2 \quad \Rightarrow \quad m_1 = \frac{m_2}{3} = 100g$

b)

$$V_{cm} = \frac{(m_1 - m_2)v}{m_1 + m_2} = \frac{200g}{400g} \times 2\,m/s$$

$$= 1\,m/s$$

4