Masta Copy

Name:

PHYSICS 231

Midterm 3, November 25 , 2002, 10:00-10:50 AM

Instructor: P. Q. Hung

10 points for each problem. READ the problems carefully. SHOW YOUR WORK. CLEARLY WRITE DOWN THE STEPS USED IN OBTAINING THE ANSWER(S). DO NOT JUST write the answers down. ANSWERS WITHOUT EXPLANATION WILL BE GIVEN NO CREDIT. PLEASE USE A SCRATCH PAPER SEPARATELY!

Miscellaneous formulae (not necessarily applied to all problems below):

$v_{xf} = v_{xi} + a_x t$; $v_{yf} = v_{yi} + a_y t$

$x_f - x_i = v_{xi} t + \frac{1}{2} a_x t^2$; $y_f - y_i = v_{yi} t + \frac{1}{2} a_y t^2$

$v_{xf}^2 = v_{xi}^2 + 2\, a_x (x_f - x_i)$; $v_{yf}^2 = v_{yi}^2 + 2\, a_y (y_f - y_i)$

$R = \frac{v_i^2 \sin 2\theta_i}{g}$; $a_r = \frac{v^2}{r}$

$\vec{F}_{net} = m\, \vec{a}$; $W = \int F(x) dx$;

$W = K_f - K_i$; $E = K + U$; $U = mgh$; $U = \frac{1}{2} k x^2$.

$\vec{R}_{C.M.} = \frac{m_1 \vec{r}_1 + .. + m_N \vec{r}_N}{m_1 + .. + m_N}$; $\vec{V}_{C.M.} = d\vec{R}_{C.M.}/dt$

$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2 m_2}{m_1 + m_2} v_{2i}$; $v_{2f} = \frac{2 m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i}$

$I = \sum_i m_i r_i^2$; $I = \int r^2 dm$; $\sum \tau = I \alpha$.

$\vec{L} = \vec{r} \times \vec{p}$; $L = I\omega$

$E = K + U = \frac{1}{2} m v^2 - \frac{GMm}{r}$

$g = \frac{GM_{earth}}{R_{earth}^2} = 9.8 m/s^2$ $G = 6.673 \times 10^{-11} N.m^2/kg^2$.

$P = F/A$; $P = P_0 + \rho g h$; $P + \frac{1}{2} \rho v^2 + \rho g y = constant$

1) A horizontal vinyl record of mass 0.10 kg and radius 0.10 m rotates freely about a vertical axis through its center with an angular speed of 4.7 rad/s. The rotational inertia of the record about its axis of rotation is $5 \times 10^{-4} kg.m^2$. A wad of wet putty of mass 0.02 kg drops vertically onto the record from above and sticks to the *edge* of the record. What is the angular speed of the record immediately after the putty sticks to it? (10 pts)

Let $I = 5\times10^{-4}\ kg.m^2$; $m = 0.02 kg$; $R = 0.10 m$

Conservation of angular momentum:

$$I\omega_0 = (I + mR^2)\,\omega_f$$

$$\Rightarrow \omega_f = \left(\frac{I}{I+mR^2}\right)\omega_0 = \left(\frac{5\times10^{-4}}{5\times10^{-4}+0.02\times10^{-2}}\right)\omega_0$$

$$= \frac{5}{7}\,\omega_0 \simeq 3.4\ rad/s$$

2) What is the escape speed on a SPHERICAL asteroid whose radius R is 500 km and whose gravitational acceleration at the SURFACE is $3.0 m/s^2$? (5 pts) At what speed does a particle leave the surface of the asteroid such that it reaches a MAXIMUM HEIGHT (measured from the surface) of $h = R/2$, where R is the radius of the asteroid? (5 pts) (Hint: Use energy conservation).

a) Escape speed from:

$$\frac{1}{2} m v_{esc}^2 - \frac{MmG}{R} = 0 \qquad g_{ast} = 3 m/s^2$$

$$\Rightarrow v_{esc}^2 = \frac{2MG}{R} = 2\underbrace{\left(\frac{MG}{R^2}\right)}_{g_{ast}} R = 2 g_{ast} R$$

$$v_{esc} = \sqrt{2 g_{ast} R} = \sqrt{2 \times 3 m/s^2 \times 5 \times 10^5 m} = 1732 \ m/s$$

b)

$$\frac{1}{2} m v^2 - \frac{GMm}{R} = 0 - \frac{GMm}{R+h}$$

$$\Rightarrow v^2 = 2\left(\frac{GM}{R} - \frac{GM}{R+h}\right) = \frac{2GM}{R}\left(1 - \frac{R}{R+h}\right)$$

$$= \underbrace{2\left(\frac{GM}{R^2}\right) R}_{v_{esc}^2} \left(\frac{h}{R+h}\right)$$

$$\Rightarrow v = v_{esc} \sqrt{\frac{h}{R+h}} = \frac{1}{\sqrt{3}} v_{esc} \simeq 1000 \ m/s$$

3) A cube of unknown material with a side dimension of 30 cm floats in water with the distance from the horizontal top surface of the cube to the water level being $h = 7cm$. What is the density of the cube? The density of water is $\rho_{water} = 1.00 \times 10^3 kg/m^3$ (10 pts)

Condition for floating:

$$\rho_{cube} V_{total}\, g = \rho_w V_{submerged}\, g$$

$$\Rightarrow \rho_{cube} (30\,cm)^3 = \rho_w (30\,cm)^2 \times 23\,cm$$

$$\Rightarrow \rho_{cube} = \left(\frac{23}{30}\right) \rho_w \simeq 0.767 \rho_w$$

$$\simeq 0.767 \times 10^3 \, kg/m^3$$