Lecture Supplement 13

***P28.14** *When S is open*, R_1, R_2, R_3 are in series with the battery. Thus:

$$R_1 + R_2 + R_3 = \frac{6\text{ V}}{10^{-3}\text{ A}} = 6\text{ k}\Omega. \tag{1}$$

When S is closed in position 1, the parallel combination of the two R_2's is in series with R_1, R_3, and the battery. Thus:

$$R_1 + \frac{1}{2}R_2 + R_3 = \frac{6\text{ V}}{1.2\times 10^{-3}\text{ A}} = 5\text{ k}\Omega. \tag{2}$$

When S is closed in position 2, R_1 and R_2 are in series with the battery. R_3 is shorted. Thus:

$$R_1 + R_2 = \frac{6\text{ V}}{2\times 10^{-3}\text{ A}} = 3\text{ k}\Omega. \tag{3}$$

From (1) and (3): $R_3 = 3\text{ k}\Omega$.

Subtract (2) from (1): $R_2 = 2\text{ k}\Omega$.

From (3): $R_1 = 1\text{ k}\Omega$.

Answers: $\boxed{R_1 = 1.00\text{ k}\Omega, R_2 = 2.00\text{ k}\Omega, R_3 = 3.00\text{ k}\Omega}$.

P28.20

$+15.0 - (7.00)I_1 - (2.00)(5.00) = 0$

$5.00 = 7.00 I_1$ so $\boxed{I_1 = 0.714\text{ A}}$

$I_3 = I_1 + I_2 = 2.00\text{ A}$

$0.714 + I_2 = 2.00$ so $\boxed{I_2 = 1.29\text{ A}}$

$+\varepsilon - 2.00(1.29) - 5.00(2.00) = 0$ $\boxed{\varepsilon = 12.6\text{ V}}$

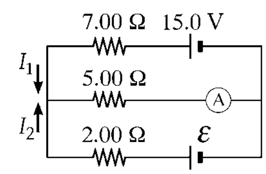

FIG. P28.20

P28.71 (a) After steady-state conditions have been reached, there is no DC current through the capacitor.

Thus, for R_3: $\boxed{I_{R_3} = 0\ (\text{steady-state})}$.

For the other two resistors, the steady-state current is simply determined by the 9.00-V emf across the 12-kΩ and 15-kΩ resistors in series:

For R_1 and R_2:

$$I_{(R_1+R_2)} = \frac{\varepsilon}{R_1 + R_2} = \frac{9.00\text{ V}}{(12.0\text{ k}\Omega + 15.0\text{ k}\Omega)} = \boxed{333\ \mu\text{A}\ (\text{steady-state})}.$$

(b) After the transient currents have ceased, the potential difference across C is the same as the potential difference across $R_2 (= IR_2)$ because there is no voltage drop across R_3. Therefore, the charge Q on C is

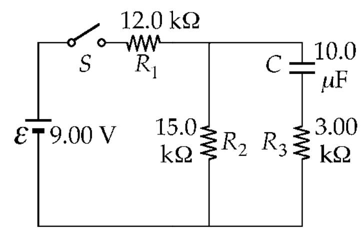

FIG. P28.71(b)

$$Q = C(\Delta V)_{R_2} = C(IR_2) = (10.0\ \mu\text{F})(333\ \mu\text{A})(15.0\ \text{k}\Omega)$$
$$= \boxed{50.0\ \mu\text{C}}.$$

(c) When the switch is opened, the branch containing R_1 is no longer part of the circuit. The capacitor discharges through $(R_2 + R_3)$ with a time constant of $(R_2 + R_3)C = (15.0\ \text{k}\Omega + 3.00\ \text{k}\Omega)(10.0\ \mu\text{F}) = 0.180\ \text{s}$. The initial current I_i in this discharge circuit is determined by the initial potential difference across the capacitor applied to $(R_2 + R_3)$ in series:

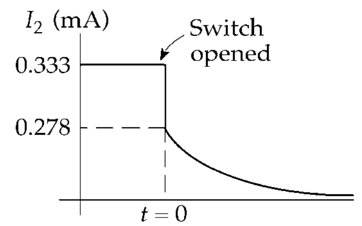

FIG. P28.71(c)

$$I_i = \frac{(\Delta V)_C}{(R_2 + R_3)} = \frac{IR_2}{(R_2 + R_3)} = \frac{(333\ \mu\text{A})(15.0\ \text{k}\Omega)}{(15.0\ \text{k}\Omega + 3.00\ \text{k}\Omega)} = 278\ \mu\text{A}.$$

Thus, when the switch is opened, the current through R_2 changes instantaneously from $333\ \mu\text{A}$ (downward) to $278\ \mu\text{A}$ (downward) as shown in the graph. Thereafter, it decays according to

$$I_{R_2} = I_i e^{-t/(R_2+R_3)C} = \boxed{(278\ \mu\text{A})\,e^{-t/(0.180\ \text{s})}\ \ (\text{for } t > 0)}.$$

(d) The charge q on the capacitor decays from Q_i to $\frac{Q_i}{5}$ according to

$$q = Q_i e^{-t/(R_2+R_3)C}$$
$$\frac{Q_i}{5} = Q_i e^{(-t/0.180\ \text{s})}$$
$$5 = e^{t/0.180\ \text{s}}$$
$$\ln 5 = \frac{t}{180\ \text{ms}}$$
$$t = (0.180\ \text{s})(\ln 5) = \boxed{290\ \text{ms}}$$

Sign convention for the second rule:

- If a resistor R is traversed in the direction of the current; $\Delta V = -IR$. (Current flows from a higher potential to a lower potential)

- If a resistor R is traversed in the direction opposite the current; $\Delta V = IR$.

- If a source of emf (assumed to have no internal resistance) is traverses in from $-$ to $+$; $\Delta V = \varepsilon$.

- If a source of emf (assumed to have no internal resistance) is traverses in from $+$ to $-$; $\Delta V = -\varepsilon$.

- if the battery has an internal resistance r, it has to be treated as a regular resistance in series with the battery; $\Delta V = \varepsilon - Ir$.