Light: particles or waves ?

The particle (photon) model of light and the wave model of light complement each other; some experiments can only be described by the wave model while some others can only be described by the particle model.

The Wave Properties of Particles

Louis de Broglie Postulated: All forms of matter have both wave and particle properties, just like photons do.

We know that for a photon: $p = \frac{h}{\lambda}$

So for any particle: $\lambda = \frac{h}{p} = \frac{h}{\gamma m v}$

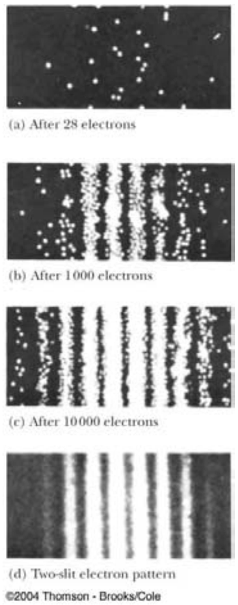
de Broglie wave length

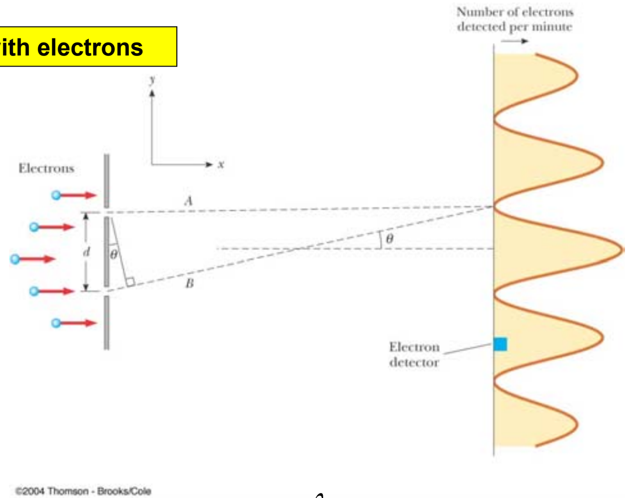
And the frequency of a particle: $f = \frac{E}{h}$

Wave-Particle duality

Soon after de Broglie's proposal, Davidson and Germer showed that electrons show diffraction effects and measured the wave length of electrons !

Double-Slit experiment with electrons





$$d \sin\theta = \frac{\lambda}{2}$$

$$\sin\theta \approx \theta = \frac{h}{2pd}$$

34. Calculate the de Broglie wavelength for an electron that has kinetic energy (a) 50.0 eV and (b) 50.0 keV.

Are these electrons moving at relativistic velocities ?

The rest energy of an electron = $m_e c^2$ = 8.2 x 10^{-14} J

= 8.2 x 10^{-14} / 1.6 x 10^{-19} eV

= 0.511 MeV

So the total energy of the electron in the two given cases:

(a) E= 511000 + 50 eV = 0.51105 MeV

(b) E =511000 + 50000 eV = 0.551 MeV

And γ for the two cases are:

(a) $\gamma = \frac{E}{E_R} = 1.0001$ and (b) $\gamma = \frac{E}{E_R} = 1.08$

34. Calculate the de Broglie wavelength for an electron that has kinetic energy (a) 50.0 eV and (b) 50.0 keV.

(a) $$\frac{p^2}{2m} = (50.0)\left(1.60 \times 10^{-19}\ \text{J}\right)$$

$$p = 3.81 \times 10^{-24}\ \text{kg}\cdot\text{m/s}$$

$$\lambda = \frac{h}{p} = \boxed{0.174\ \text{nm}}$$

About 200 times smaller than the wavelength of light

(b) $$\frac{p^2}{2m} = \left(50.0 \times 10^{3}\right)\left(1.60 \times 10^{-19}\ \text{J}\right)$$

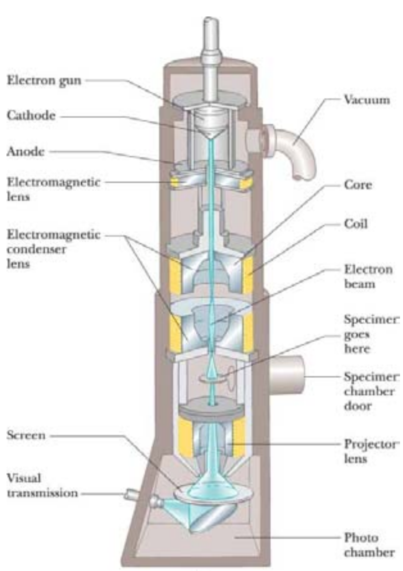
$$p = 1.20 \times 10^{-22}\ \text{kg}\cdot\text{m/s}$$

$$\lambda = \frac{h}{p} = 5.49 \times 10^{-12}\ \text{m}$$

The relativistic answer is slightly more precise:

$$\lambda = \frac{h}{p} = \frac{hc}{\left[\left(mc^2 + K\right)^2 - m^2c^4\right]^{1/2}} = \boxed{5.37 \times 10^{-12}\ \text{m}}$$

Electron Microscope



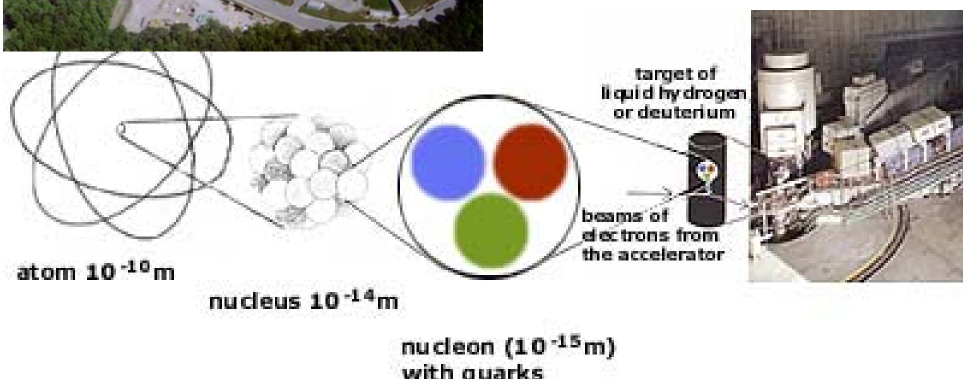
(a)

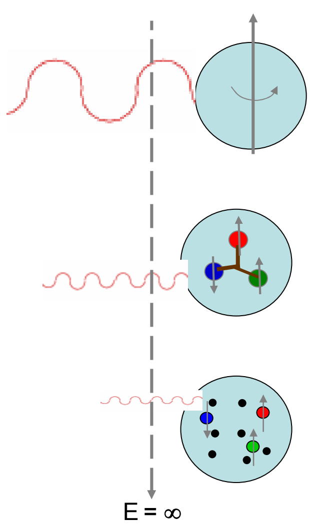
About 100-200 times better resolution than an optical microscope

Bigger (and much more expensive) "Electron Microscope"

pc = 6000 MeV

$$\lambda = \frac{hc}{pc} \approx 7 \times 10^{-17}\, m$$



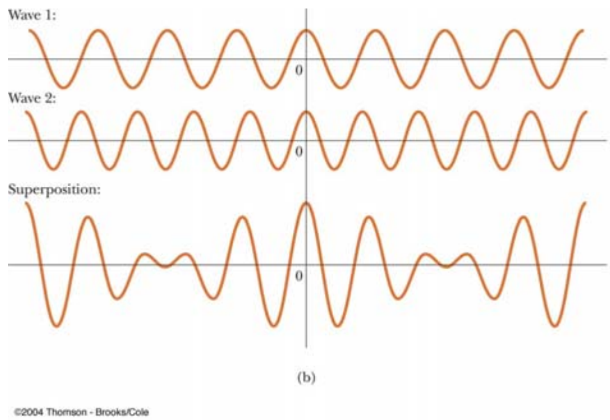


The Quantum Particle: Wave Packet

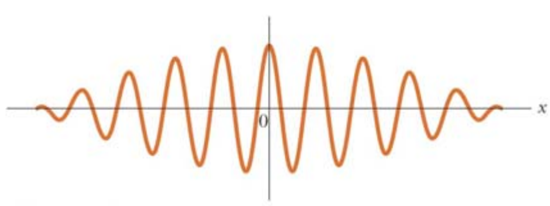
An ideal particle would have zero size and would be localized in space

An ideal wave has a single frequency and is infinitely long.

Consider combining two waves with slightly different frequencies:



When a large number of waves with varying frequencies are combined, there is destructive interference everywhere except near *x=0*



The result is a **wave packet** that is localized in space

The quantum particle is a wave packet.

Uncertainty Principal

Werner Heisenberg used quantum theoretical arguments so show that it is impossible to make simultaneous measurements of a particle's position and momentum with infinite accuracy. This is known as the **Heisenberg Uncertainty Principal**

If a measurement of the position of a particle is made with uncertainty Δx and a simultaneous measurement of its x component of measurement made with uncertainty Δp_x, the product is the two uncertainties can never be smaller than $\hbar/2$

$$\Delta x \Delta p_x \geq \frac{\hbar}{2}$$

$$(\hbar = h / 2\pi)$$