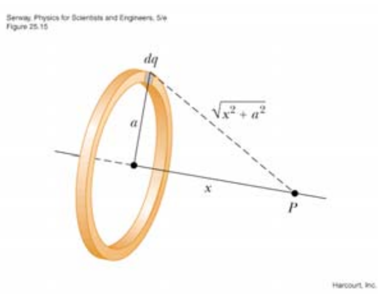
Example 25.5:

- Find the electric potential at a point P located on the perpendicular axis of uniformly charged ring of radius a and total charge Q.

  Let us orient the ring so that the perpendicular axis is along the x direction and point P is at a distance x from the center of the ring.



$$V = k_e \int \frac{dq}{r} \tag{22}$$

$$= k_e \int \frac{dq}{\sqrt{(x^2+a^2)}} \tag{23}$$

x, and a are constants
$$= \frac{k_e}{\sqrt{(x^2+a^2)}} \int dq \tag{24}$$

$$= \frac{k_e Q}{\sqrt{(x^2+a^2)}} \tag{25}$$

Find the electric field at point P

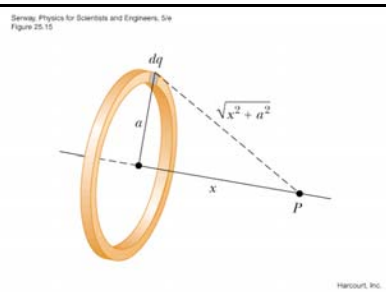
$$\mathbf{E} = -\frac{dV}{dx}\hat{\mathbf{i}} - \frac{dV}{dy}\hat{\mathbf{j}} - \frac{dV}{dz}\hat{\mathbf{j}} \tag{26}$$

$$\frac{dV}{dx} = -\frac{k_e Q x}{(x^2+a^2)^{3/2}} \tag{27}$$

$$\frac{dV}{dy} = 0 \tag{28}$$

$$\frac{dV}{dz} = 0 \tag{29}$$

$$\Rightarrow \mathbf{E} = -\frac{dV}{dx} = \frac{k_e Q x}{(x^2+a^2)^{3/2}}\hat{\mathbf{i}} \tag{30}$$



- Where is the potential maximum ?

  At the maximum:

$$\frac{dV}{dx} = 0 \tag{31}$$

$$-\frac{k_e Q x}{(x^2+a^2)^{3/2}} = 0 \tag{32}$$

$$\Rightarrow x = 0 \tag{33}$$

The potential is maximum at the center of the center of the ring.

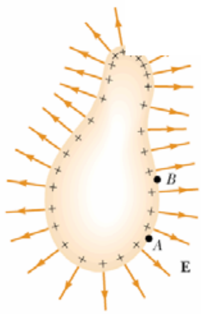
Potential due to a charged conductor

- Potential difference between points A and B on the surface of the conductor:
- Select a path along the conductor. $\Rightarrow \mathbf{E} \perp d\mathbf{s}$

$$V_B - V_A = -\int_A^B \mathbf{E} \cdot d\mathbf{s} = 0 \quad (1)$$

- This works for any two points on the surface.
- $\mathbf{E} = 0$ inside the conductor.
- $V_C - V_A = 0$ for any point inside the conductor too.
- $\Rightarrow$ **Every point of a conductor in equilibrium is at the same electric potential**

Serway, Physics for Scientists and Engineers, 5/e
Figure 25.20



- Electric filed lines from a sharp point of a conductor (small radius of curvature) spreads our quicker than the field lines from an almost flat conductor (large curvature of radius).
- If both surfaces are part of the same conductor, they should both be at the same potential.
- Amount of energy needed to bring a charge from ∞ to each point must be the same.
- $\Rightarrow$ **The electric field near the sharp point has to be higher than the electric filed near a surface with a higher radius of curvature.**
- **Since, $E = \frac{\sigma}{\epsilon_0}$ just outside a conducting surface, the charge density near the sharp point is higher than the charge density near a surface with a higher radius of curvature.**

Example 25.9

Two spherical conductors are separated by a large distance and have the indicated charges. They are connected by a thin conducting wire. Find the magnitudes of the electric fields at the surface of the spheres.

- Since they are connected by a conductor, the two spheres are at the same potential:

$$V = k_e \frac{q_1}{r_1} = k_e \frac{q_2}{r_2} \qquad (2)$$

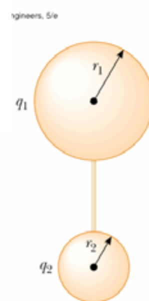
$$\Rightarrow \frac{q_1}{q_2} = \frac{r_1}{r_2} \qquad (3)$$

but,

$$E_1 = k_e \frac{q_1}{r_1^2} \quad ; \quad E_2 = k_e \frac{q_2}{r_2^2} \qquad (4)$$

$$\Rightarrow \frac{E_1}{E_2} = \frac{q_1}{q_2} \frac{r_2^2}{r_1^2} \qquad (5)$$

$$\frac{E_1}{E_2} = \frac{r_2}{r_1} \qquad (6)$$

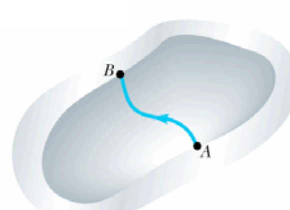


Cavity within a conductor

- for any two points on the surface of the cavity, $V_B - V_A = 0$

$$0 = V_B - V_A = -\int_A^B \mathbf{E} \cdot d\mathbf{s} \qquad (7)$$

- This is to work for every path from A to B.
- $\Rightarrow$ **E** is zero everywhere inside the cavity.
- A closed conducting surface creates a field free region inside.



Problem 25.43:

25.43 (a) $[\alpha] = \left[\frac{\lambda}{x}\right] = \frac{\text{C}}{\text{m}} \cdot \left(\frac{1}{\text{m}}\right) = \boxed{\frac{\text{C}}{\text{m}^2}}$

(b) $V = k_e \int \frac{dq}{r} = k_e \int \frac{\lambda\, dx}{r} = k_e \alpha \int_0^L \frac{x\, dx}{(d+x)} = \boxed{k_e \alpha \left[L - d \ln\left(1 + \frac{L}{d}\right)\right]}$