Gen Packing Fraction and Single Arm Dilution Factor

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The packing fraction of the target, *pf*, is the volume ratio of target material to all material in the target cell. The packing fraction takes into account the fact that target material, in the form of small beads or crystals, does not fill up the entire volume of the target cavity, and therefore scattering from within the cell is not only from the target material. Rather, there is a significant probability that the electron scattered from a helium nucleus, which fills the gaps between target beads. Other materials in the target cell, such as NMR coils, must also be taken into account. If contributions from these other materials within the target cell are known, a true target thickness can be determined.

The method of determining this packing fraction is by comparison of data rates between the polarized target and a target of known thickness. This target of known thickness was a target of solid ^{12}C, specially machined for our target insert. The thickness of this carbon target was 0.6858 cm. This value was chosen so as to make the radiation length of the carbon target (carbon plus helium) roughly the same as a deuteron target of 60% packing fraction. This serves to make radiative corrections similar between the two targets. Scattering rates between our scant few carbon runs were compared to polarized target runs, providing a rate ratio (RR).

$$RR = \frac{Rate_{ND_3}}{Rate_{carbon}} \tag{1}$$

This rate ratio can then be compared to a model of radiated rates from the deuteron target and from the carbon target, with packing fraction left as a variable.

$$RR = \frac{R_D \cdot pf + R_{^{15}N} \cdot pf + R_{tgthe} \cdot (1 - pf) + R_{NMR} + \Sigma R_{unpol}}{R_{^{12}C} + R_{ctgthe} + \Sigma R_{unpol}} \tag{2}$$

This requires a model of cross sections for all the materials present. Also, since the rates obtained from data are naturally radiated, we must either compare these rates to modelled rates that have been both internally and externally radiated, or we must determine radiative correction factors to transform the data rates to Born rates.

Model

In order to obtain cross sections for various nuclei, the O'Connell & Lightbody code, QFS, was used. QFS models the (e,e') cross sections, building the total cross section from five individual reaction mechanisms, quasielastic scattering, scattering from two interacting nucleons, delta production, higher resonances, and deep inelastic scattering [1]. It is intended to predict cross sections within 20% over a range of 0.5 to 5.0 GeV incident electrons. Comparing Born cross sections produced by QFS to data from previous experiments at similar kinematics to our experiment shows that QFS is fairly accurate for nuclei with A $\geq$ 4.

The Born cross sections provided by QFS are used as input into `external_gen.f`, which generates internal radiative corrections in the plane-wave approximation and full external radiative corrections. The output of `external_gen.f` is a data file containing Born, internal, and external cross sections for each material in the target composition file, at each energy in the ruplan file. The target composition file contains the list of all materials traversed by the beam as it travels through the target together with the thickness, density, concentration, mass, and atomic number for each material. A sample target composition file can be found in Appendix A. The runplan file contains the nominal incident beam energy, electron scattering angle, and a range of scattered electron energy values that covers our range of interest.

The output of `external_gen.f` is then compiled by corfac.f, which takes each cross section at each scattered energy and weights it by the material's relative density per molecular weight. This gives the total radiated rate at each scattered energy. This radiated rate is then compared to the Born rate at each scattered energy to give the correction factor at that energy. This is done for both ammonia target and carbon target. The resultant radiative correction is then simply,

$$R_{corr} = \frac{\frac{R_{tgt_{unrad}}}{R_{tgt_{rad}}}}{\frac{R_{c12_{unrad}}}{R_{c12_{rad}}}} \tag{3}$$

which can be used to convert the data rate ratio to a rate ratio of Born rates.

$$RR_c = R_{corr} \cdot RR \tag{4}$$

Calculating Packing Fraction

Armed with equations (4) and using QFS to generate Born cross sections for each target material, we can solve for pf in equation (1).

$$pf = \frac{RR_c(R_c + R_{ctgthe} + R_{hex} + R_{Al}) - (R_{tgthe} + R_{hex} + R_{Al} + R_{Cu} + R_{Ni})}{R_D + R_{^{15}N} - R_{tgthe}} \tag{5}$$

The subscript "hex" denotes the helium outside the target cavity that the beam must travel through. Copper and Nickel are included because the NMR coils were CuNi tubing. The rates of each of these materials is assumed to be the product of the Born cross section and the number density, N. It was assumed that any flux factor involved in computing an actual rate would cancel from both numerator and denominator. Each number density was computed as follows:

$$N = conc \cdot \frac{\rho}{MW} \cdot N_A \cdot length \tag{6}$$

"conc" is the relative concentration of the material, which was 1.0 for everything with the exception of deuteron which was 3.0 (ignoring impurities), since there are 3 deuterons for every nitrogen in $^{15}ND_3$.

Material	Ratio of N_X to N_D
Deuterium	1.0
Nitrogen	0.3333
Carbon	0.2860
Target Helium	0.2403
External Helium	0.0801
Aluminum	0.0146
Copper	0.0031
Nickel	0.0014

Table 1: Relative Number Densities for Materials in the Target Composition File

A value for packing fraction at each scattered energy can then be calculated. Since the packing fraction is a physical quantity describing the volume occupied by the target material, it should be constant across E'. Therefore, it is reasonable to average the values from each energy bin, provided the values are roughly constant, to obtain a more precise value for packing fraction.

The data analyzed to date in this effort has been limited. Only a few carbon runs exist for Q^2=0.5 data. Using run 22279, a carbon run on the dummy (large radius) target, with both rasters on, and comparing to target runs 22282-22284, I obtain a value for packing fraction between 0.53 and 0.54 with a relative error of 2%. Data from our Q^2=1.0 kinematics has not yet been analyzed.

Dilution Factor

If packing fraction is known, the single arm dilution factor is readily calculable. The dilution factor is the ratio of rates from polarized materials in the target to rates from all materials (polarized and unpolarized) in the target.

$$f = \frac{R_D \cdot pf \cdot R_{corr2}}{R_D \cdot pf + R_{^{15}N} \cdot pf + R_{tgthe}(1-pf) + R_{Al} + R_{Cu} + R_{Ni}} \tag{7}$$

This new radiative correction factor is similar to R_{corr} but it transforms the deuteron-only to whole target ratio from Born to radiated, so that it may be applied directly to data rates. Using the given packing fraction of approximately 0.535, the dilution factor roughly takes the shape of the deuterium cross section across the scattered energy range, peaking at 0.37 at the quasielastic peak. This value will not change much with small changes to the packing fraction.

Work Outstanding

QFS is limited in its ability to model deuteron cross sections. We are also hampered by a lack of deuteron data in our kinematic region with which to compare to the output of QFS. Several errors were found in the original QFS code, which have been corrected, but it is felt that more errors may exist, yet to be found. While QFS seems to model materials of higher atomic number quite well, deuteron is the most important. Plots of the modelled rate versus the data rate shows a clear difference around the quasielastic peak. If the strength of the deuteron signal is simply shifted around, then this won't impact our values for packing fraction much since the averaging across the energy range should take care of it.

A mass-production of rate ratios needs to be performed to check the packing fraction for each run (or at least a good sampling of all runs) to check for any time correlation effects, such as settling of the target material. Packing fraction can also be calculated from rate ratios of polarized target to empty (helium) target. This might be a good check on our carbon runs, although no empty runs were taken with a large diameter target cell. However, since we have so few carbon runs, it would be nice to have some check on those results.

Double-arm dilution factor needs to be computed. This requires code that can generate double-arm cross sections and radiated cross sections (for radiative corrections). Once cross sections and radiative corrections are obtained for the double-arm results, the same formula for dilution factor can be used.

References

1. J.W. Lightbody, Jr. & J.S. O'Connell, Computers in Physics May/June 1997 57-64

Appendix A

Target Composition File: ^{15}ND$_3$

	material	thick	unit	density	X0	conc.	flag	Z	A	MW	ZCOR
1.	beam exit	0.0150	in	1.850	65.190	1.00	w	4	9	9.012	0.00
2.	air gap	3.0000	cm	.00129	36.660	1.000	w	7	14	14.421	0.00
3.	OVC entr	0.0080	in	2.700	24.011	1.000	w	13	27	26.982	0.20
4.	LN2 can	0.0015	in	2.700	24.011	1.000	w	13	27	26.982	0.30
5.	4K shield	0.0015	in	2.700	24.011	1.000	w	13	27	26.982	0.70
6.	tailpiece	0.0020	in	2.700	24.011	1.000	w	13	27	26.982	0.90
7.	LHe	0.5000	cm	0.145	94.322	1.000	w	2	4	4.003	1.00
8.	endcap entr	0.0015	in	2.700	24.011	1.000	w	13	27	26.982	1.00
9.	tgt 15N	3.0000	cm	0.917	43.255	0.010	t	7	15	15.000	1.00
10.	tgt H3	3.0000	cm	0.917	43.255	0.010	t	1	1	1.008	1.00
11.	tgt 14N	3.0000	cm	0.867	40.862	0.000	t	7	14	14.003	1.00
12.	tgt H3	3.0000	cm	0.867	40.862	0.000	t	1	1	1.008	1.00
13.	tgt 15N	3.0000	cm	1.056	50.500	0.970	t	7	15	15.000	1.00
14.	tgt D3	3.0000	cm	1.056	50.500	2.970	t	1	2	2.014	1.00
15.	tgt 14N	3.0000	cm	1.056	48.107	0.020	t	7	14	14.003	1.00
16.	tgt D3	3.0000	cm	1.056	48.107	0.020	t	1	2	2.014	1.00
17.	tgt He	3.0000	cm	0.145	94.322	1.000	t	2	4	4.003	1.00
18.	NMR Al	0.0000	cm	2.700	24.011	1.000	w	13	27	26.982	1.00
19.	NMR Cu	0.01008	cm	8.960	12.860	1.000	w	29	64	63.546	1.00
20.	NMR Ni	0.00433	cm	8.760	12.680	1.000	w	28	59	58.693	1.00
21.	endcap exit	0.0015	in	2.700	24.011	1.000	w	13	27	26.982	1.00
22.	LHe	0.5000	cm	0.145	94.322	1.000	w	2	4	4.003	1.00
23.	tailpiece	0.0020	in	2.700	24.011	1.000	w	13	27	26.982	0.90
24.	4K shield	0.0015	in	2.700	24.011	1.000	w	13	27	26.982	0.70
25.	LN2 can	0.0015	in	2.700	24.011	1.000	w	13	27	26.982	0.30
26.	OVC entr	0.0080	in	2.700	24.011	1.000	w	13	27	26.982	0.20
27.	air gap	3.0000	cm	.00129	36.660	1.000	w	7	14	14.421	0.00
28.	beam exit	0.0150	in	1.850	65.190	1.00	w	4	9	9.012	0.00

Table 2: "X0" is the radiation length in units of gm/cm^2. "flag" indicates whether the material is a target material (which must be corrected by packing fraction) or a wall component. "ZCOR" is a z-correction factor (yet to be implemented) that takes into account that as you move away from z=0, the liklihood of a scattering event being detected by the HMS diminishes. The current ZCOR numbers are only a rough estimate.

Similar files have been created for $^{15}NH_3$ and ^{12}C.

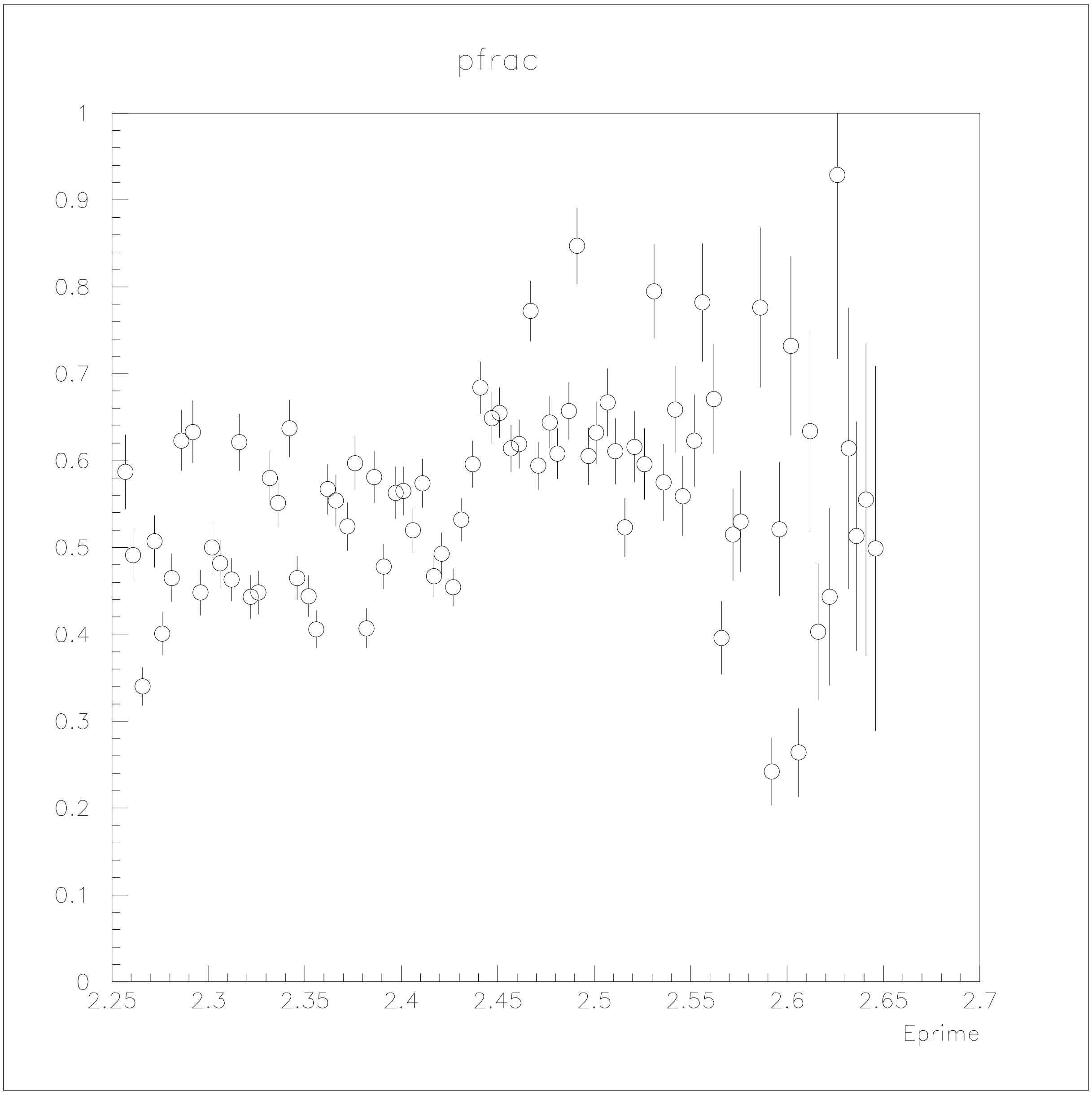

Figure 1: Packing Fraction for $^{15}ND_3$. This plot looks like crap right now because of the mismatch between model and data rates for ND_3. The scatter is due to bin size of roughly 2 MeV (which was used for diagnostics since bin size of 15 MeV hides much of the fluctuation of the packing fraction). A revamped QFS that accurately models the deuteron should clean this up.

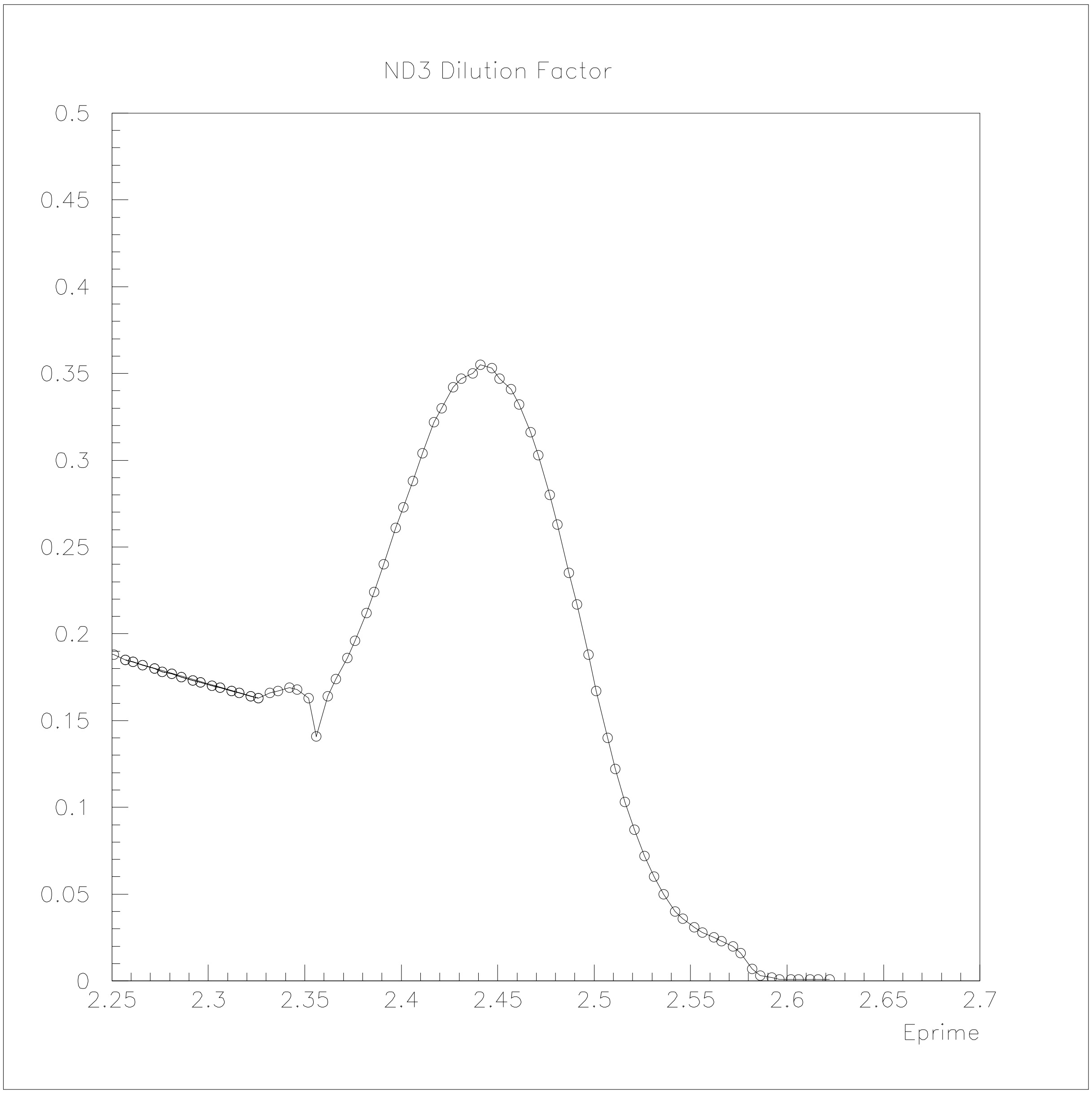

Figure 2: Single Arm Dilution Factor for $^{15}ND_3$. Disjoints in plot are caused by flaws in the QFS model program and will be fixed up shortly.

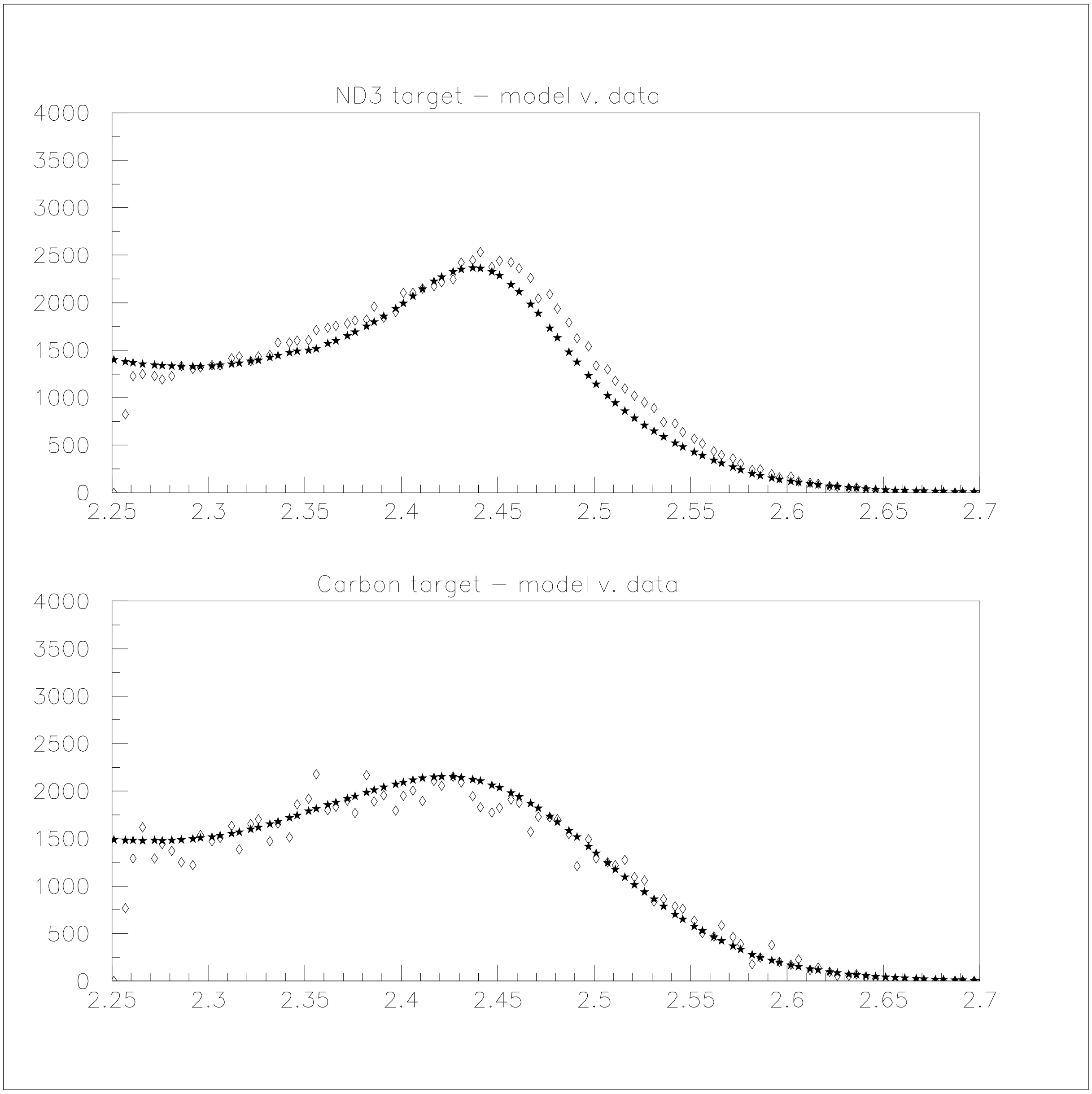

Figure 3: Comparison of rates between model and data. The model for the carbon target fits very well to the data from carbon run 22279. However, the deuteron target does not fit as well. Since the carbon fit is good, it is believed that the error in the deuteron model is in the modelling of the deuteron rates, rather than from poor accounting of background contributions.