A Survey of the Rare Pion and Muon decays

Maxim A. Bychkov
University of Virginia

- About Us and Our Physics Goals
- Apparatus and Method
- Results

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Current PIBETA collaboration members:

V. A. Baranov,[c] W. Bertl,[b] M. Bychkov,[a] Yu.M. Bystritsky,[c] E. Frlež,[a]
N.V. Khomutov,[c] A.S. Korenchenko,[c] M. Korolija,[f] T. Kozlowski,[d]
N.P. Kravchuk,[c] N.A. Kuchinsky,[c] D. Mzhavia,[c,e] A. Palladino,[a]
D. Počanić,[a] P. Robmann,[g] O.A. Rondon-Aramayo,[a]
A.M. Rozhdestvensky,[c] T. Sakhelashvili,[b] S. Scheu,[g] V.V. Sidorkin,[c]
L.C. Smith,[a] U. Straumann,[g] I. Supek,[f] Z. Tsamalaidze,[e]
A. van der Schaaf,[g] B. A. VanDevender,[a] E.P. Velicheva,[c]
V.V. Volnykh,[c] and Y. Wang[a]

[a] *Dept of Physics, Univ of Virginia, Charlottesville, USA*

[b] *Paul Scherrer Institut, Villigen PSI, Switzerland*

[c] *Joint Institute for Nuclear Research, Dubna, Russia*

[d] *Institute for Nuclear Studies, Swierk, Poland*

[e] *IHEP, Tbilisi, State University, Tbilisi, Georgia*

[f] *Rudjer Bošković Institute, Zagreb, Croatia*

[g] *University of Zürich, Zürich, Switzerland*

Physics Goals

We study rare decays of pions and muons:

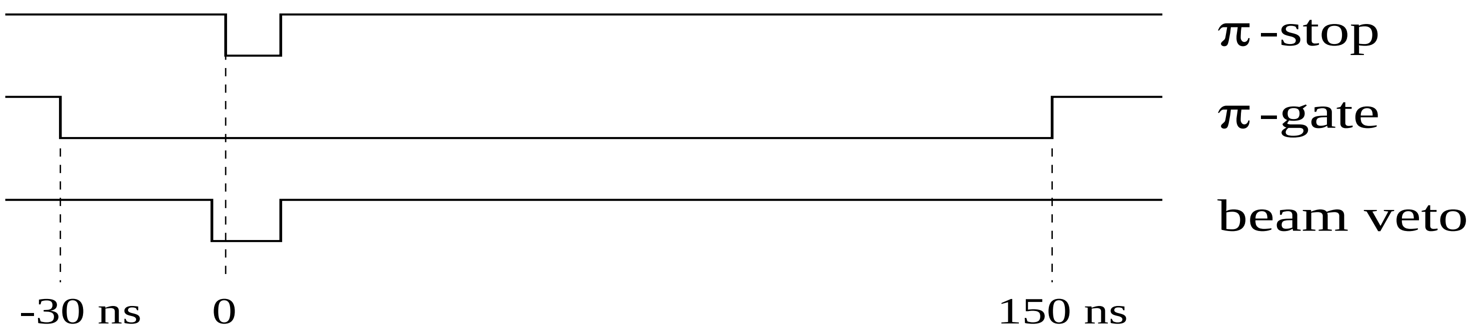
Mode	Fraction$(\Gamma_j/\Gamma)\%$	Subsequently
$\pi^+ \to \mu^+\nu_\mu$	(99.98770 ± 0.00004)	$\mu^+ \to e^+\overline{\nu}_e\nu_\mu$ $(\approx 100\%)$
		$\mu^+ \to e^+\overline{\nu}_e\nu_\mu\gamma \sim 10^{-3}(k)$
$\pi^+ \to \mu^+\nu_\mu\gamma$	$\sim 10^{-4}$ (k)	
$\pi^+ \to e^+\nu_e$	$(1.230 \pm 0.004) \times 10^{-4}$	
$\pi^+ \to e^+\nu_e\gamma$	$\sim 10^{-7}$ (k)	
$\pi^+ \to \pi^0 e^+\nu_e$	$(1.036 \pm 0.006) \times 10^{-8}$	$\pi^0 \to 2\gamma$ (98.8%)
		$\pi^0 \to e^+e^-\gamma$ (1.2%)
$\pi^+ \to e^+\nu_e e^+e^-$	$(3.2 \pm 0.5) \times 10^{-9}$	

Physics Goals

- Study β decay of the pion $\pi^+ \to \pi^0 e^+ \nu_e$
  - SM check related to CKM unitarity
- Study pion radiative decay $\pi^+ \to e^+ \nu_e \gamma$
  - Structure of the pion, check of the CVC hypothesis, deviations from $V - A$ form of $\mathcal{L}_{\rm weak}$
- Study muon radiative decay $\mu^+ \to e^+ \nu \overline{\nu} \gamma$
  - Precise test of the weak interaction, deviations from $V - A$ form of $\mathcal{L}_{\rm weak}$
- Study nonradiative decay $\pi^+ \to e^+ \nu_e$
  - electron-muon universality
  - pseudoscaler coupling in $\mathcal{L}_{\rm weak}$
  - massive neutrino

Experimental Method

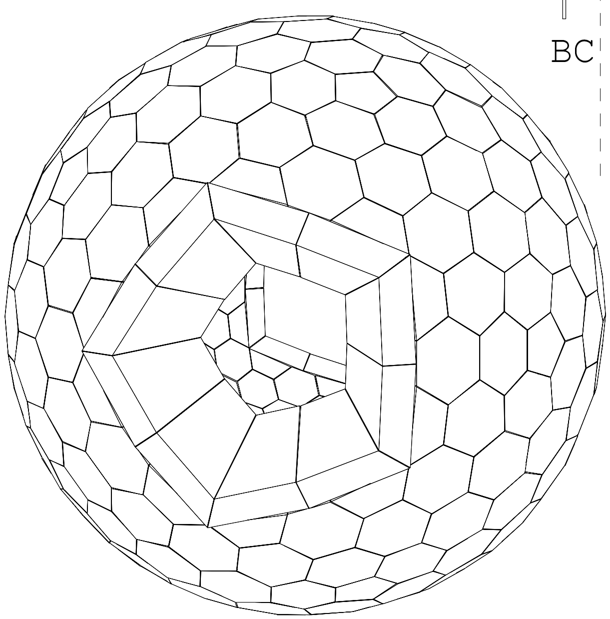
- π^+s are stopped in the active target and subsequently decay at rest in the lab.
- The beam veto suppresses events resulting from strong interactions in the degrader. It lasts 10 ns and is initiated by a π-stop signal.
- π-stop signal opens a 180 ns wide π-gate during which all events are collected.

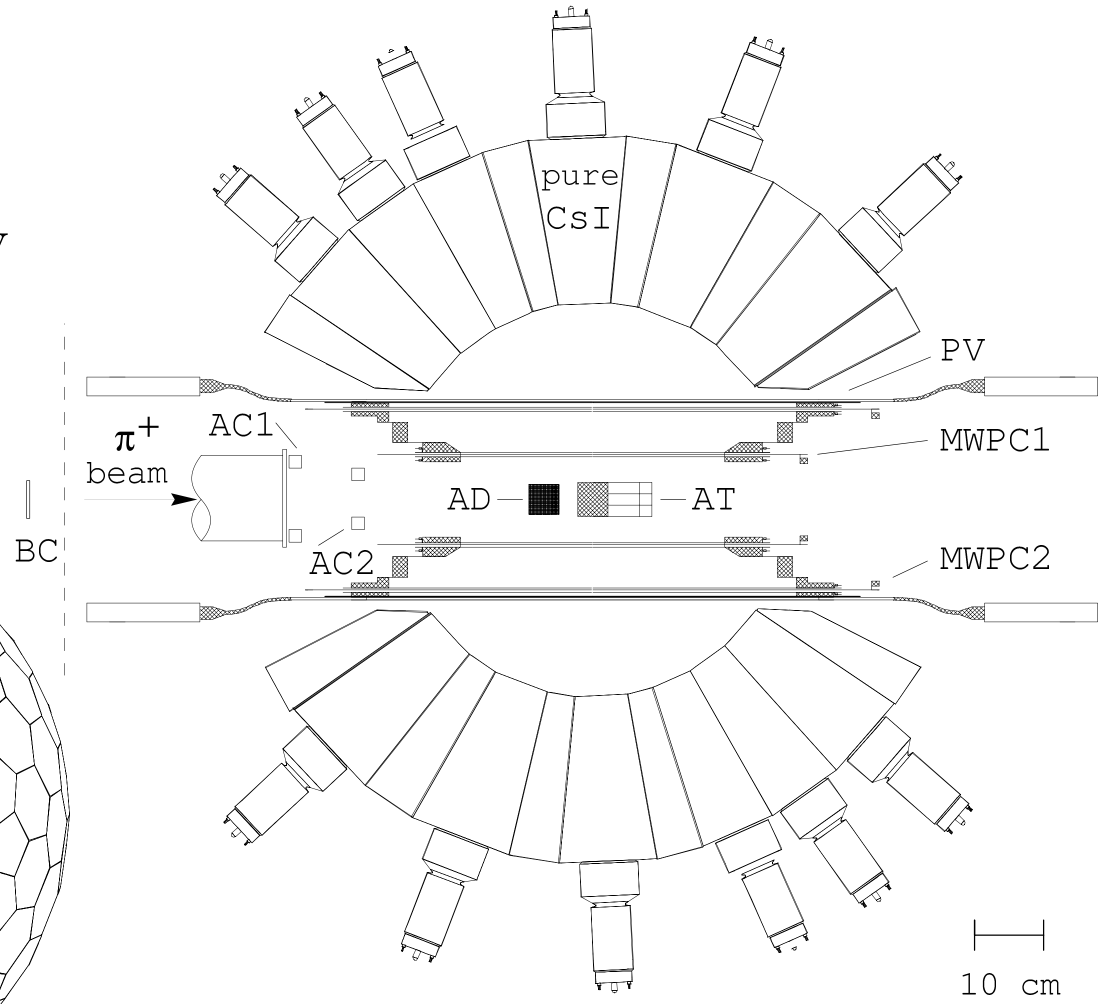


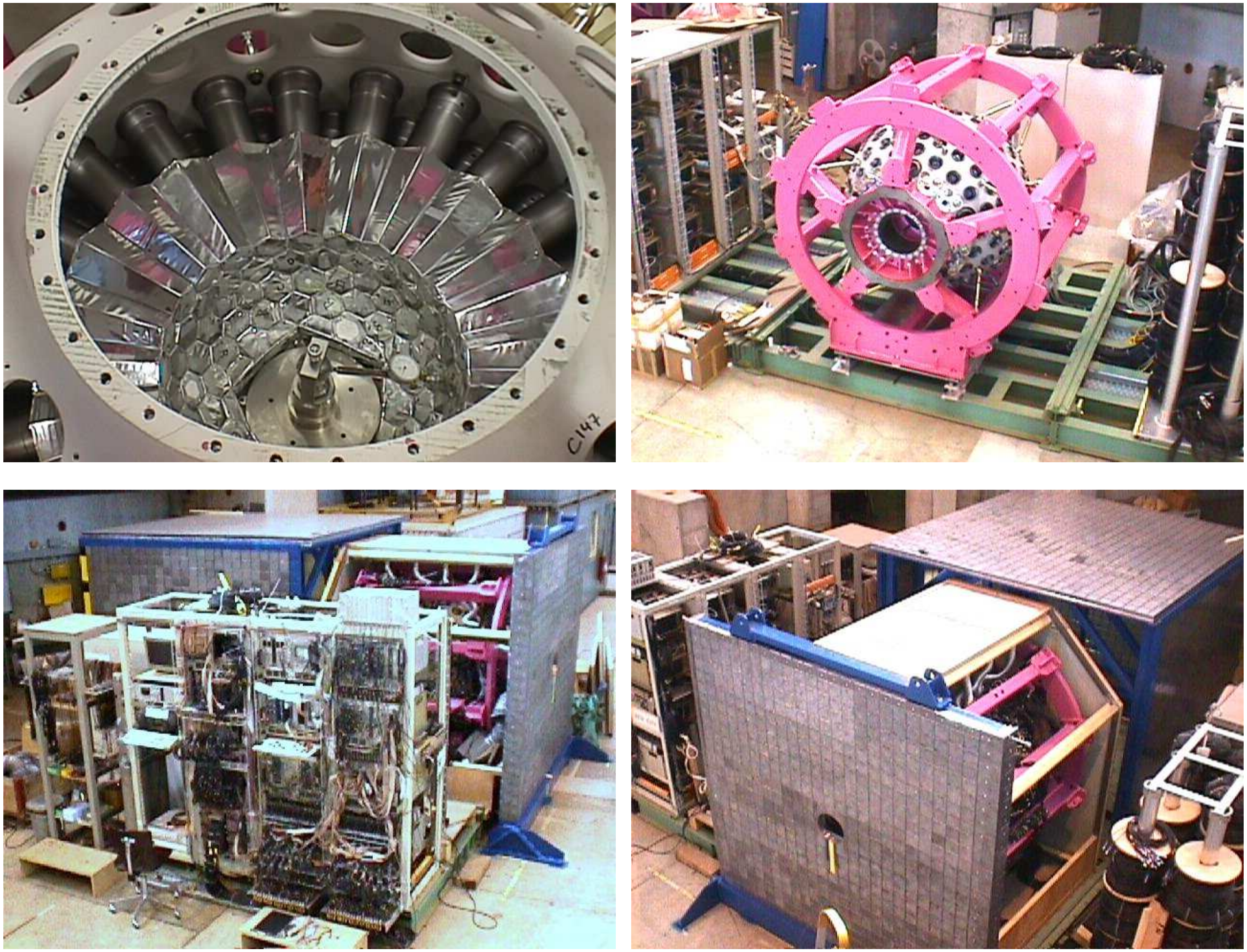
- All pion decays normalized to $\pi^+ \to e^+\nu_e$ decays and muon decays to $\mu^+ \to e^+\nu\bar{\nu}$.

The PIBETA Apparatus:

- ∘ stopped π^+ beam
- ∘ segmented active tgt.
- ∘ 240-det. CsI(p) calo.
- ∘ central tracking
- ∘ digitized PMT signals
- ∘ stable temp./humidity
- ∘ cosmic μ antihouse



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Data Analysis: Method

In order to reduce the systematic uncertainties we use $\pi^+ \rightarrow \mathrm{e}^+ \nu_\mathrm{e}$ $(\pi_{\mathrm{e}2})$ decay for normalization:

$$\Gamma^{\mathrm{exp}}_{\mathrm{decay}} = \Gamma_{\pi_{\mathrm{e}2}} \cdot \frac{A_{\pi_{\mathrm{e}2}} \cdot N_{\mathrm{decay}}}{N_{\pi_{\mathrm{e}2}} \cdot A_{\mathrm{decay}}}$$

$\Gamma_{\pi_{\mathrm{e}2}}$ is branching ratio of $\pi^+ \rightarrow \mathrm{e}^+ \nu_\mathrm{e}$ decay

N_{decay} is the number of events detected for a given decay

A_{decay} is the acceptance for the same decay

Pion Decay: $\pi^+ \longrightarrow \mathrm{e}^+\nu_\mathrm{e}$ $(\pi_{\mathrm{e}2})$

Available Results: $\pi^+ \to e^+\nu_e$ Decay

Marciano and Sirlin, [PRL **71** (1993) 3629]:

$$\frac{\Gamma(\pi \to e\bar{\nu}(\gamma))}{\Gamma(\pi \to \mu\bar{\nu}(\gamma))}_{\text{calc}} = (1.2352 \pm 0.0005) \times 10^{-4}$$

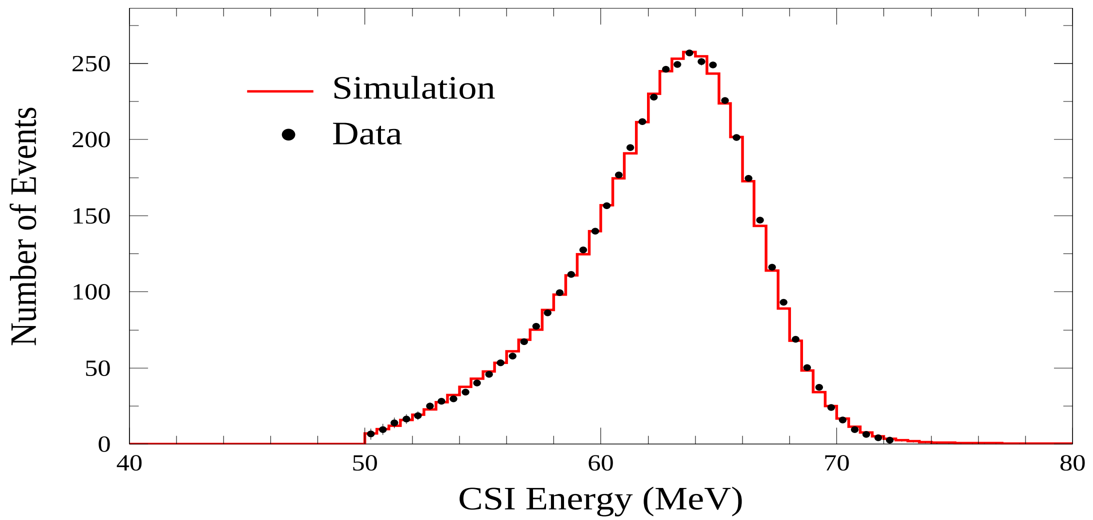
Decker and Finkemeier, [NP B **438** (1995) 17]:

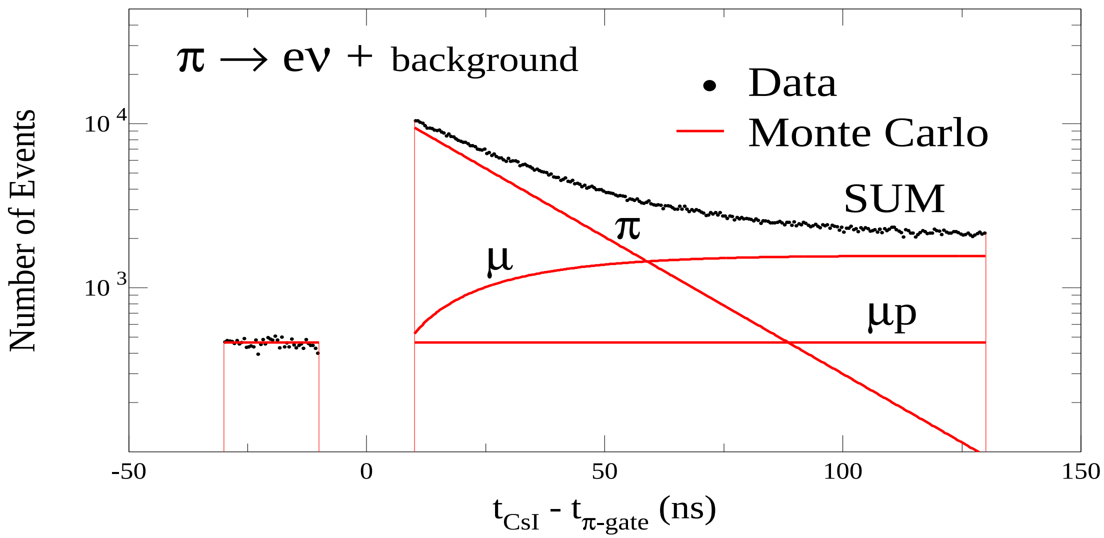
$$\frac{\Gamma(\pi \to e\bar{\nu}(\gamma))}{\Gamma(\pi \to \mu\bar{\nu}(\gamma))}_{\text{calc}} = (1.2356 \pm 0.0001) \times 10^{-4}$$

Experiment, world average (PDG 2006):

$$\frac{\Gamma(\pi \to e\bar{\nu}(\gamma))}{\Gamma(\pi \to \mu\bar{\nu}(\gamma))}_{\text{exp}} = (1.230 \pm 0.004) \times 10^{-4}$$

Data Analysis: $\pi^+ \rightarrow e^+\nu_e$





The Pion Beta Decay: $\pi^+ \rightarrow \pi^0 \mathrm{e}^+ \nu_\mathrm{e}$ **(**π_β **)**

Theoretical Description: The Pion Beta Decay

$\pi^{\pm} \to \pi^{0} e^{\pm} \nu$:   BR $\simeq 1 \times 10^{-8}$,   pure vector trans.: $0^{-} \to 0^{-}$.

Theoretical decay rate at tree level:

$$\frac{1}{\tau_0} = \frac{G_F^2 |V_{ud}|^2}{30\pi^3} \left(1 - \frac{\Delta}{2M_+}\right)^3 \Delta^5 f(\epsilon, \Delta)$$

$$= 0.40692\,(22) |V_{ud}|^2 \, (\mathrm{s}^{-1}) \ .$$

With radiative and loop corrections: $\dfrac{1}{\tau} = \dfrac{1}{\tau_0}(1+\delta)$, so that the branching ratio becomes:

$$BR(\pi\beta) = \frac{\tau_+}{\tau_0}(1+\delta) = 1.0593\,(6) \times 10^{-8} (1+\delta) |V_{ud}|^2 \ .$$

Experimental State of Things

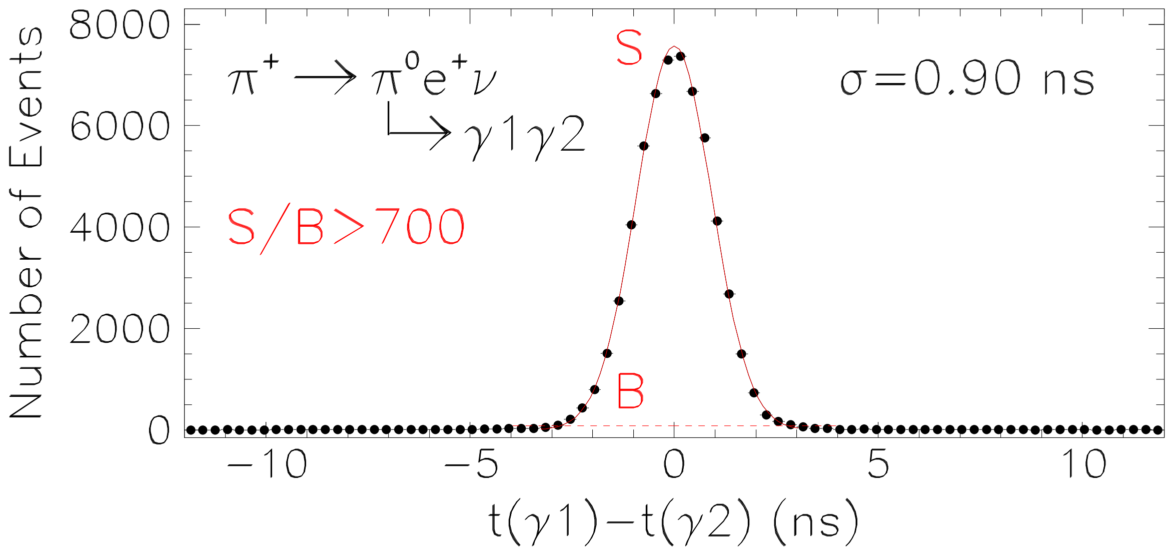
Best result until recently: [McFarlane et al., PRD 32 (1985) 547.]

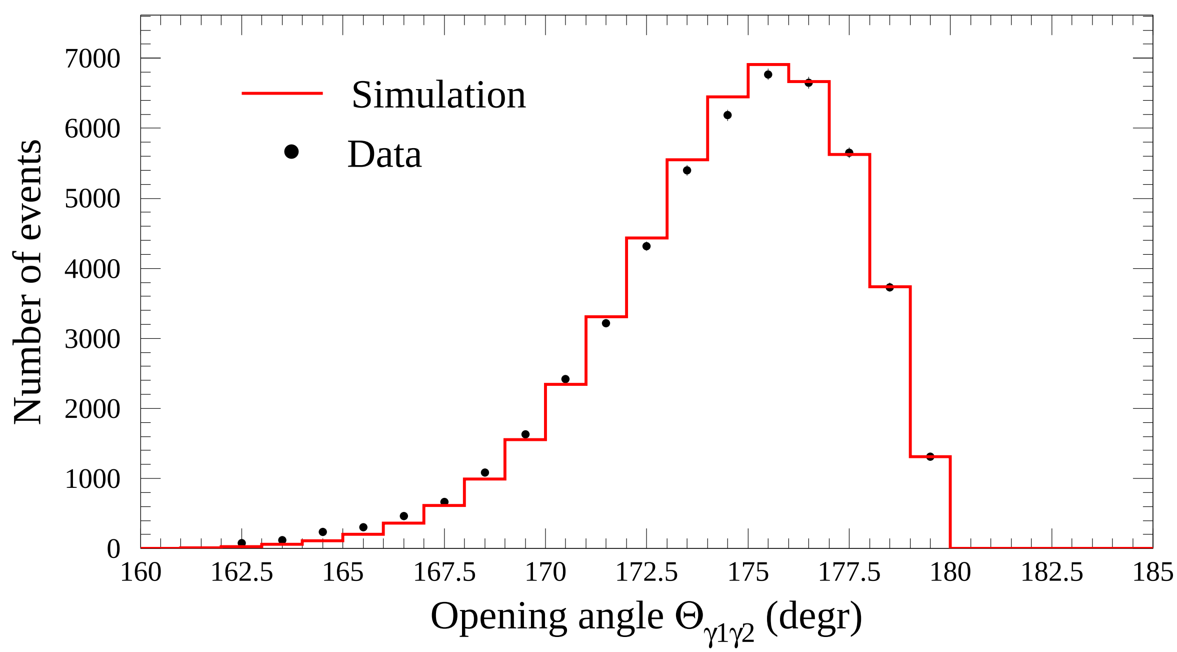
$$BR(\pi^+ \to \pi^0 e^+ \nu) = (1.026 \pm 0.039) \times 10^{-8}, \text{ (i.e., } \sim 4\,\%)$$

Accuracy: $\leq 1\,\%$	check CVC and rad. corrections
$\sim 0.5\,\%$	add to SAF & n_β input to V_{ud}
$< 0.3\,\%$	check for failure of CKM unitarity:

Extracting $|V_{ud}|$ from pion β decay is very lucrative.

The $\pi^+ \rightarrow \pi^0 e^+ \nu_e$ decay





PIBETA Current Result for $\pi^+ \to \pi^0 e^+ \nu_e$ Decay [PRL **93**, 181803 (2004)]

$$B_{\pi\beta}^{\rm exp} = [1.040 \pm 0.004\,({\rm stat}) \pm 0.004\,({\rm syst})] \times 10^{-8}\,,$$

$$B_{\pi\beta}^{\rm exp} = [1.036 \pm 0.004\,({\rm stat}) \pm 0.004\,({\rm syst}) \pm 0.003\,(\pi_{\rm e2})] \times 10^{-8}\,,$$

McFarlane et al. [PRD 1985]: $B = (1.026 \pm 0.039) \times 10^{-8}$

SM Prediction (PDG, 2004):

$$B = \quad 1.038 - 1.041 \times 10^{-8} \qquad (90\%\ {\rm C.L.})$$

$$(1.005 - 1.007 \times 10^{-8} \qquad \text{excl. rad. corr.})$$

PDG 2004: $V_{ud} = 0.9738(5)$

PIBETA current: $V_{ud} = 0.9748(25)$ or $V_{ud} = 0.9728(30)$.

Radiative Pion Decay: $\pi^+ \rightarrow e^+ \nu_e \gamma$ ($\pi_{e2\gamma}$)

Theoretical Description: $\pi^+ \to e^+\nu_e\gamma$ Decay

$$\frac{d^2\Gamma^{\mathrm{theor}}}{dxdy} = \frac{d^2\Gamma_{IB}}{dxdy} + \frac{d^2\Gamma_{SD}}{dxdy} + \frac{d^2\Gamma_{\mathrm{int}}}{dxdy} = \frac{\alpha}{2\pi}\boxed{\Gamma_{\pi\to e\nu}}\Bigg\{ IB(x,y) +$$

$$\frac{m_\pi^2}{4\boxed{m_e^2}}\left(\frac{F_V}{f_\pi}\right)^2 \left[(1+\gamma)^2 SD^+(x,y) + (1-\gamma)^2 SD^-(x,y)\right] +$$

$$\frac{F_V}{f_\pi}\left[(1+\gamma) SD^+_{\mathrm{int}}(x,y) + (1-\gamma) SD^-_{\mathrm{int}}(x,y)\right]\Bigg\} \ ,$$

where IB, $SD^{\pm}$, $SD^{\pm}_{\mathrm{int}}$ are analytical functions of $x \equiv 2E_\gamma/m_{\pi^+}$ and $y \equiv 2E_e/m_{\pi^+}$

$F_A(a) = F_A(0)(1 + a\cdot(1-x)^2)$ is the axial-vector form factor

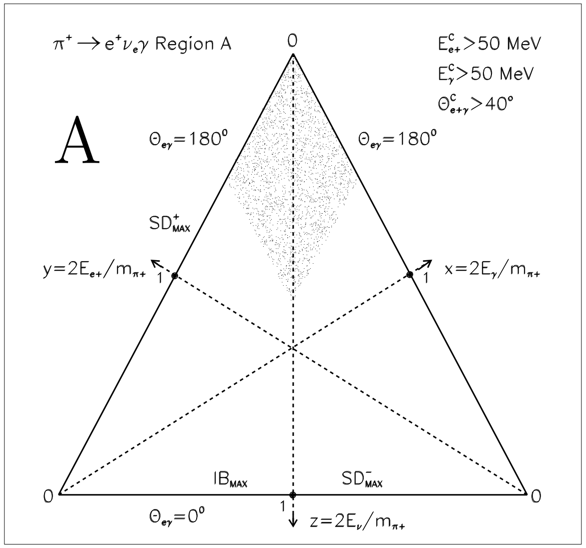
$F_V(a) = F_V(0)(1 + a\cdot(1-x)^2)$ is the vector form factor

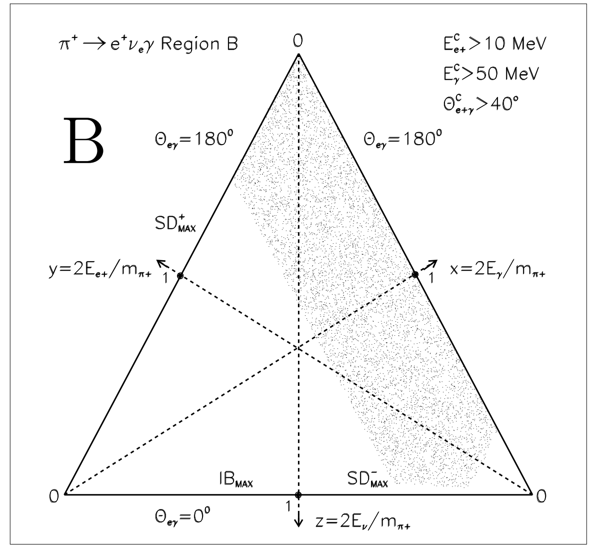
$\gamma \ = \ F_A/F_V \sim$ pion polarizability α_E

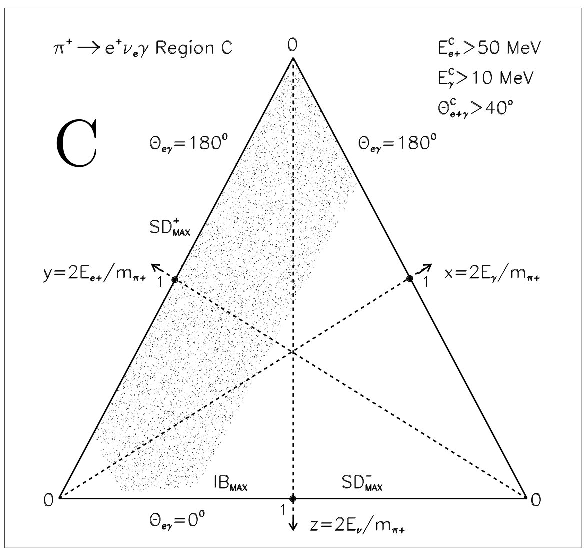
Available Data on Pion Form Factors

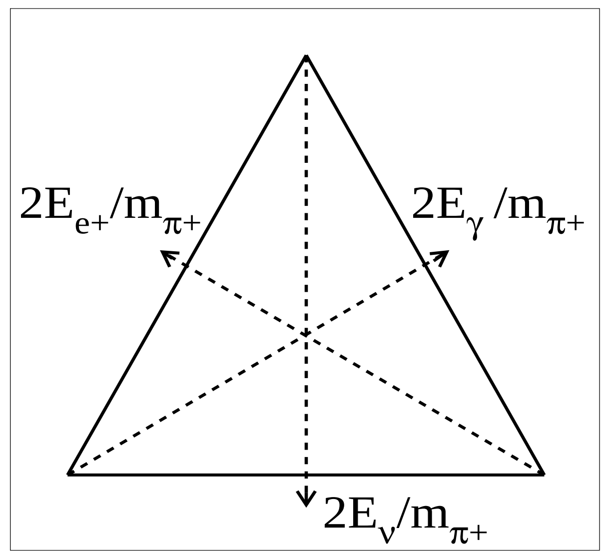
$$|F_V| \stackrel{\text{cvc}}{=} \frac{1}{\alpha}\sqrt{\frac{2\hbar}{\pi\tau_{\pi^0}m_{\pi^0}}} = 0.0259(5)\ .$$

$F_A \times 10^4$	reference	
106 ± 60	Bolotov et al. (1990)	●
135 ± 16	Bay et al. (1986)	
60 ± 30	Piilonen et al. (1986)	
110 ± 30	Stetz et al. (1979)	
116 ± 16	world average (PDG 2002)	


$\pi^+ \rightarrow e^+\nu_e\gamma$ Region A
E^c_{e+}>50 MeV
E^c_γ>50 MeV
$\Theta^c_{e+\gamma}$>40°
A
$\Theta_{e\gamma}$=180°
$\Theta_{e\gamma}$=180°
SD^+_{MAX}
$y=2E_{e+}/m_{\pi+}$
$x=2E_\gamma/m_{\pi+}$
IB_{MAX}
SD^-_{MAX}
$\Theta_{e\gamma}$=0°
$z=2E_\nu/m_{\pi+}$


$\pi^+ \rightarrow e^+\nu_e\gamma$ Region B
E^c_{e+}>10 MeV
E^c_γ>50 MeV
$\Theta^c_{e+\gamma}$>40°
B
$\Theta_{e\gamma}$=180°
$\Theta_{e\gamma}$=180°
SD^+_{MAX}
$y=2E_{e+}/m_{\pi+}$
$x=2E_\gamma/m_{\pi+}$
IB_{MAX}
SD^-_{MAX}
$\Theta_{e\gamma}$=0°
$z=2E_\nu/m_{\pi+}$


$\pi^+ \rightarrow e^+\nu_e\gamma$ Region C
E^c_{e+}>50 MeV
E^c_γ>10 MeV
$\Theta^c_{e+\gamma}$>40°
C
$\Theta_{e\gamma}$=180°
$\Theta_{e\gamma}$=180°
SD^+_{MAX}
$y=2E_{e+}/m_{\pi+}$
$x=2E_\gamma/m_{\pi+}$
IB_{MAX}
SD^-_{MAX}
$\Theta_{e\gamma}$=0°
$z=2E_\nu/m_{\pi+}$


$2E_{e+}/m_{\pi+}$
$2E_\gamma/m_{\pi+}$
$2E_\nu/m_{\pi+}$

$\pi^+ \rightarrow e^+\nu_e\gamma$: Data analyses 1999-2001

Results of the SM fit

[Phys. Rev. Lett. **93**, 181804 (2004)]

Best-fit $\pi \to e\nu\gamma$ branching ratios obtained with:
$F_V = 0.0259$ (fixed) and $F_A = 0.0115(4)$ (fit)
$\chi^2/\text{d.o.f.} = 25.4$.
Radiative corrections are included in the calculations.

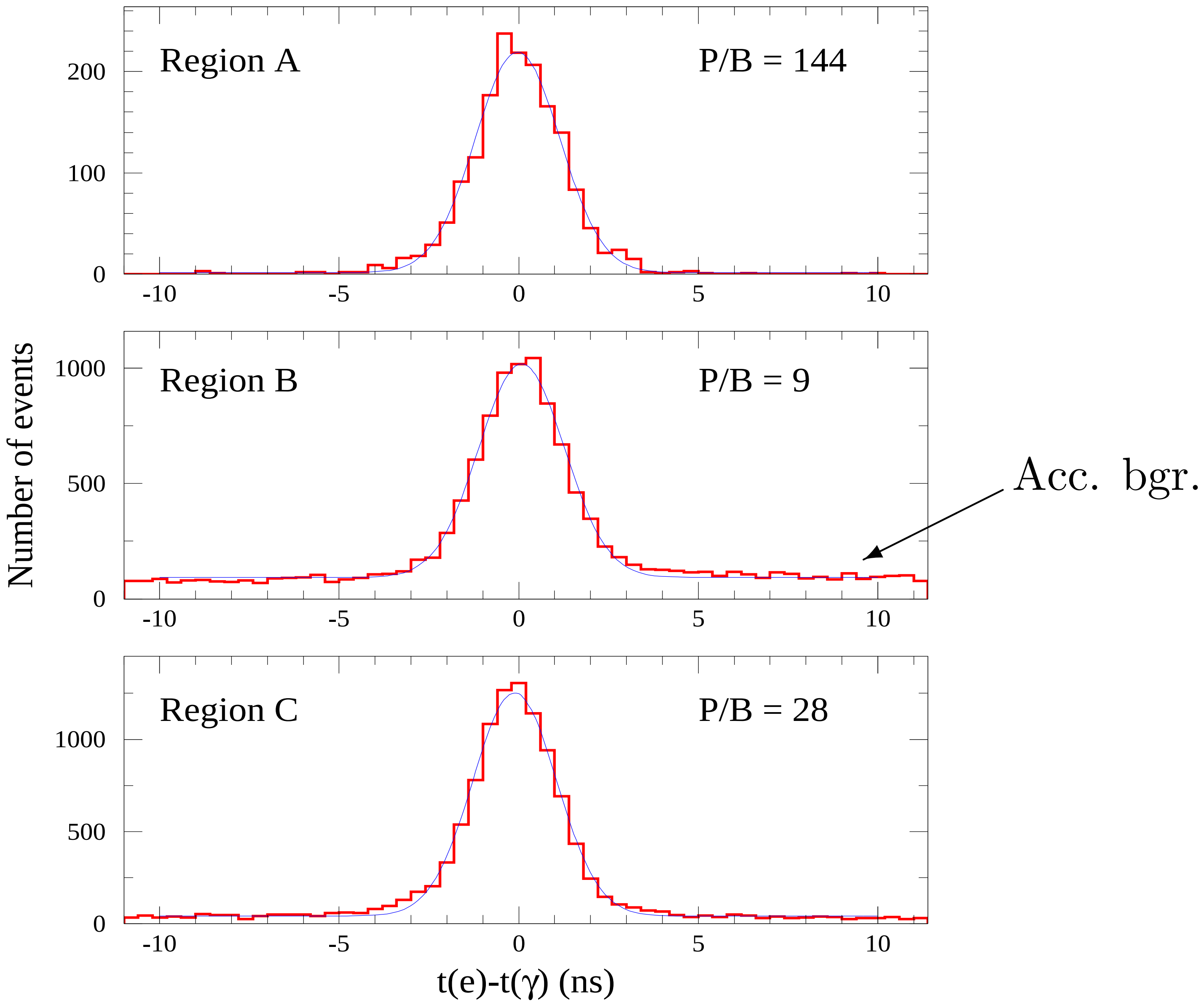
$E_{e^+}^{\min}$ (MeV)	$E_\gamma^{\min}$ (MeV)	$\theta_{e\gamma}^{\min}$	B_{exp} ($\times 10^{-8}$)	B_{the} ($\times 10^{-8}$)
50	50	–	2.71(5)	2.583(1)
10	50	40°	11.6(3)	14.34(1)
50	10	40°	39.1(13)	37.83(1)

$\pi^+ \rightarrow e^+ \nu_e \gamma$:

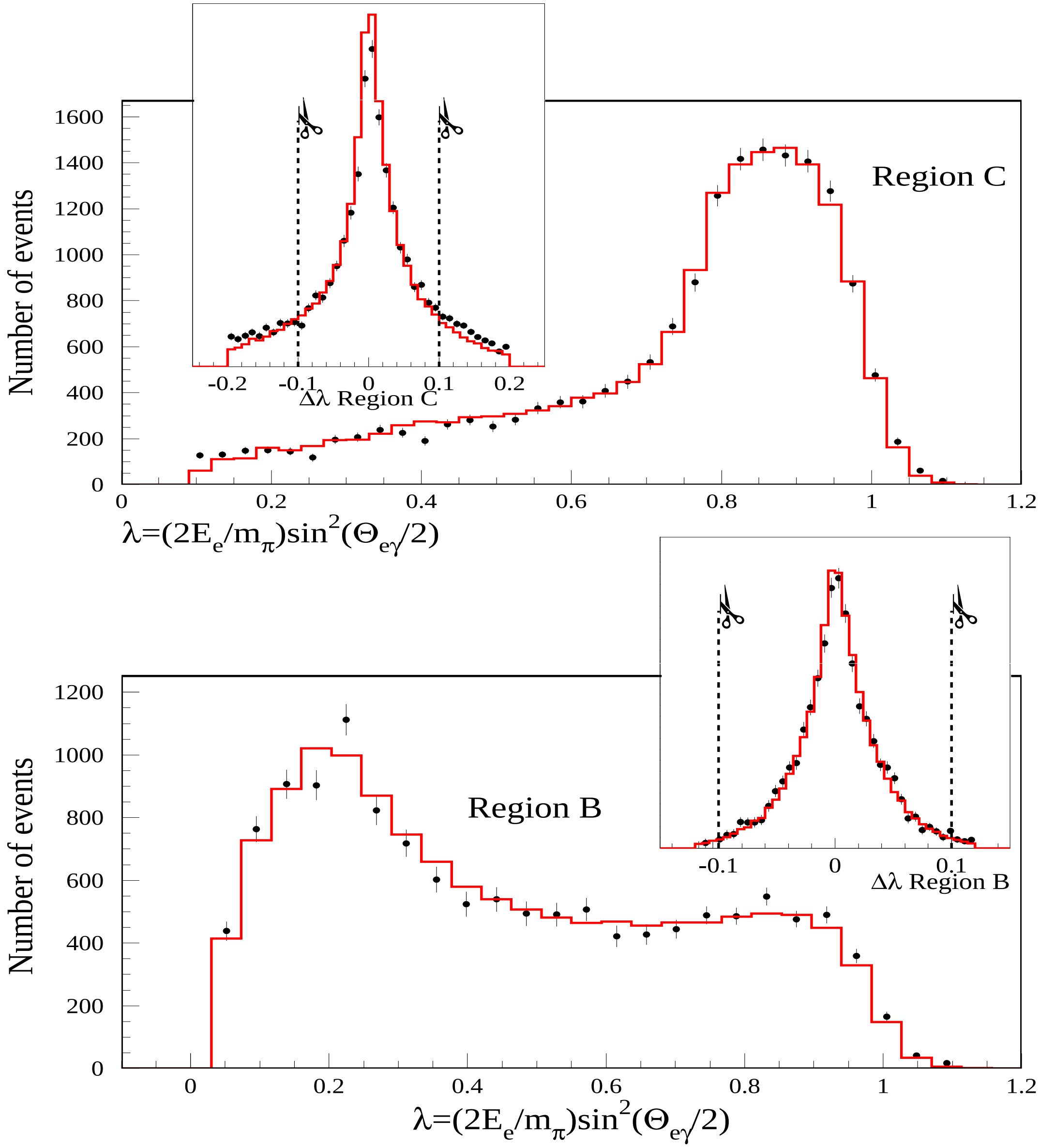
Data analyses 2004

AND

1999-2001 revisited



Region A
P/B = 144
Region B
P/B = 9
Region C
P/B = 28
Number of events
t(e)-t(γ) (ns)
Acc. bgr.


Number of events
Region C
λ=(2E_e/m_π)sin²(Θ_eγ/2)
Δλ Region C
Number of events
Region B
λ=(2E_e/m_π)sin²(Θ_eγ/2)
Δλ Region B

Preliminary Results of the combined analyses

$$F_A = 0.0115 \pm 0.0004 \qquad '04$$

$$F_A = 0.0106 \pm 0.0005 \qquad '99 -' 01$$

$$F_A = 0.0111 \pm 0.0003 \qquad '99 -' 01^{\text{A only}}$$

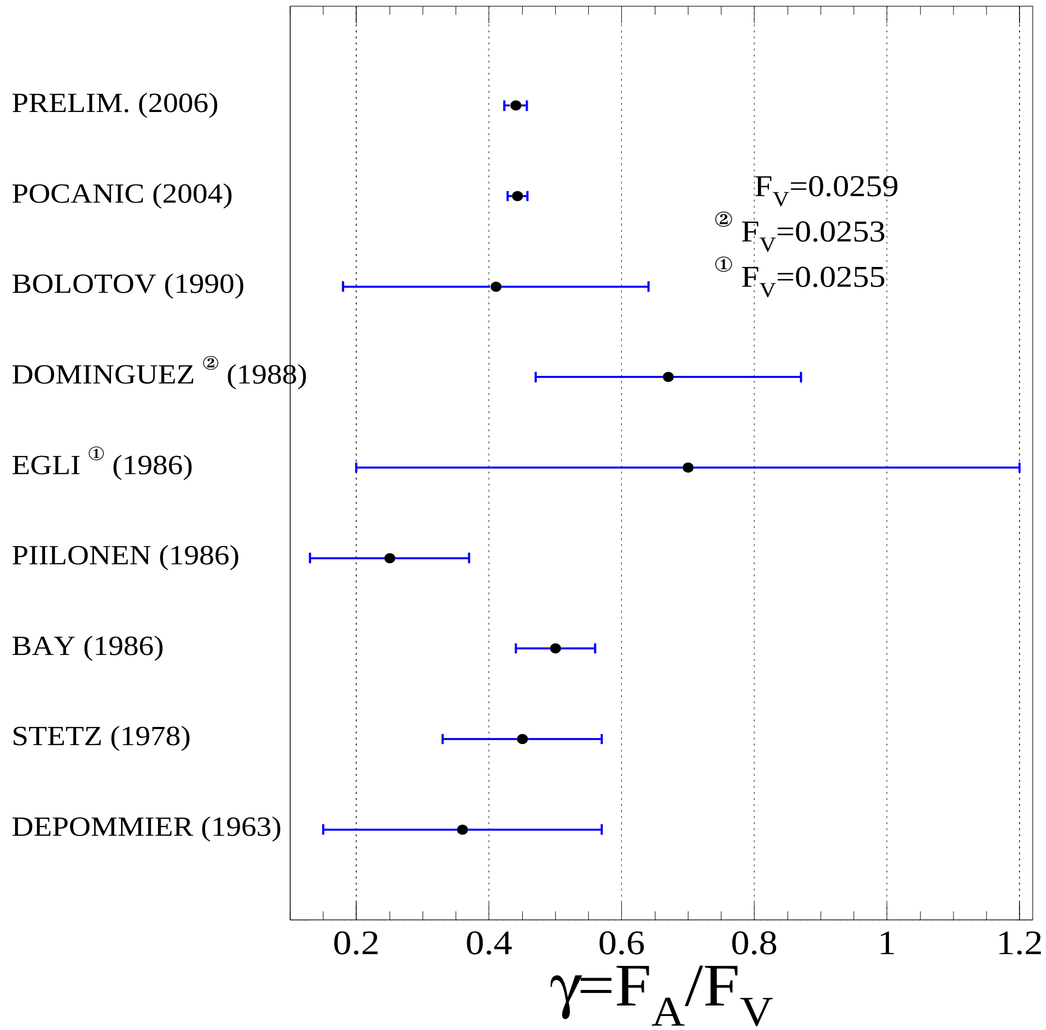
$$a = 0.241 \pm 0.093$$

$$|F_T| \; < \; 5.1 \cdot 10^{-4} \quad 90\% \text{ CL}$$

$$F_V = 0.0259 \text{ CVC fixed}$$

Carefull systematic checks remove the anomaly. Overall precision $\geq 5\times$

Preliminary Results and Comparison



The Radiative Muon Decay: $\mu^+ \to e^+\nu\overline{\nu}\gamma$

Theoretical Description: $\mu^+ \rightarrow e^+ \nu\bar{\nu}\gamma$ Decay

$$\frac{d^3\Gamma(x,y,\theta)}{dx\,dy\,2\pi\,d(cos\theta)} = f_1(x,y,\theta) + \overline{\eta} f_2(x,y,\theta) + (1 - \frac{4}{3}\rho) f_3(x,y,\theta)$$

where f_is are polynomials in

$x = 2E_{e^+}/m_\mu$, $y = 2E_\gamma/m_\mu$, $cos\theta = \hat{p}_{e^+} \cdot \hat{p}_\gamma$ and $\Delta = 1 - \beta cos\theta$ and

$$\rho = \frac{3}{4} - \frac{3}{4}\left[|g^V_{LR}|^2 + |g^V_{RL}|^2 + 2|g^T_{LR}|^2 + 2|g^T_{RL}|^2 + \mathrm{Re}(g^S_{RL} g^{T*}_{RL} + g^S_{LR} g^{T*}_{LR})\right]$$

$$\overline{\eta} = \left(|g^V_{RL}|^2 + |g^V_{LR}|^2\right) + \frac{1}{8}\left(|g^S_{LR} + 2g^T_{LR}|^2 + |g^S_{RL} + 2g^T_{RL}|^2\right) + 2\left(|g^T_{LR}|^2 + |g^T_{RL}|^2\right).$$

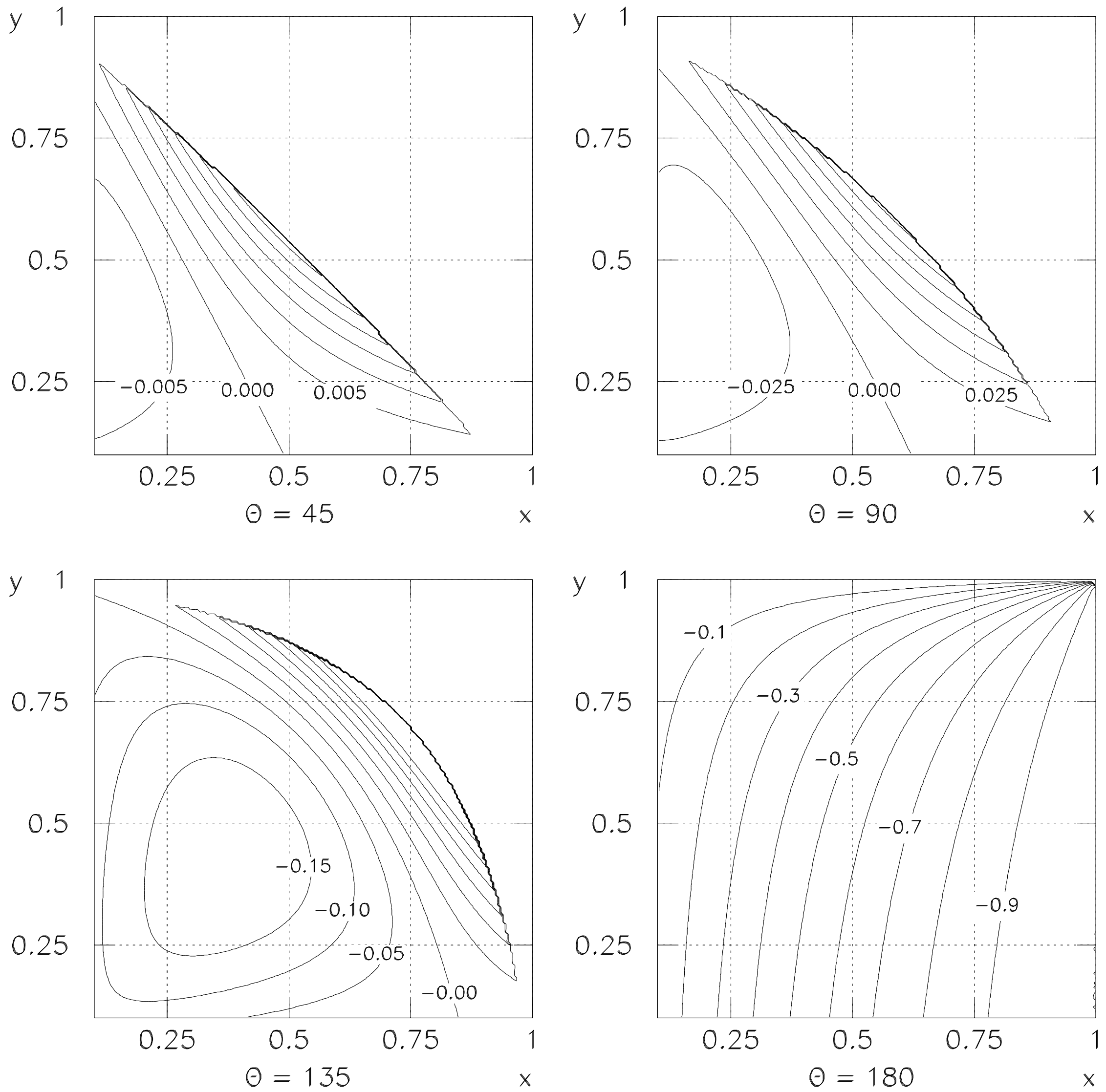
Available Data on Michel parameters

$$\rho \stackrel{\mathrm{SM}}{=} 3/4 \ , \ \overline{\eta} \stackrel{\mathrm{SM}}{=} 0.$$

ρ	reference
0.7508 ± 0.0011	Musser et al. (2005)
0.7518 ± 0.0026	Derenzo et al. (1969)
0.7509 ± 0.0010	world average (PDG 2006)

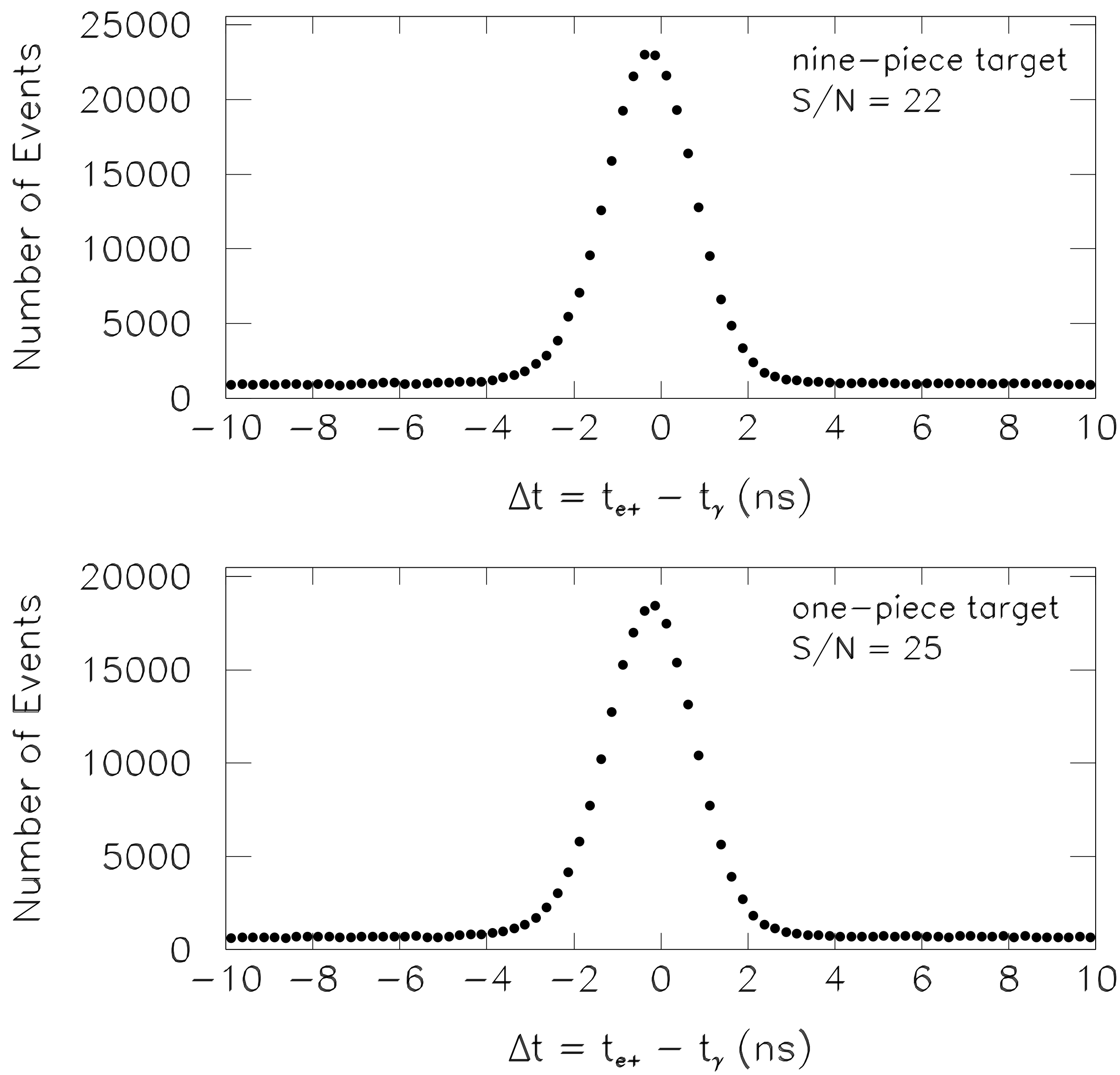
$\overline{\eta}$	reference
-0.014 ± 0.090	Eichenberger et al. (1984)
0.09 ± 0.14	Bogart et. al. (1967)
0.02 ± 0.08	world average (PDG 2006)

$\mu^+ \to e^+ \nu \bar{\nu} \gamma$: f_2/f_1 for various values of θ

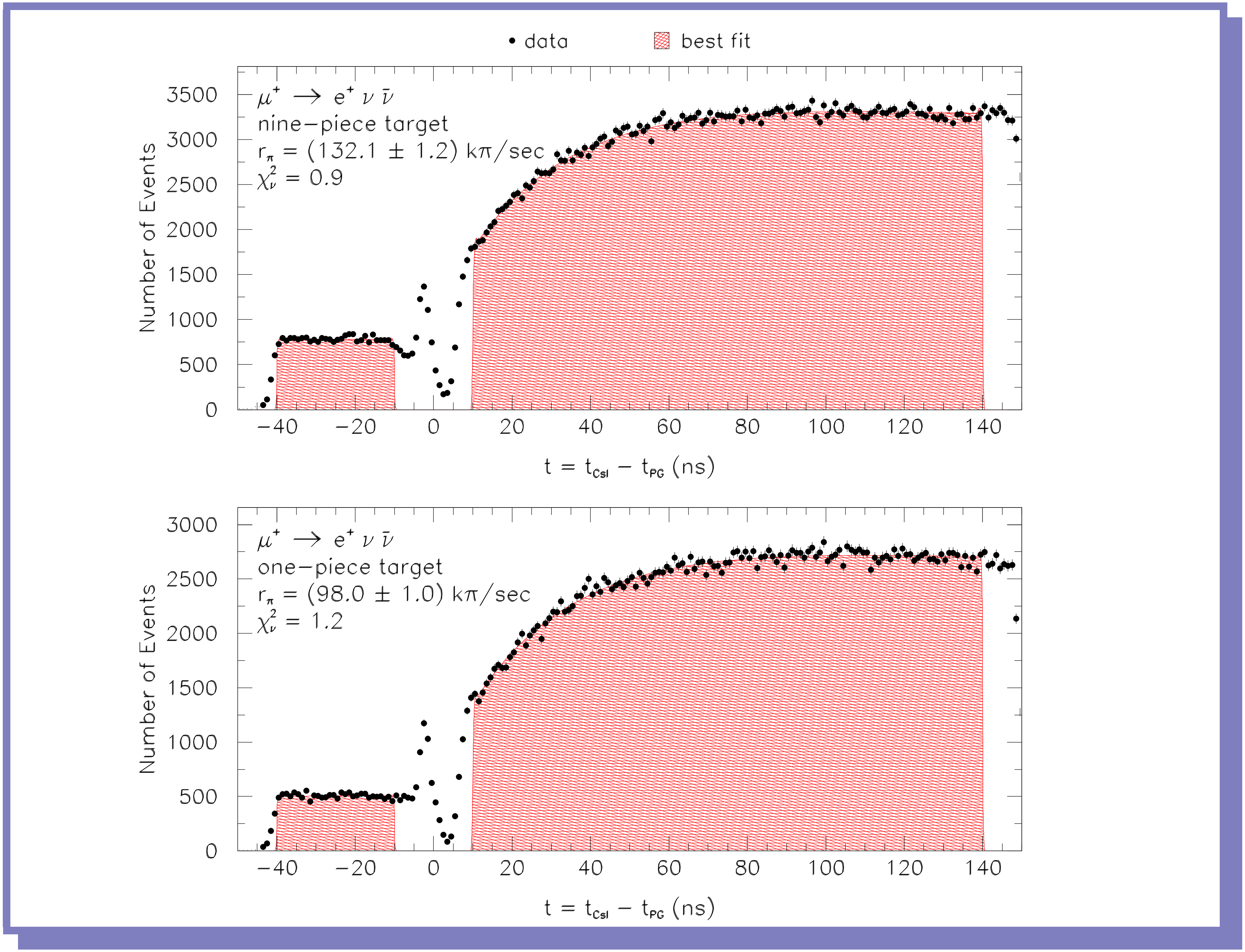


Data Analysis: $\mu^+ \to e^+ \nu \bar{\nu} \gamma$

[B. VanDevender]

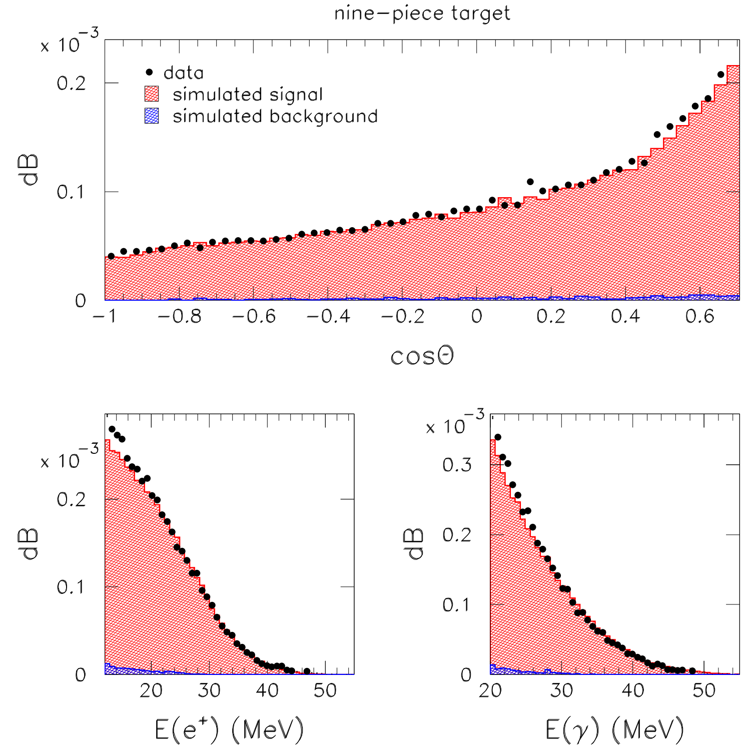


Number of Events
25000
20000
15000
10000
5000
0
−10 −8 −6 −4 −2 0 2 4 6 8 10
nine−piece target
S/N = 22
Δt = t_e+ − t_γ (ns)
Number of Events
20000
15000
10000
5000
0
−10 −8 −6 −4 −2 0 2 4 6 8 10
one−piece target
S/N = 25
Δt = t_e+ − t_γ (ns)



• data
best fit
$\mu^+ \rightarrow e^+ \nu \bar{\nu}$
nine−piece target
r_π = (132.1 ± 1.2) kπ/sec
χ^2_ν = 0.9
Number of Events
0
500
1000
1500
2000
2500
3000
3500
−40
−20
0
20
40
60
80
100
120
140
t = $t_{CsI} - t_{PG}$ (ns)
$\mu^+ \rightarrow e^+ \nu \bar{\nu}$
one−piece target
r_π = (98.0 ± 1.0) kπ/sec
χ^2_ν = 1.2
Number of Events
0
500
1000
1500
2000
2500
3000
−40
−20
0
20
40
60
80
100
120
140
t = $t_{CsI} - t_{PG}$ (ns)

$\mu^+ \to e^+\nu\bar{\nu}\gamma$ Differential Branching Ratio



$$B^{\rm exp} = [4.40 \pm 0.02\ (\text{stat.}) \pm 0.09\ (\text{syst.})] \times 10^{-3}$$

$$B^{\rm theo} = 4.30 \times 10^{-3}\ (E_\gamma > 10\,\text{MeV},\ \theta > 30^\circ)$$

$15\times$ increase in accuracy

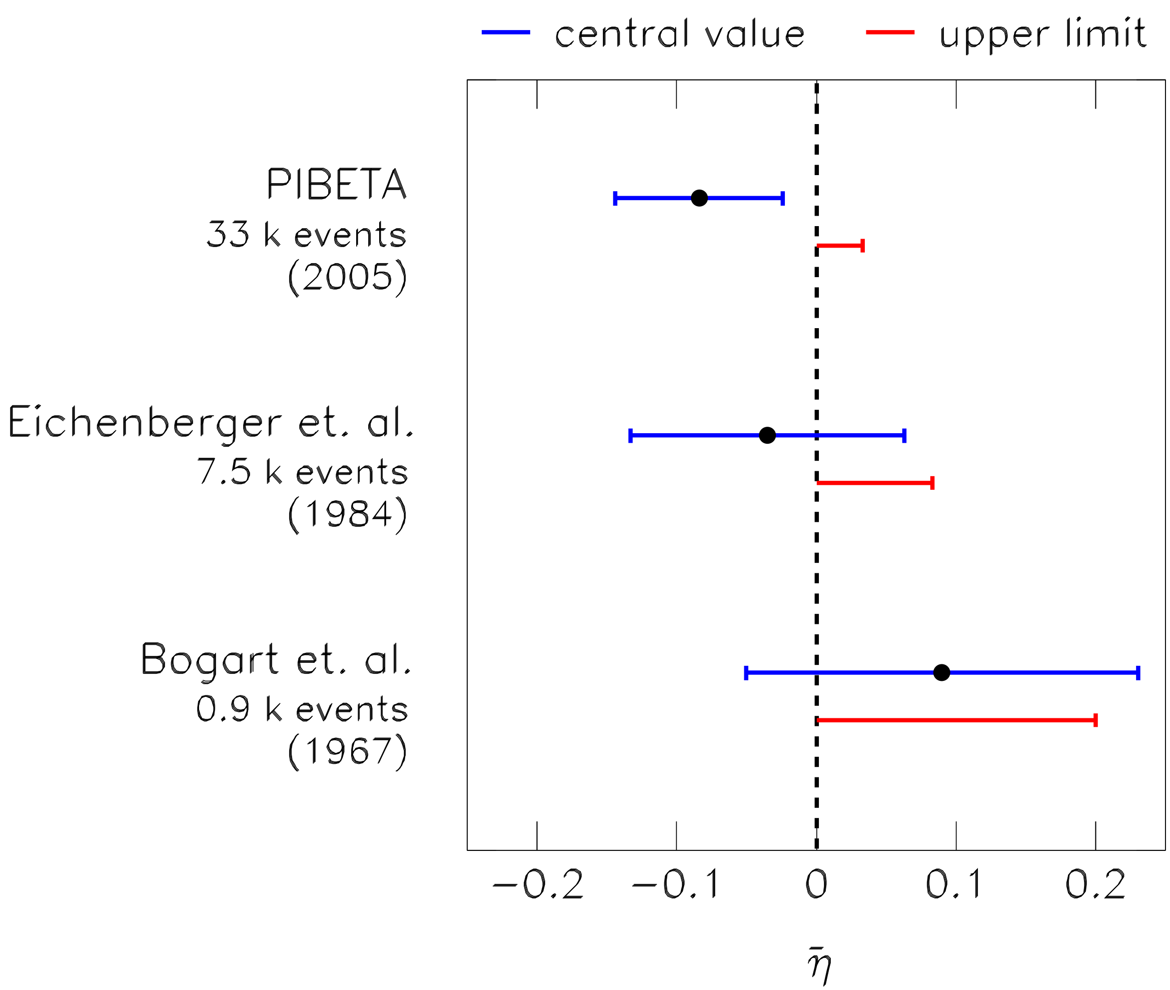
Final Results for $\overline{\eta}$ and ρ

data set	$\overline{\eta}$	ρ
nine-piece target	-0.066 ± 0.070	0.750 ± 0.010
	-0.065 ± 0.065	0.75 (fixed)
one-piece target	-0.115 ± 0.085	0.751 ± 0.011
	-0.111 ± 0.077	0.75 (fixed)

Combined: $\overline{\eta} = -0.084 \pm 0.050$(stat.)$\pm 0.034$(syst.)

$\Rightarrow \overline{\eta} \leq 0.033$ (68 % c.l.) or $\overline{\eta} \leq 0.060$ (90 % c.l.)

Experimental History of $\overline{\eta}$



Conclusions

1. In recent years the PIBETA collaboration has:
   - increased the world data set on π_β, $\pi_{e2\gamma}$, $\mu \to e\nu\bar{\nu}\gamma$ decays by ~ 2 orders of magnitude or more,
   - increased the precision of BR's of π_β, $\pi_{e2\gamma}$ decays $4-7\times$,
   - resolved the $\pi_{e2\gamma}$ anomaly
   - obtained new values of the μ decay parameters $\bar{\eta}$ and ρ from the $\mu \to e\nu\bar{\nu}\gamma$ data
2. Other forthcoming developments:
   - PEN proposal π_{e2} BR with 0.05% precision accepted in '06 (R-05-01)
   - Combine all available data on $\pi_{e2\gamma}$. Dalitz version ?
   - PEN experiment will duplicate our '04 data, with yet lower backgrounds.

http://pibeta.phys.virginia.edu

http://pen.phys.virginia.edu