

# Meson correlation functions in a QCD plasma

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## Summary

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## Introduction and outlook

- Meson spectral functions (**MSF**) in different channels are of interest for the study of mesonic properties and behaviour in the deconfined phase of QCD: the quark-gluon plasma. The survival of  $q\bar{q}$  bound states above the critical temperature  $T_c$  may change the expected pattern of mesonic spectra in the analysis of Relativistic Heavy Ion collisions.
- MSF are explored with different techniques: effective models (e.g. NJL, PNJL), lattice calculations, perturbative approaches, etc. The Hard Thermal Loop (HTL) approximation, together with Next to Leading corrections, is employed here to evaluate MSF, at temperatures above  $T_c$  and zero chemical potential.

- The HTL approach is based on the separation of different momentum scales:

⇒ **Hard scale:**  $k \sim T$ , for plasma particles

⇒ **Soft scale:**  $k \sim gT$ , for collective modes.

This separation strictly holds in weak coupling regime ( $g \ll 1$ ). At the temperatures of present experimental interest (where  $g \sim 1$ ) this separation could be less clear. Quark-gluon interaction (thermal average) is of order  $gT$ , thus negligible for hard particles, but comparable with kinetic term for soft particles: resummation is needed, keeping only terms up to  $g^2T^2$ .

- The HTL approximation reproduces quite well lattice data for the thermodynamics of the QGP phase, at  $T \geq 2.5T_c$ . Analogous test can be performed for the MSF, since a direct comparison with experimental data is not doable.

## Thermal meson correlation functions from lattice data

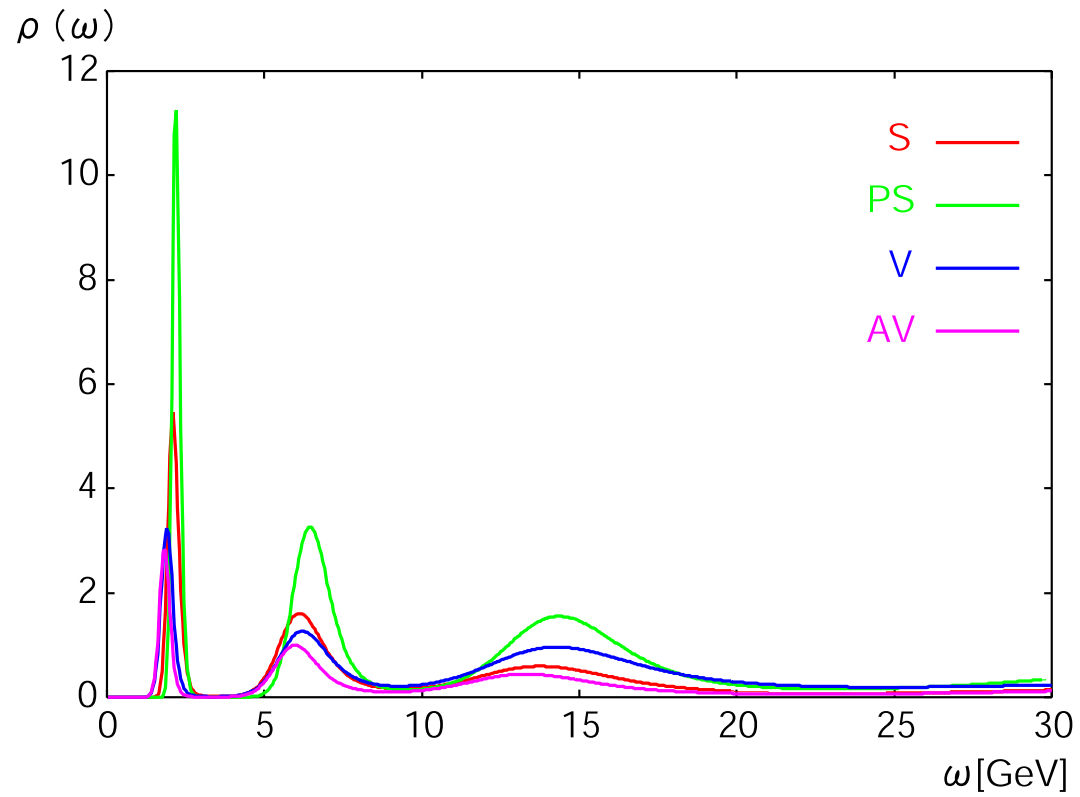
Thermal meson propagator along the (imaginary) temporal direction:

$$G_M(-i\tau, \mathbf{p}) = \int_0^{+\infty} d\omega \sigma_M(\omega, \mathbf{p}) \frac{\cosh(\omega(\tau - \beta/2))}{\sinh(\omega\beta/2)} .$$

where  $\sigma_M(\omega, \mathbf{p})$  is the thermal meson spectral function.

- $G_M(-i\tau, \mathbf{p} = \mathbf{0})$  is measured on the lattice for a finite set of values of  $\tau$  ( $\sim 20$ ).
- $\sigma_M(\omega, \mathbf{0})$  has to be reconstructed (Maximum Entropy Method usually employed).

Lattice Meson Spectral Functions above  $T_C$  (T. Hatsuda, hep-lat/0509306 and references therein.)



$s\bar{s}$  dimensionless spectral function at  $T = 1.38T_c$ . Peak position at  $\omega = 2.4m_\phi^{(T=0)}$ .

## Thermal meson correlation function

Consider the current operator, carrying the quantum numbers of a meson ( $\Gamma_M = 1, \gamma^5, \gamma^\mu, \gamma^\mu \gamma^5$  for the different channels):

$$J_M(-i\tau, \mathbf{x}) = \bar{q}(-i\tau, \mathbf{x}) \Gamma_M q(-i\tau, \mathbf{x}) ,$$

and the fluctuation operator  $\tilde{J}_M$  (average over the grand canonical ensemble):

$$\tilde{J}_M(-i\tau, \mathbf{x}) = J_M(-i\tau, \mathbf{x}) - \langle J_M(-i\tau, \mathbf{x}) \rangle ,$$

### Thermal meson 2 point correlation function

$$\begin{aligned} G_M(-i\tau, \mathbf{x}) &= \langle \tilde{J}_M(-i\tau, \mathbf{x}) \tilde{J}_M^\dagger(0, \mathbf{0}) \rangle \\ &= \frac{1}{\beta} \sum_{n=-\infty}^{+\infty} \int \frac{d^3 p}{(2\pi)^3} e^{-i\omega_n \tau} e^{i\mathbf{p} \cdot \mathbf{x}} \chi_M(i\omega_n, \mathbf{p}) \end{aligned}$$

with  $\tau \in [0, \beta = 1/T]$  and  $\omega_n = 2n\pi T$  ( $n = 0, \pm 1, \pm 2 \dots$ ).

Spectral representation for the meson propagator in momentum space:

$$\chi_M(i\omega_n, \mathbf{p}) = - \int_{-\infty}^{+\infty} d\omega \frac{\sigma_M(\omega, \mathbf{p})}{i\omega_n - \omega} \quad \Rightarrow \quad \sigma_M(\omega, \mathbf{p}) = \frac{1}{\pi} \text{Im} \chi_M(\omega + i\eta, \mathbf{p}).$$

$\sigma_M$  being the corresponding **spectral function**.

**Thermal meson propagator in mixed representation:**

$$G_M(-i\tau, \mathbf{p}) = \frac{1}{\beta} \sum_{n=-\infty}^{+\infty} e^{-i\omega_n \tau} \chi_M(i\omega_n, \mathbf{p}) = -\frac{1}{\beta} \sum_{n=-\infty}^{+\infty} e^{-i\omega_n \tau} \int_{-\infty}^{+\infty} d\omega \frac{\sigma_M(\omega, \mathbf{p})}{i\omega_n - \omega},$$

sum over the Matsubara frequencies are performed with a standard contour integration in the complex  $\omega$  plane:

$$G_M(-i\tau, \mathbf{p}) = \int_0^{+\infty} d\omega \sigma_M(\omega, \mathbf{p}) \frac{\cosh[\omega(\tau - \beta/2)]}{\sinh(\omega\beta/2)} \equiv \int_0^{+\infty} d\omega \sigma_M(\omega, \mathbf{p}) K(\omega, \tau).$$



## Free spectral functions

In Fourier space the free mesonic 2 point correlation function reads

$$\chi_M(i\omega_l, \mathbf{p}) = -2N_c \frac{1}{\beta} \sum_{n=-\infty}^{+\infty} \int \frac{d^3k}{(2\pi)^3} \text{Tr}[\Gamma_M S_F(i\omega_n, \mathbf{k}) \gamma^0 \Gamma_M^\dagger \gamma^0 S_F(i\omega_n - i\omega_l, \mathbf{k} - \mathbf{p})],$$

where  $\omega_l = 2l\pi T$  (mesonic frequency), while  $\omega_n = (2n+1)\pi T$ ;  $2N_c$  comes from trace over light flavours and colours.

The spectral representation of **free fermion propagator** is

$$S_F(i\omega_n, \mathbf{p}) = - \int_{-\infty}^{+\infty} dp_0 \frac{\rho_F(p_0, \mathbf{p})}{i\omega_n - p_0} .$$

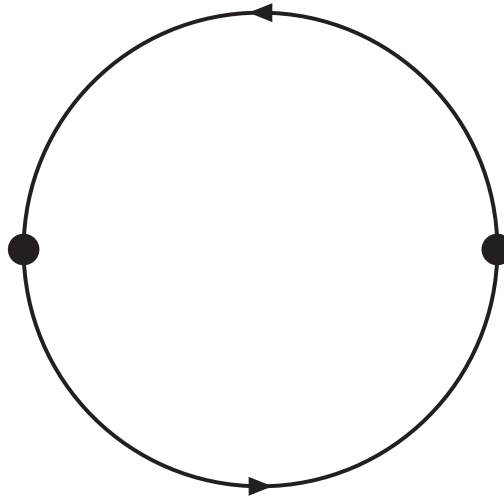
with the spectral function

$$\rho_F(p) = \epsilon(p_0) (\not{p} + m) \delta(p^2 - m^2)$$

The free mesonic spectral function ( $\mathbf{p} = 0$ ) is then

$$\sigma_M^{\text{free}}(\omega, \mathbf{0}) = \frac{N_c}{4\pi^2} \theta(\omega - 2m) \sqrt{1 - \left(\frac{2m}{\omega}\right)^2} \omega^2 \tanh(\omega/4T) \left( a + b \left(\frac{2m}{\omega}\right)^2 \right),$$

where  $(a, b) = (1, -1), (1, 0), (-2, -1), (-2, 3)$  in the scalar, pseudoscalar, vector and pseudovector channels, respectively.



## HTL spectral functions: quark propagator

HTL quark propagator (for quarks with **soft** momentum):

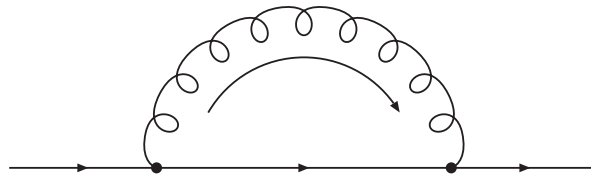
$${}^*S(\omega, \mathbf{p}) = {}^*\Delta_+(\omega, p) \frac{\gamma^0 - \boldsymbol{\gamma} \cdot \hat{\mathbf{p}}}{2} + {}^*\Delta_-(\omega, p) \frac{\gamma^0 + \boldsymbol{\gamma} \cdot \hat{\mathbf{p}}}{2}$$

with

$${}^*\Delta_{\pm}(\omega, p) = \frac{-1}{\omega \mp p - \frac{m_q^2}{2p} \left[ \left( 1 \mp \frac{\omega}{p} \right) \ln \frac{\omega + p}{\omega - p} \pm 2 \right]},$$

quark thermal mass  $m_q = g(T)T/\sqrt{6}$ ,

$g(T)$  gauge running coupling evaluated at renormalization scale  $\mu \sim T$ .



Alternatively, by setting

$${}^* \Delta_{\pm}(z, p) = - \int_{-\infty}^{+\infty} d\omega \frac{\rho_{\pm}(\omega, p)}{z - \omega} \quad \Rightarrow \quad \rho_{\pm}(\omega, p) = \frac{1}{\pi} \text{Im } {}^* \Delta_{\pm}(\omega + i\eta, p) ,$$

the HTL quark propagator has the spectral representation:

$${}^* S(i\omega_n, \mathbf{p}) = - \int_{-\infty}^{+\infty} d\omega \frac{\rho_{\text{HTL}}(\omega, \mathbf{p})}{i\omega_n - \omega} ,$$

with the HTL quark spectral function

$$\rho_{\text{HTL}}(\omega, \mathbf{p}) = \frac{\gamma^0 - \boldsymbol{\gamma} \cdot \hat{\mathbf{p}}}{2} \rho_+(\omega, p) + \frac{\gamma^0 + \boldsymbol{\gamma} \cdot \hat{\mathbf{p}}}{2} \rho_-(\omega, p) .$$

The explicit expression of the HTL quark spectral function reads

$$\rho_{\pm}(\omega, k) = \frac{\omega^2 - k^2}{2m_q^2} [\delta(\omega - \omega_{\pm}) + \delta(\omega + \omega_{\mp})] + \beta_{\pm}(\omega, k) \theta(k^2 - \omega^2)$$

with

$$\beta_{\pm}(\omega, k) = -\frac{m_q^2}{2} \frac{\pm\omega - k}{\left[ k(-\omega \pm k) + m_q^2 \left( \pm 1 - \frac{\pm\omega - k}{2k} \ln \frac{k+\omega}{k-\omega} \right) \right]^2 + \left[ \frac{\pi}{2} m_q^2 \frac{\pm\omega - k}{k} \right]^2}.$$

HTL spectral function has two pieces:

- pole term in time-like domain  $\omega > k$

1. **quasiparticle** from  ${}^* \Delta_+^{-1}(\omega_+(k), k) = 0$

with asymptotic behaviour for  $k \gg m_q$ :

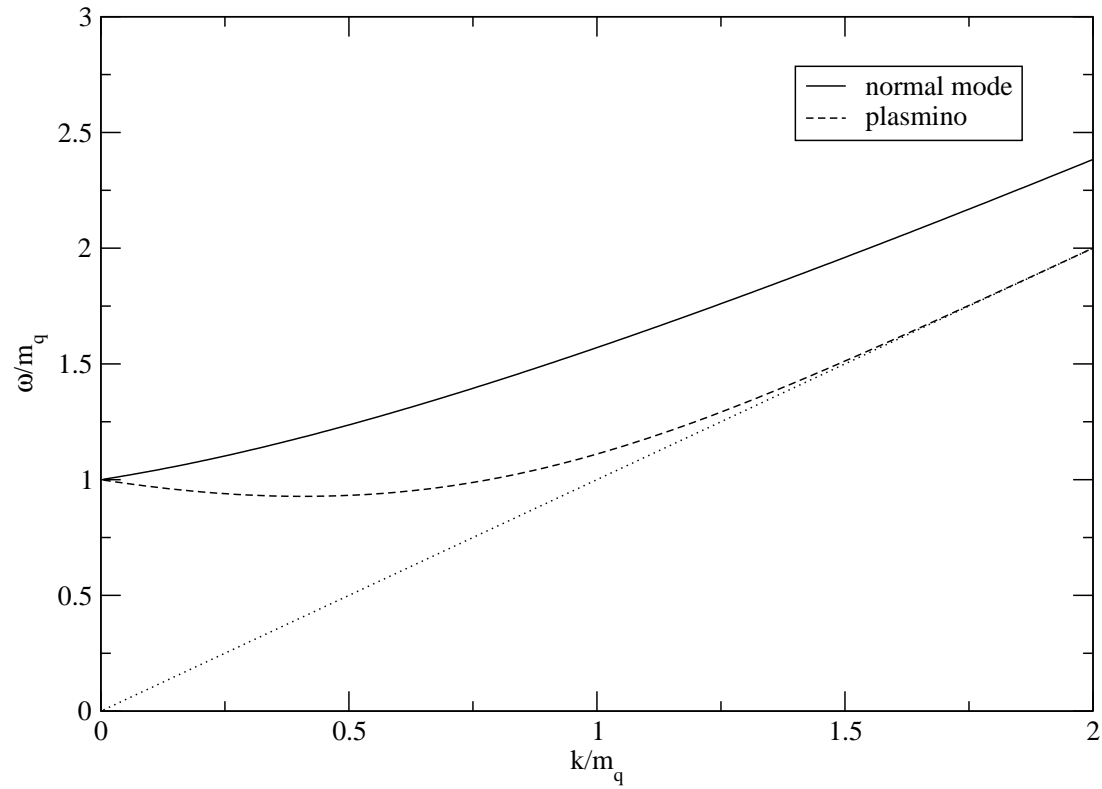
$$\omega_+(k) \simeq \sqrt{k^2 + \hat{m}_\infty^2} \quad \hat{m}_\infty^2 = 2m_q^2 = \frac{g^2 T^2}{3}$$

2. **plasmino** from  ${}^* \Delta_-^{-1}(\omega_-(k), k) = 0$

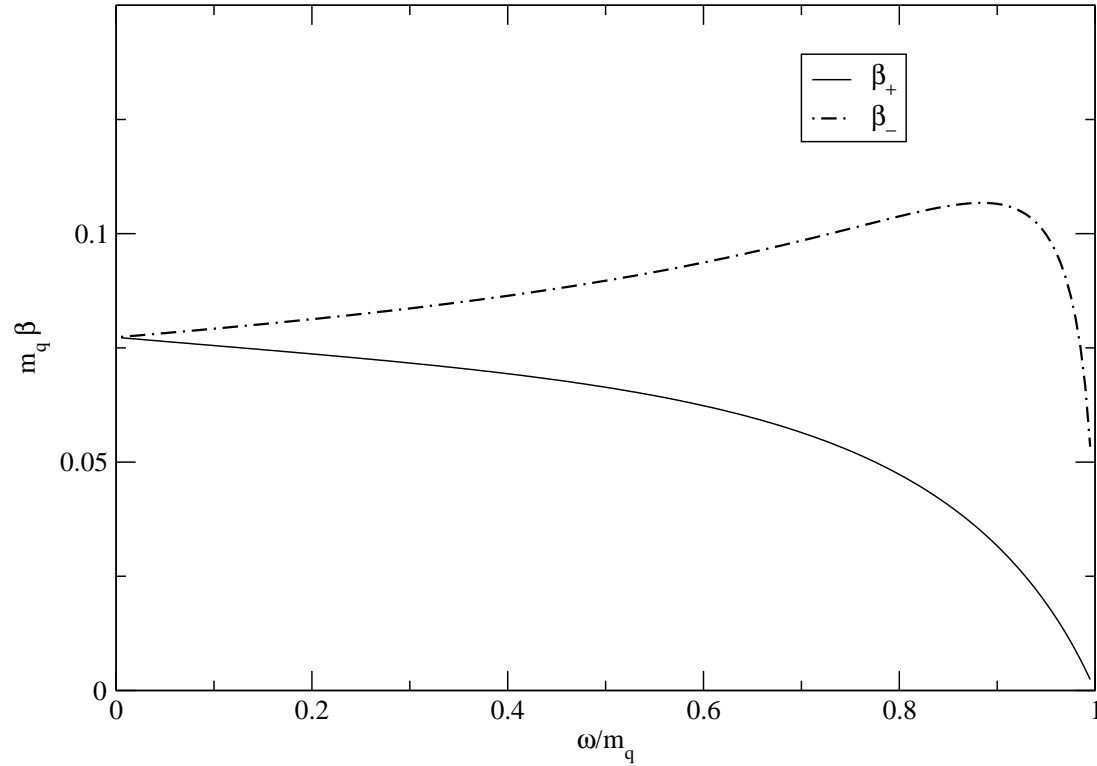
with asymptotic behaviour for  $k \gg m_q$ :

$$\omega_-(k) \simeq k + 2k \exp\left(-\frac{2k^2 + m_q^2}{m_q^2}\right)$$

- a continuum term ( $\beta_{\pm}$ ) in space-like domain  $\omega < k$



Dispersion relations corresponding to the quasiparticle poles of the HTL fermion propagator in the time-like domain.



Dimensionless (continuum) spectral function  $m_q \cdot \beta_{\pm}(\omega, k)$  for space-like momenta at  $k = m_q$  as a function of  $\omega/m_q$ . The maximum of  $\beta_-$  stems from the second zero of the function  $\text{Re}({}^* \Delta_-^{-1})$ , occurring in the space-like region, but it does not correspond to a quasi-particle excitation.

## HTL mesonic correlator in PS channel

The HTL approximation for the meson 2-point function in the pseudoscalar channel, employs for the fermionic lines the HTL resummed fermion propagators. NB: the pseudoscalar vertex has no HTL correction.

$$\begin{aligned}\chi^{\text{ps}}(i\omega_l, \mathbf{p}) &= 2N_c \frac{1}{\beta} \sum_{n=-\infty}^{+\infty} \int \frac{d^3 k}{(2\pi)^3} \text{Tr}[\gamma^5 \star S(i\omega_n, \mathbf{k}) \gamma^5 \star S(i\omega_n - i\omega_l, \mathbf{k} - \mathbf{p})] \\ &= 2N_c \frac{1}{\beta} \sum_{n=-\infty}^{+\infty} \int \frac{d^3 k}{(2\pi)^3} \int_{-\infty}^{+\infty} d\omega_1 \int_{-\infty}^{+\infty} d\omega_2 \frac{1}{i\omega_n - \omega_1} \frac{1}{i\omega_n - i\omega_l - \omega_2} \times \\ &\quad \times \text{Tr}[\gamma^5 \rho_{\text{HTL}}(\omega_1, \mathbf{k}) \gamma^5 \rho_{\text{HTL}}(\omega_2, \mathbf{q})]\end{aligned}$$

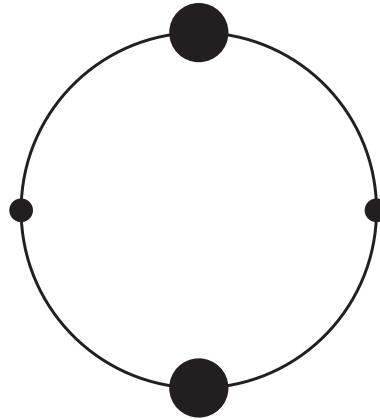
with  $\mathbf{q} = \mathbf{k} - \mathbf{p}$ .



## Pseudoscalar meson spectral function

$$\begin{aligned} \sigma^{\text{PS}}(\omega, \mathbf{p}) &= 2N_c \int \frac{d^3k}{(2\pi)^3} (e^{\beta\omega} - 1) \int_{-\infty}^{+\infty} d\omega_1 \int_{-\infty}^{+\infty} d\omega_2 \tilde{n}(\omega_1) \tilde{n}(\omega_2) \delta(\omega - \omega_1 - \omega_2) \times \\ &\times \left\{ (1 + \hat{\mathbf{k}} \cdot \hat{\mathbf{q}}) [\rho_+(\omega_1, k) \rho_+(\omega_2, q) + \rho_-(\omega_1, k) \rho_-(\omega_2, q)] + \right. \\ &\left. + (1 - \hat{\mathbf{k}} \cdot \hat{\mathbf{q}}) [\rho_+(\omega_1, k) \rho_-(\omega_2, q) + \rho_-(\omega_1, k) \rho_+(\omega_2, q)] \right\} \end{aligned}$$

In the above  $\tilde{n}(\omega) = [1 + e^{\beta\omega}]^{-1}$  is the Fermi distribution and the identity  $\rho_+(-\omega, k) = \rho_-(\omega, k)$  has been used.



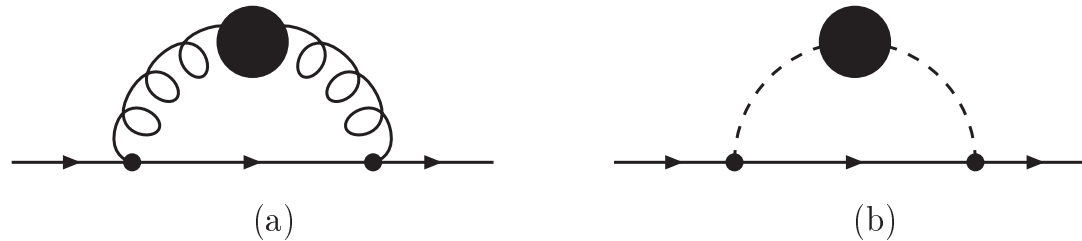
### The case $\mathbf{p} = 0$ : beyond HTL

For  $\mathbf{p} = 0$  the above formula reduces to:

$$\begin{aligned} \sigma^{\text{PS}}(\omega, \mathbf{0}) &= \frac{2N_c}{\pi^2} (e^{\beta\omega} - 1) \int_0^{+\infty} dk k^2 \int_{-\infty}^{+\infty} d\omega_1 \int_{-\infty}^{+\infty} d\omega_2 \tilde{n}(\omega_1) \tilde{n}(\omega_2) \\ &\quad \delta(\omega - \omega_1 - \omega_2) [\rho_+(\omega_1, k) \rho_+(\omega_2, k) + \rho_-(\omega_1, k) \rho_-(\omega_2, k)] . \end{aligned}$$

The fermionic momentum inside the spectral function is integrated over **all scale of momenta** (hard and soft), but HTL approximation valid to dress propagation of **soft modes**, not of hard ones.

In **Next to Leading Approximation (NLA)** the self energy of hard quarks is corrected by the interaction with a **soft gluon (HTL dressed)**, longitudinal and transverse:



Next to Leading corrections to a hard quark propagator, arising from the interaction with a soft transverse (a) and longitudinal (b) gluon: for these HTL resummed propagators are used. Two diagrams receive contributions from all orders in perturbation theory.

NLA taken into account (approximately), with [constant shift of the asymptotic quark mass](#),

$$\delta m_\infty^2 = -\frac{1}{2\pi} g^2 \frac{4}{3} T \hat{m}_D$$

with Debye screening mass:

$$\hat{m}_D = \sqrt{\frac{N_c}{3} + \frac{N_f}{6}} gT$$

Then

$$\begin{aligned} m_\infty^2 &= \frac{1}{3} g^2 T^2 - \frac{1}{2\pi} \frac{4}{3} \sqrt{\frac{N_c}{3} + \frac{N_f}{6}} g^3 T^2 \\ &= \hat{m}_\infty^2 - \frac{2}{3\pi} \sqrt{N_c + \frac{N_f}{2}} g^2 T \hat{m}_\infty \end{aligned}$$

Notice that  $g \geq 1$ , hence this leads to unphysical negative mass values at large  $T$ . Suggestion: take instead the positive root of the “self-consistent like” equation: [Ref. Blaizot, Iancu et al.]

$$m_{NLA}^2 = \hat{m}_\infty^2 - \frac{2}{3\pi} g^2 \sqrt{N_c + \frac{N_f}{2}} T m_{NLA}$$

( $\Rightarrow$ ) right coefficients of the terms of order  $g^3$  to the entropy and baryon density.

**Strategy:** fix an intermediate scale of momenta  $\Lambda = \sqrt{2\pi T \hat{m}_D}$  and perform the integration over the fermionic loop ( $k$ ) as follows

- For  $k < \Lambda$  keep full HTL description of quark propagator
- For  $k > \Lambda$  consider only physical modes (quarks and transverse gluons) with the asymptotic mass  $m_{NLA}$ .

Spectral representation for the **NLA quark propagator**

$$S^{\text{NLA}}(i\omega_n, \mathbf{k}) = - \int_{-\infty}^{+\infty} d\omega \frac{\rho^{\text{NLA}}(\omega, \mathbf{k})}{i\omega_n - \omega},$$

with

$$\rho^{\text{NLA}}(\omega, \mathbf{k}) = \frac{\gamma^0 - \boldsymbol{\gamma} \cdot \hat{\mathbf{k}}}{2} \rho_+^{\text{NLA}}(\omega, k) + \frac{\gamma^0 + \boldsymbol{\gamma} \cdot \hat{\mathbf{k}}}{2} \rho_-^{\text{NLA}}(\omega, k).$$

**Requirements** for the NLA quark spectral function  $\rho_{\pm}^{\text{NLA}}$

- for small momenta it reduces to the HTL one;
- for large momenta it yields a spectrum dominated by an undamped excitation whose dispersion relation approaches  $\epsilon_k^{\text{NLA}} = \sqrt{k^2 + m_{NLA}^2}$ ,

- it obeys the “sum rule”

$$\int_{-\infty}^{+\infty} d\omega \rho_{\pm}(\omega, k) = 1$$

(stemming from equal-time anticommutation relations)

- the associated quark propagator anti-commutes with  $\gamma^5$ . Indeed chirality is not destroyed by thermal masses (gap mass  $m_q$  and asymptotic mass  $m_{\infty}$ ) arising from interactions with thermal bath.

Then

$$\rho_{\pm}^{\text{NLA}}(\omega, k) = \theta(\Lambda - k)\rho_{\pm}(\omega, k) + \theta(k - \Lambda)\delta(\omega \mp \epsilon_k^{\text{NLA}})$$

**Pseudo-scalar meson spectral function** (for  $\omega > 0, \mathbf{p} = 0$ ):

$$\sigma_{\text{NLA}}^{\text{ps}}(\omega, \mathbf{0}) = \frac{2N_c}{\pi^2} (e^{\beta\omega} - 1) \int_0^{+\infty} dk k^2 \int_{-\infty}^{+\infty} d\omega_1 \int_{-\infty}^{+\infty} d\omega_2 \tilde{n}(\omega_1) \tilde{n}(\omega_2) \\ \delta(\omega - \omega_1 - \omega_2) [\rho_+^{\text{NLA}}(\omega_1, k) \rho_+^{\text{NLA}}(\omega_2, k) + \rho_-^{\text{NLA}}(\omega_1, k) \rho_-^{\text{NLA}}(\omega_2, k)] .$$

## More precisely...

To avoid overcounting of the same physical mode (in a small range around  $\Lambda$ ) and to interpolate smoothly between the soft and hard regimes we introduce two additional cutoffs  $\Lambda_1$  and  $\Lambda_2$  defined as follows:

$$\begin{aligned}\omega_+(\Lambda_1) &= \sqrt{\Lambda^2 + m_{NLA}^2} \\ \omega_+(\Lambda) &= \sqrt{\Lambda_2^2 + m_{NLA}^2} .\end{aligned}$$

Leading to

$$\rho_{\pm}^{\text{NLA}}(\omega, k) = \begin{cases} \rho_{\pm}(\omega, k) & \text{if } k < \Lambda_1 \\ \cos^2(\alpha(k))\rho_{\pm}(\omega, k) + \sin^2(\alpha(k))\delta(\omega \mp \epsilon_k^{\text{NLA}}) & \text{if } \Lambda_1 < k < \Lambda_2 \\ \delta(\omega \mp \epsilon_k^{\text{NLA}}) & \text{if } k > \Lambda_2 \end{cases}$$

with

$$\alpha(k) = \frac{\pi}{2} \cdot \frac{k - \Lambda_1}{\Lambda_2 - \Lambda_1}$$

## Numerical Results

### Results at $p = 0$

Different contributions to the HTL pseudoscalar spectral function:

$$\sigma_{\text{HTL}}^{\text{ps}}(\omega, \mathbf{0}) = \sigma^{\text{pp}}(\omega, \mathbf{0}) + \sigma^{\text{pc}}(\omega, \mathbf{0}) + \sigma^{\text{cc}}(\omega, \mathbf{0}) .$$

- Pole-pole (pp)

$$\begin{aligned} \sigma^{\text{pp}}(\omega) = & \frac{N_c}{2\pi^2} \frac{e^{\beta\omega} - 1}{m_q^4} \left[ \tilde{n}^2(\omega_+(k_1)) (\omega_+^2(k_1) - k_1^2)^2 \frac{k_1^2}{2|\omega'_+(k_1)|} + \right. \\ & + 2 \sum_{k_2} \tilde{n}(\omega_+(k_2)) [1 - \tilde{n}(\omega_-(k_2))] (\omega_+^2(k_2) - k_2^2) (\omega_-^2(k_2) - k_2^2) \frac{k_2^2}{|\omega'_+(k_2) - \omega'_-(k_2)|} + \\ & \left. + \sum_{k_3} \tilde{n}^2(\omega_-(k_3)) (\omega_-^2(k_3) - k_3^2)^2 \frac{k_3^2}{2|\omega'_-(k_3)|} \right] \end{aligned}$$



It contains well defined physical processes:

1. annihilation of a normal quark ( $q_+$ ) anti-quark pair into a meson ( $M$ ) at rest

$$q_+ + \bar{q}_+ \longrightarrow M ,$$

2. decay of a normal quark mode into a plasmino mode ( $q_+$ ) with same momentum and a meson at rest

$$q_+ \longrightarrow q_- + M ,$$

3. annihilation of a plasmino anti-plasmino pair into a meson

$$q_- + \bar{q}_- \longrightarrow M$$

- Pole-cut (pc)

$$\sigma^{\text{pc}}(\omega) = \frac{2N_c}{\pi^2} \frac{e^{\beta\omega} - 1}{m_q^2} \int_0^\infty dk k^2 \cdot$$

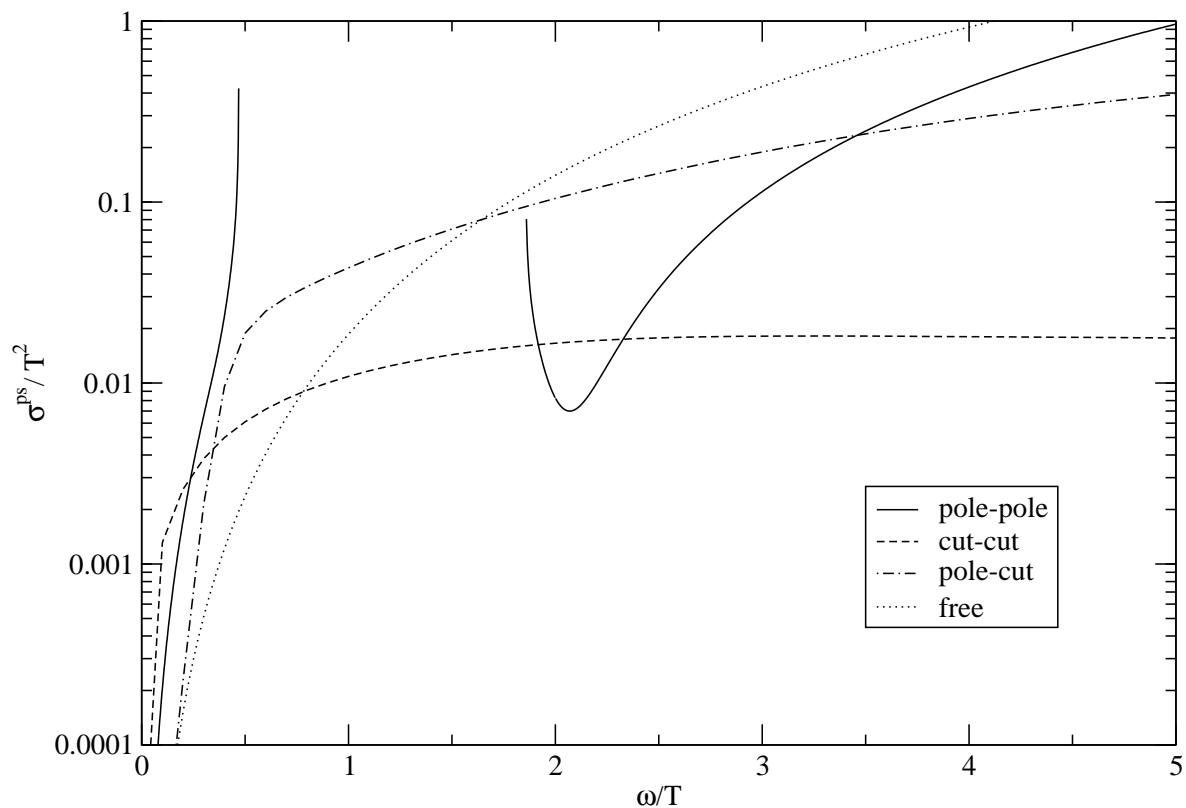
$$\cdot \left[ \theta(k^2 - (\omega - \omega_+)^2) \tilde{n}(\omega - \omega_+) \tilde{n}(\omega_+) \beta_+(\omega - \omega_+, k) (\omega_+^2 - k^2) \right.$$

$$\left. + \theta(k^2 - (\omega - \omega_-)^2) \tilde{n}(\omega - \omega_-) \tilde{n}(\omega_-) \beta_-(\omega - \omega_-, k) (\omega_-^2 - k^2) \right] .$$

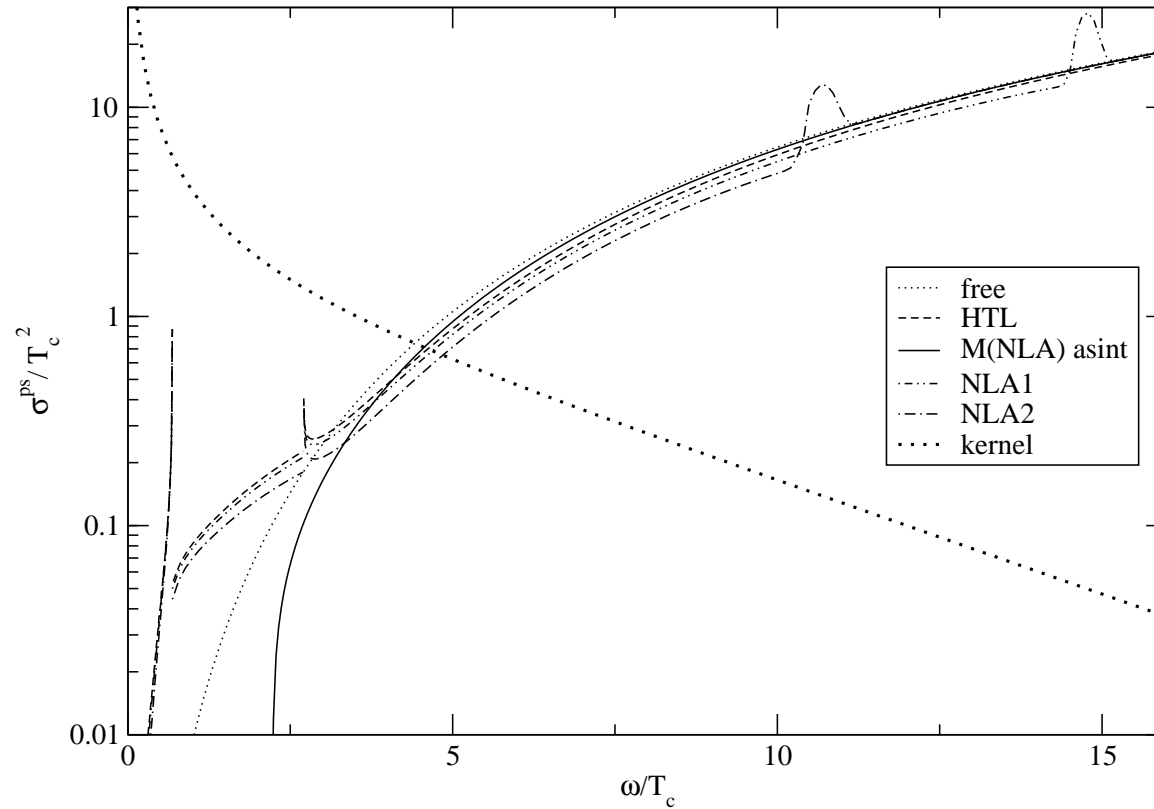
- Cut-cut (cc)

$$\sigma^{\text{cc}}(\omega) = \frac{2N_c}{\pi^2} (e^{\beta\omega} - 1) \int_0^\infty dk k^2 \int_{-k}^{+k} dx \tilde{n}(x) \tilde{n}(\omega - x) \theta(k^2 - (\omega - x)^2) \cdot$$

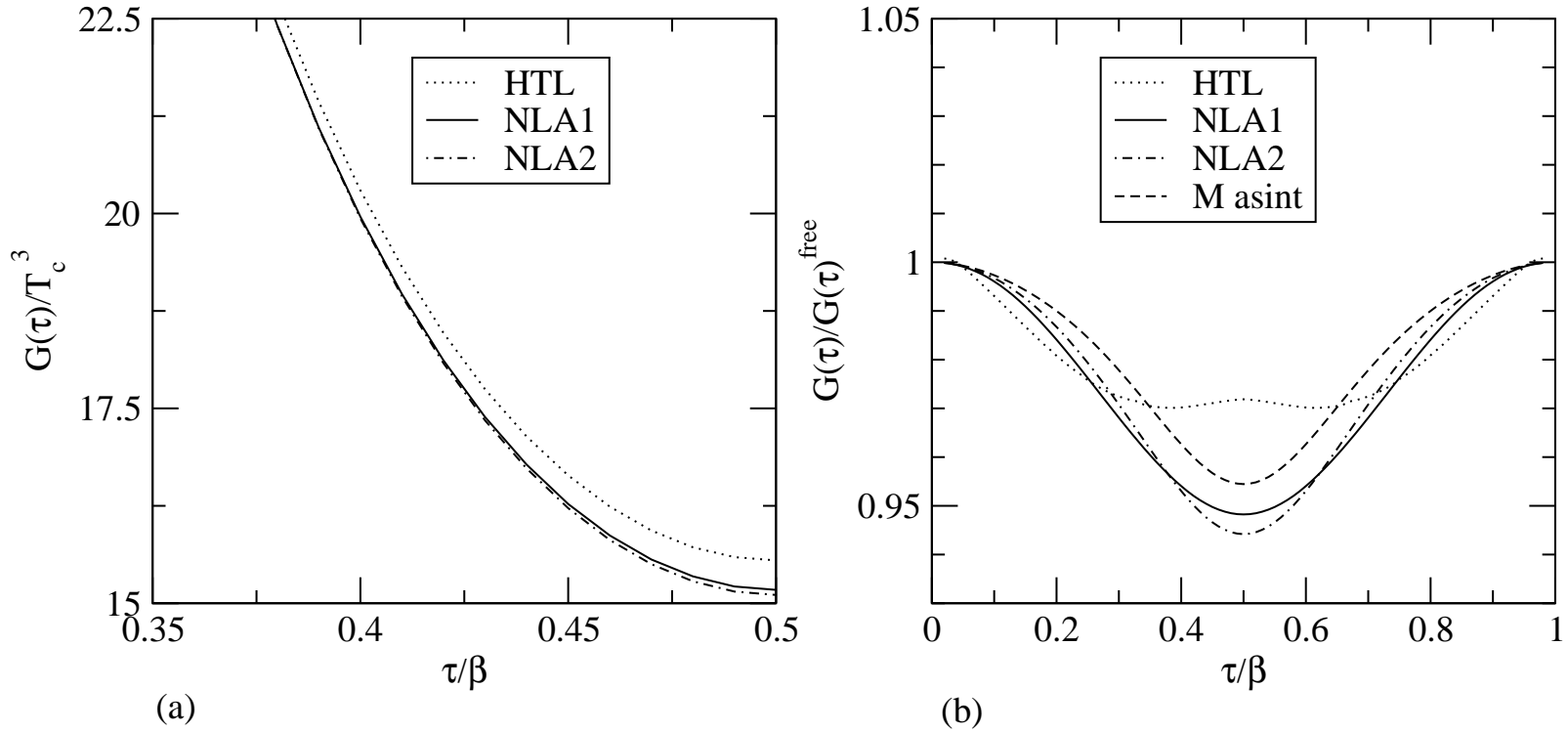
$$\cdot [\beta_+(x, k) \beta_+(\omega - x, k) + \beta_-(x, k) \beta_-(\omega - x, k)] .$$



The various contributions (pole-pole, pole-cut and cut-cut) to the dimensionless spectral function of a pseudoscalar meson  $\sigma^{\text{ps}}/T^2$  at  $P = 0$  versus  $x = \omega/T$ . **HTL approximation** both for hard and soft momenta.  $T$  is such that  $g(T) = \sqrt{6}$ , hence  $m_q = T$ . Peaks at  $\omega/T \simeq 0.47$  and  $1.86$  are the Van Hove singularities.



Zero momentum pseudoscalar spectral function  $\sigma^{ps}/T_c^2$  versus  $\omega/T_c$  in different approximations: free result, HTL, **NLA** and quarks with a thermal mass  $m_{NLA}$ . NLA1 corresponds to the choice  $\Lambda = \sqrt{2\pi T \hat{m}_D}$ , NLA2 to  $\Lambda = \sqrt{\pi T \hat{m}_D}$ . The plot refers to  $T = 2T_c$ .



(a): Behaviour of  $G(\tau)/T_c^3$  vs  $\tau/\beta$ . (b): Behaviour of  $G(\tau)/G^{\text{free}}(\tau)$  vs  $\tau/\beta$ . NLA1 corresponds to  $\Lambda = \sqrt{2\pi T \hat{m}_D}$ , NLA2 corresponds to  $\Lambda = \sqrt{\pi T \hat{m}_D}$ . In panel (b) is also displayed the result obtained in the case of quarks with a thermal mass  $m = m_{\text{NLA}}$ . The curves are given for  $T = 2T_c$ .

## Results at $p \neq 0$ (HTL approximation)

Much more cumbersome calculations, hence restricted to the pure HTL approximation. In addition to the temporal correlator  $G(-i\tau, \mathbf{p})$ , one can evaluate the **z-axis correlator** (also considered in lattice calculations)

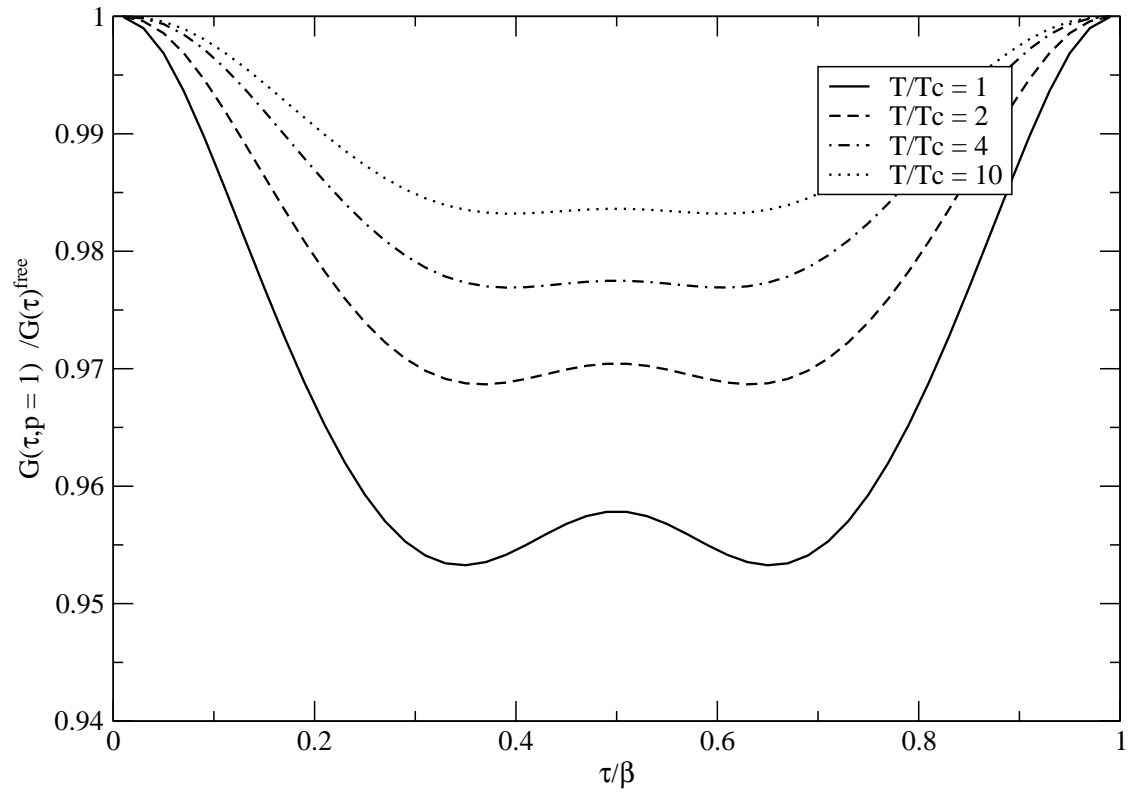
$$\mathcal{G}(z) \equiv \int_0^\beta d\tau \int d\mathbf{x}_\perp \chi_M(-i\tau, \mathbf{x}_\perp, z) = \int_{-\infty}^{+\infty} \frac{dp_z}{2\pi} e^{ip_z z} \int_{-\infty}^{+\infty} d\omega \frac{\sigma(\omega, \mathbf{p}_\perp=0, p_z)}{\omega}$$

which requires the knowledge of the finite momentum meson spectral function  $\sigma(\omega, \mathbf{p})$ .

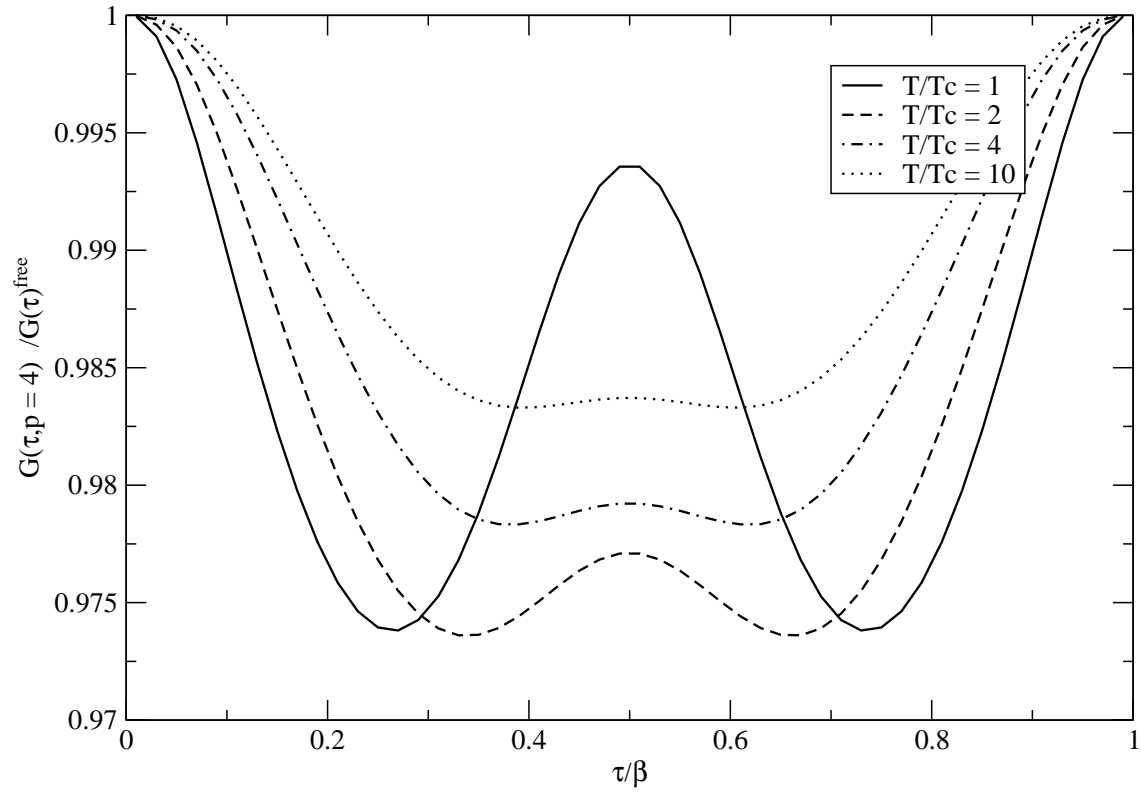
Usually the conjecture that the spatial correlations are exponentially suppressed at large  $z$ ,

$$\mathcal{G}(z) \underset{z \rightarrow +\infty}{\sim} e^{-m_{\text{scr}} z}.$$

allows to extract informations on the nature of the excitations characterizing the QGP phase.



The ratio  $G_{\text{HTL}}(\tau)/G_{\text{free}}(\tau)$  for different temperatures at  $p = 1 \text{ fm}^{-1}$ .



The ratio  $G_{\text{HTL}}(\tau)/G_{\text{free}}(\tau)$  for different temperatures at  $p = 4 \text{ fm}^{-1}$ .



## Conclusions

- We have investigated pseudoscalar mesonic thermal spectral functions in HTL approximation and (at zero momentum) in the NLA framework
- Most striking feature of HTL spectral function is the appearance of [Van Hove singularities](#), well visible at  $p = 0$  and  $T = 2T_c$ . They survive with increasing temperature, but are rapidly washed out at finite momenta.
- In the improved NLA treatment these features remain almost unaltered, particularly at low energies. The detection of Van Hove singularities would be a clear signature of deconfinement, being related to the minimum of the (collective) plasmino mode.
- However:
  - no evidence for sharp resonances in soft energy domain was found in

lattice MSF of light quarks

- the most interesting channel, for experiments, is the vector channel, associated with dilepton production.

- Vector MSF can be evaluated in HTL, but pose some problems to NLA, due to vertex corrections (identically zero in pseudoscalar channel).

Notice that in the heavy quark vector meson sector the existence of bound states above  $T_c$  has been suggested both by lattice calculations and effective potential approaches.

- Work is in progress at finite momentum, in order to obtain the spatial correlator.

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