Improved Gaussian model of QCD vacuum
Pion Distribution Amplitude and Form-Factors

A. P. Bakulev

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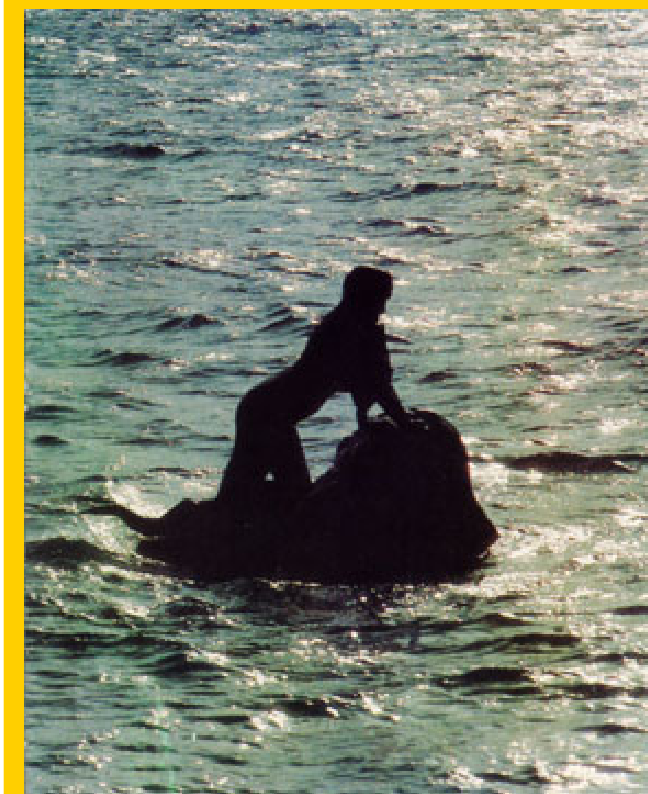

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- Improved Model for NLCs and Pion DA
- Conclusions

Collaborators & Publications

Collaborators

- S. Mikhailov BLTPh, JINR, Dubna
- A. Pimikov BLTPh, JINR, Dubna
- N. Stefanis ITP-II, Ruhr-Universität Bochum
- A. Karanikas University of Athens, Athens

Publications

- A.B., S.M., N.S. **PLB 508 (2001) 279**
- A.B., S.M., N.S. **PRD 67 (2003) 074012**
- A.B., S.M., N.S. **PLB 578 (2004) 91**
- A.B., N.S. *et al.* **PRD 70 (2004) 033014**
- A.B., N.S. **NPB 721 (2005) 50**
- A.B., A.K., N.S. **PRD 72 (2005) 074015**
- A.B., S.M., N.S. **PRD 73 (2006) 056002**

QCD SRs for π Distribution Amplitude

Pion distribution amplitude (DA)

- Matrix element of nonlocal axial current on light cone

$$\langle 0 \mid \bar{d}(z)\gamma_\mu\gamma_5 E(z,0) u(0) \mid \pi(P)\rangle\Big|_{z^2=0} = i f_\pi P_\mu \int_0^1 dx\, e^{ix(zP)}\, \varphi_\pi^{\text{Tw-2}}(x,\mu^2)$$

- **gauge-invariance** due to Fock–Schwinger string:

$$E(z,0) = \mathcal{P} e^{ig\int_0^z A_\mu(\tau) d\tau^\mu}$$

- Physical meaning of $\varphi_\pi(x;\mu^2)$ — amplitude for transition $\pi \to u + d$

Representation of Pion DA

- It is convenient to represent the pion DA:
  $$\varphi_\pi(x;\mu^2) = \varphi^{\text{As}}(x)\times$$
  $$\times\left[1+a_2(\mu^2)C_2^{3/2}(2x-1)+a_4(\mu^2)C_4^{3/2}(2x-1)+\ldots\right]$$
  where $C_n^{3/2}(2x-1)$ are the Gegenbauer polynomials (1-loop eigenfunctions of ER-BL kernel)

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$$\{a_2(\mu^2), a_4(\mu^2), \ldots\} \Leftrightarrow \varphi_\pi(x;\mu^2)$$

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- ER-BL solution at 2-loop level [Mikhailov&Radyushkin; 1986
Müller; 1994–95
A.B.&Stefanis; 2005]

Non-Local Condensates in QCD SR

- Illustration of
  NLC-model: $\langle \bar{q}(0) q(z) \rangle = \langle \bar{q}(0) q(0) \rangle e^{-|z^2| \lambda_q^2 / 8}$

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- A **single scale** parameter $\lambda_q^2 = \langle k^2\rangle$ characterizing the average momentum of quarks in QCD vacuum:

$$\lambda_q^2 = \begin{cases} 0.4 \pm 0.1 \text{ GeV}^2 & [\text{ QCD SRs, 1987 }] \\ 0.5 \pm 0.05 \text{ GeV}^2 & [\text{ QCD SRs, 1991 }] \\ 0.4 - 0.5 \text{ GeV}^2 & [\text{ Lattice, 1998-2002 }] \end{cases}$$

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- Correlation length $\lambda_q^{-1} \sim \rho$-meson size
- Possible to include second ($\Lambda \simeq 450$ **MeV**) scale with $\langle\bar{q}(0)q(z)\rangle\Big|_{|z|\gg 1 \text{ Fm}} \sim \langle\bar{q}q\rangle e^{-|z|\Lambda}$ (not included here)

Introducing NLC in QCD calculations

$$T\left(\bar{\psi}\psi\right) = \overbrace{\bar{\psi}\psi} + :\bar{\psi}\psi: \quad \textbf{(Wick theorem)}$$

$$\langle T\left(\bar{\psi}\psi\right)\rangle = i^{-1}\hat{S}_0(x) + \quad \textbf{?}$$

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$$\langle :\bar{\psi}\psi: \rangle \overset{\text{def}}{=} 0$$

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[**SVZ'79**]

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NLC QCD SR

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$F_S(z^2) + \hat{z}F_V(z^2)$

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M&R'86

Nonlocal condensat

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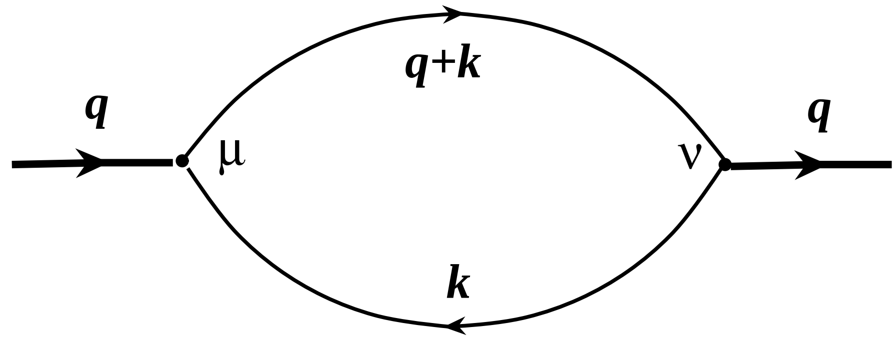
Distribution Amplitudes, Form Factors

Diagrams for $\langle T\,(J_\mu(z)J_\nu(0))\rangle$



PT

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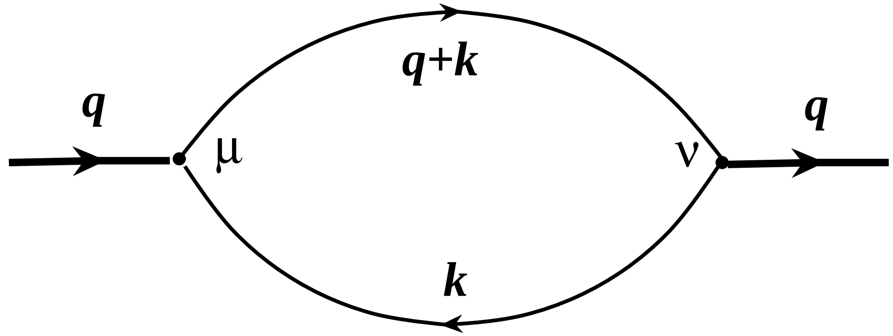
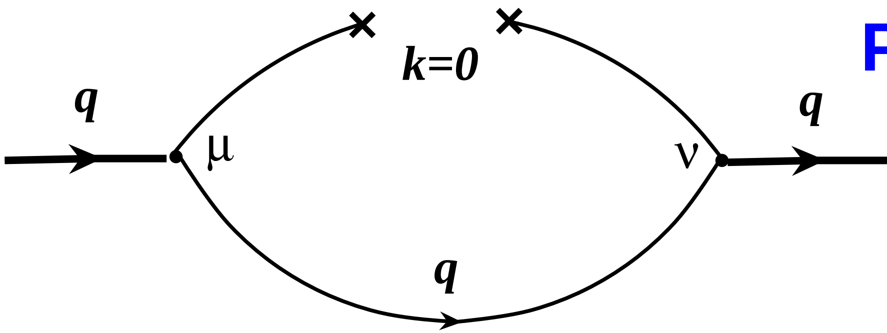


PT



SVZ SRs

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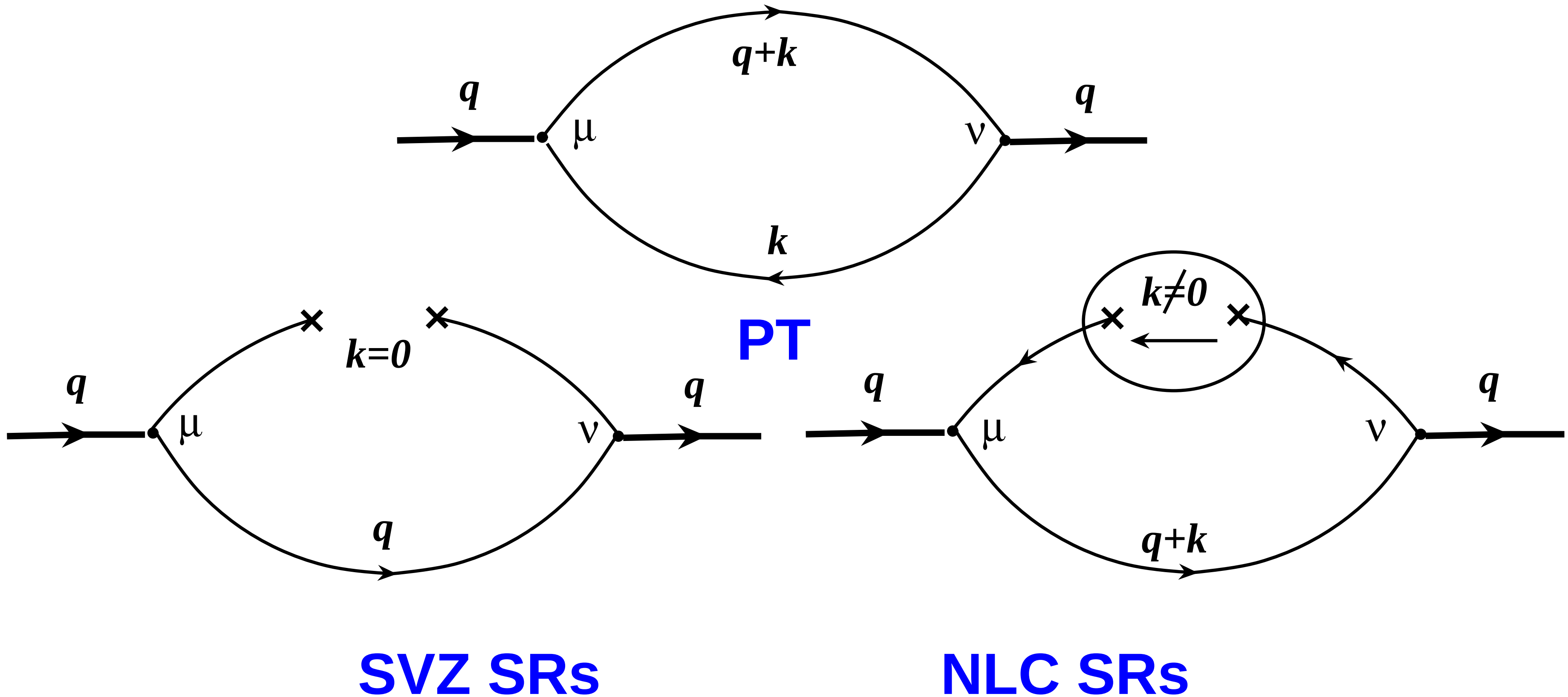

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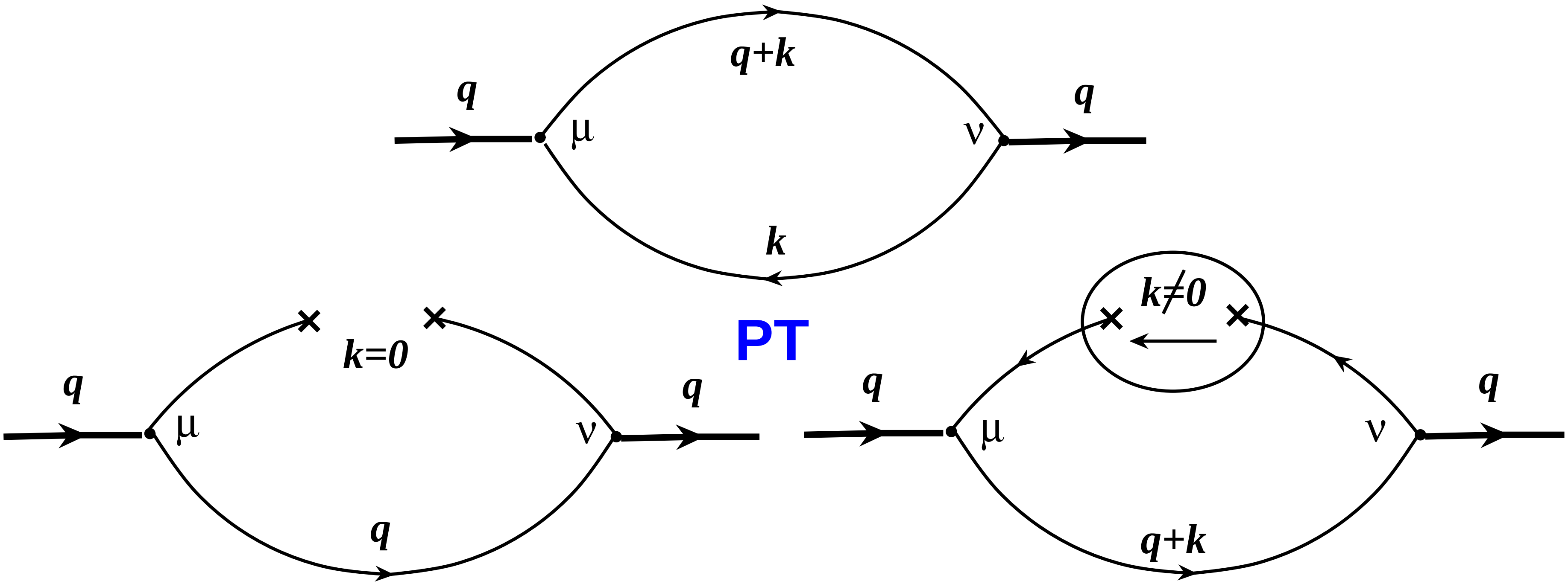

SVZ SRs **NLC SRs**

Quarks run through vacuum with nonzero momentum

$$k \neq 0: \quad \langle k^2 \rangle = \frac{\langle \bar{\psi} D^2 \psi \rangle}{\langle \bar{\psi} \psi \rangle} = \lambda_q^2 = 0.35 - 0.55\,\text{GeV}^2$$

Axial-axial correlator

We study correlator:

$$\Pi^N_{\mu\nu} = i \int d^4x\, e^{iqx} \langle 0| T \left[J^N_{\mu 5}(0) J^+_{\nu 5}(x) \right] |0\rangle$$

of two axial currents

$$J^N_{\mu 5}(0) = \bar{d}(0)\gamma_\mu\gamma_5 \left[-in\nabla \right]^N u(0)\,; \;\; J^+_{\nu 5}(x) = \bar{u}(x)\gamma_\nu\gamma_5 d(x)$$

corresponding to charged π-meson. Current $J^N_{\mu 5}(0)$ produces

$$\langle 0 \mid J^N_{\mu 5}(0) \mid \pi(P)\rangle = i f_\pi P_\mu \, (nP)^N \int_0^1 dx\, x^N \varphi_\pi(x)$$

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$$\langle 0 \mid J^N_{\mu 5}(0) \mid \pi(P)\rangle = i f_\pi P_\mu \, (nP)^N \, \langle x^N \rangle_\pi$$

NLC QCD SR for Pion DA

Here is example of QCD SR with Non-Local Condensates

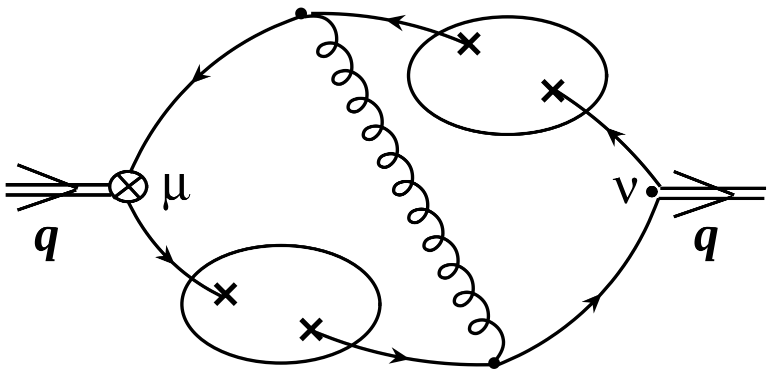
$$f_\pi^2\,\varphi_\pi(x) = \int_0^{s_0} \rho^{\text{pert}}(x;s)\,e^{-s/M^2} ds + \frac{\alpha_s GG\rangle}{24\pi M^2}\,\varphi_G(x;\Delta) + \frac{16\pi\alpha_s\langle\bar{q}q\rangle^2}{81M^4} \sum_{i=2V,3L,4Q} \varphi_i(x;\Delta)$$

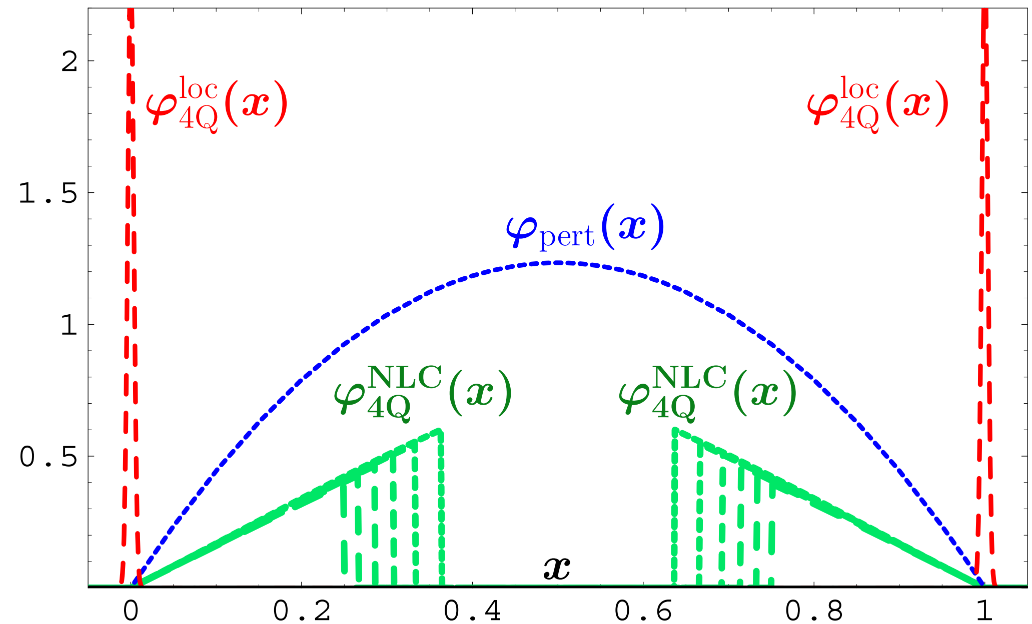
Local limit: $\lambda_q^2/M^2 \equiv \Delta \to 0$,

$$\begin{aligned}
\varphi_G(x;\Delta) &= [\delta(x)+\delta(1-x)] \\
\varphi_{2V}(x;\Delta) &= [x\delta'(1-x)+(1-x)\delta'(x)] \\
\varphi_{4Q}(x;\Delta) &= 9[\delta(x)+\delta(1-x)]
\end{aligned}$$

NLC contributions to QCD SR

Examples for Gaussian NLC with a single parameter λ_q^2



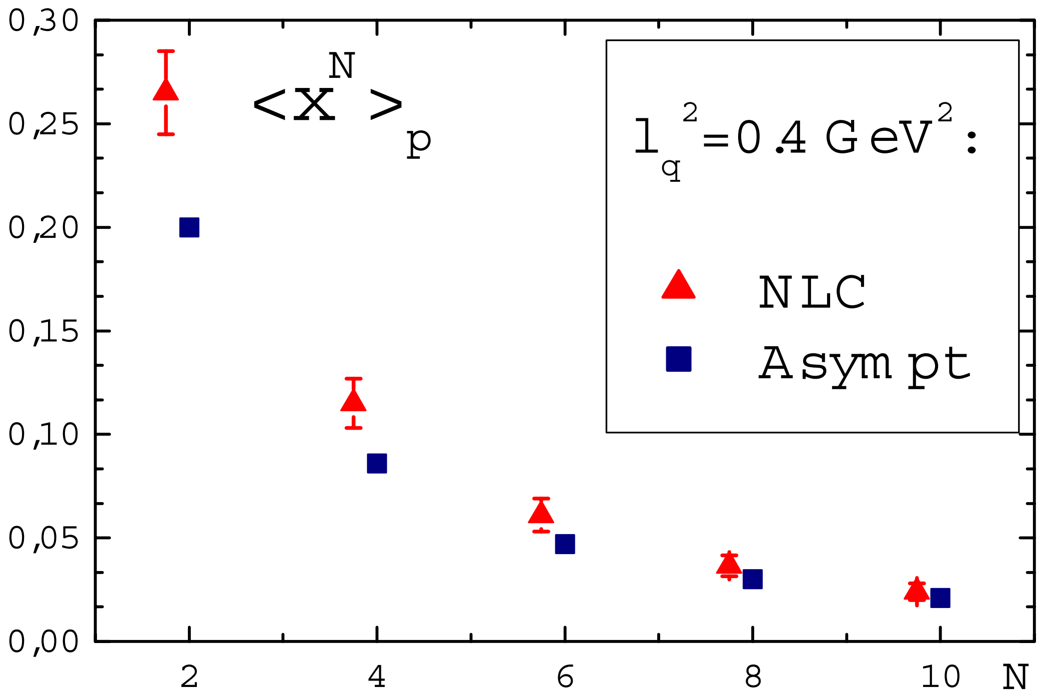


Local limit: $\lambda_q^2/M^2 \equiv \Delta \to 0$,

$$\varphi_{4Q}^{\rm loc}(x) \equiv \lim_{\Delta\to 0} \varphi_{4Q}^{\rm NLC}(x;\Delta) = 9[\delta(x) + \delta(1-x)]$$

NLC SRs for pion DA

Moments $\langle \xi^N \rangle_\pi = \int_0^1 \varphi_\pi(x)\,(2x-1)^N\,dx$ at $\mu^2 \approx 1$ **GeV^2**



from NLC SRs

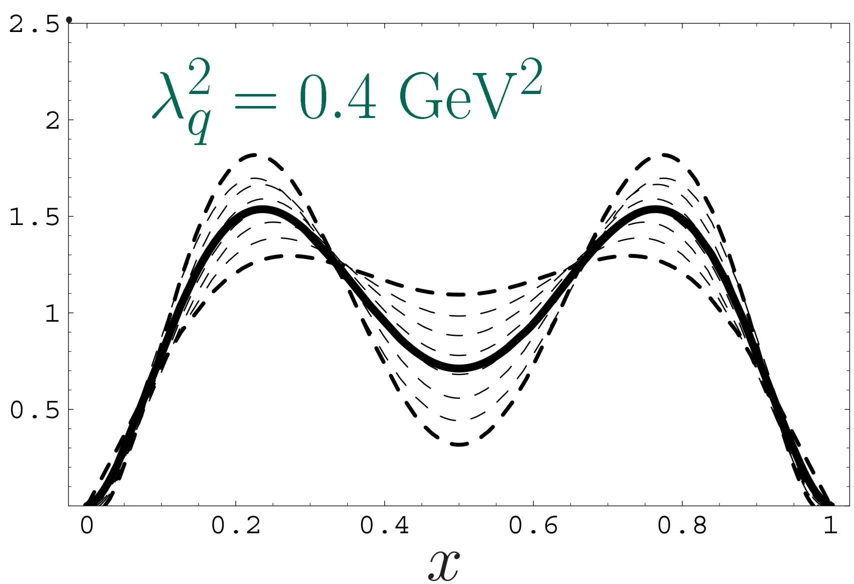
▲ **PLB 508(2001)279**

These $\langle \xi^N \rangle_\pi$ values allow one to restore DA $\varphi_\pi(x)$

NLC SRs for Pion DA

produce **bunch** of self-consistent 2-parameter models $\varphi_\pi(x)$ at $\mu^2 \simeq 1$ **GeV**^{2}:

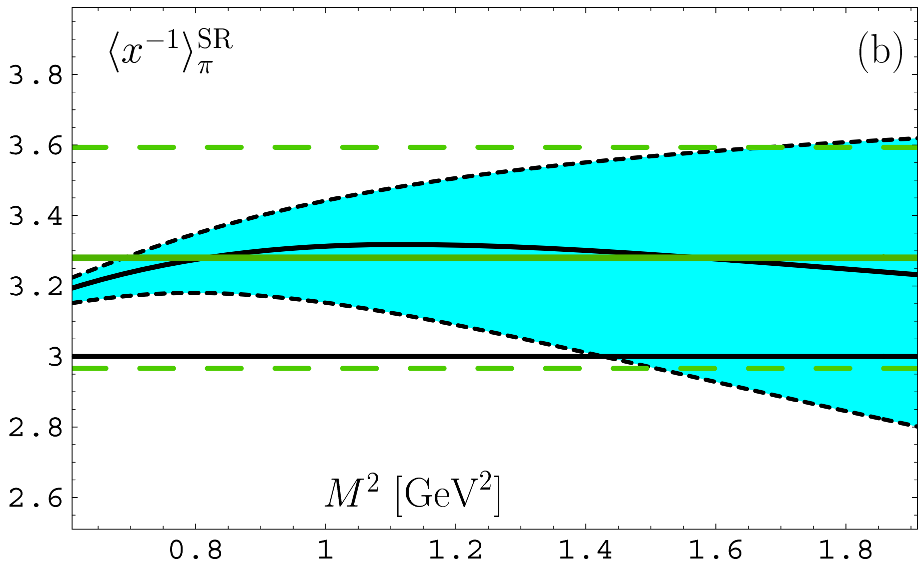
$$\varphi_\pi(x) = \varphi^{\text{As}}(x)\left[1 + a_2\, C_2^{3/2}(2x-1) + a_4\, C_4^{3/2}(2x-1)\right]$$



$$\begin{aligned} a_2^{\text{b.f.}} &= +0.188 \\ a_4^{\text{b.f.}} &= -0.130 \\ \chi^2 &\approx 0.001 \\ \langle x^{-1}\rangle^{\text{SR}} &= 3.30(30) \end{aligned}$$

NLC SR estimate of $\langle x^{-1}\rangle_\pi^{SR}$

BMS [PLB (2001)]: at $\mu^2 \simeq 1$ **GeV**^{2}



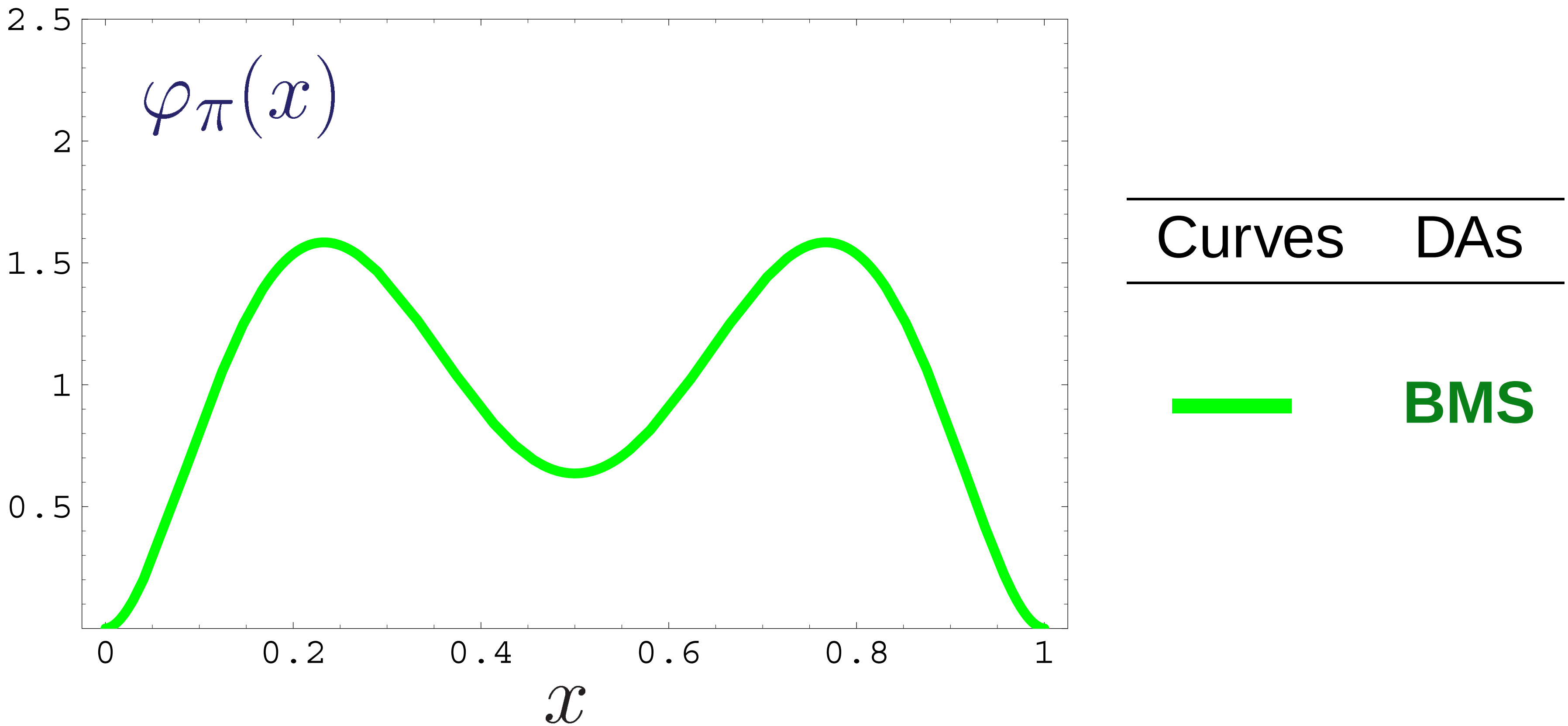
$\lambda_q^2 = 0.4$ **GeV**^{2},

$\langle x^{-1}\rangle_\pi^{\textbf{SR}} = 3.3 \pm 0.3,$

$\langle x^{-1}\rangle_\pi^{\textbf{b.f.}} = 3.17$

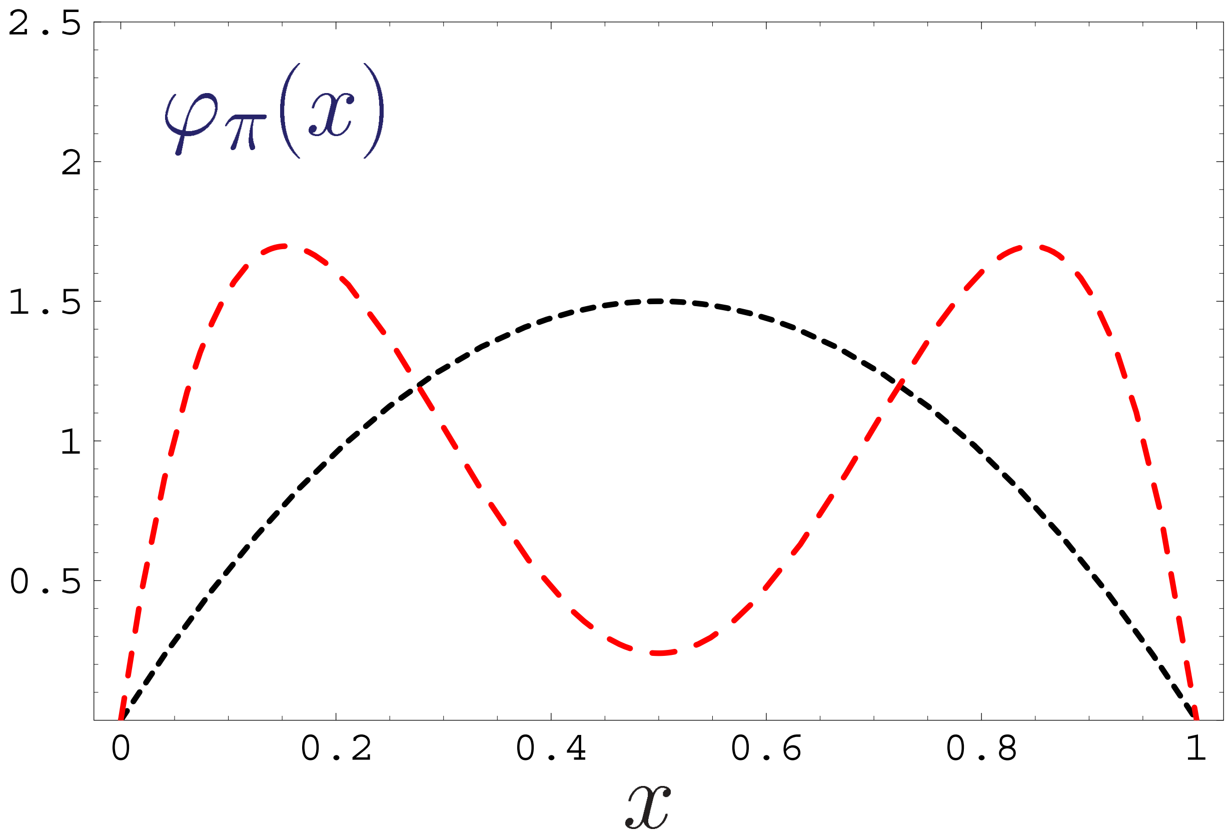
The moment $\langle x^{-1}\rangle_\pi^{\textbf{SR}}$ could be determined **only in NLC SRs** because end-point singularities absent ☛

BMS vs CZ distribution amplitude



BMS DA is end-point suppressed!

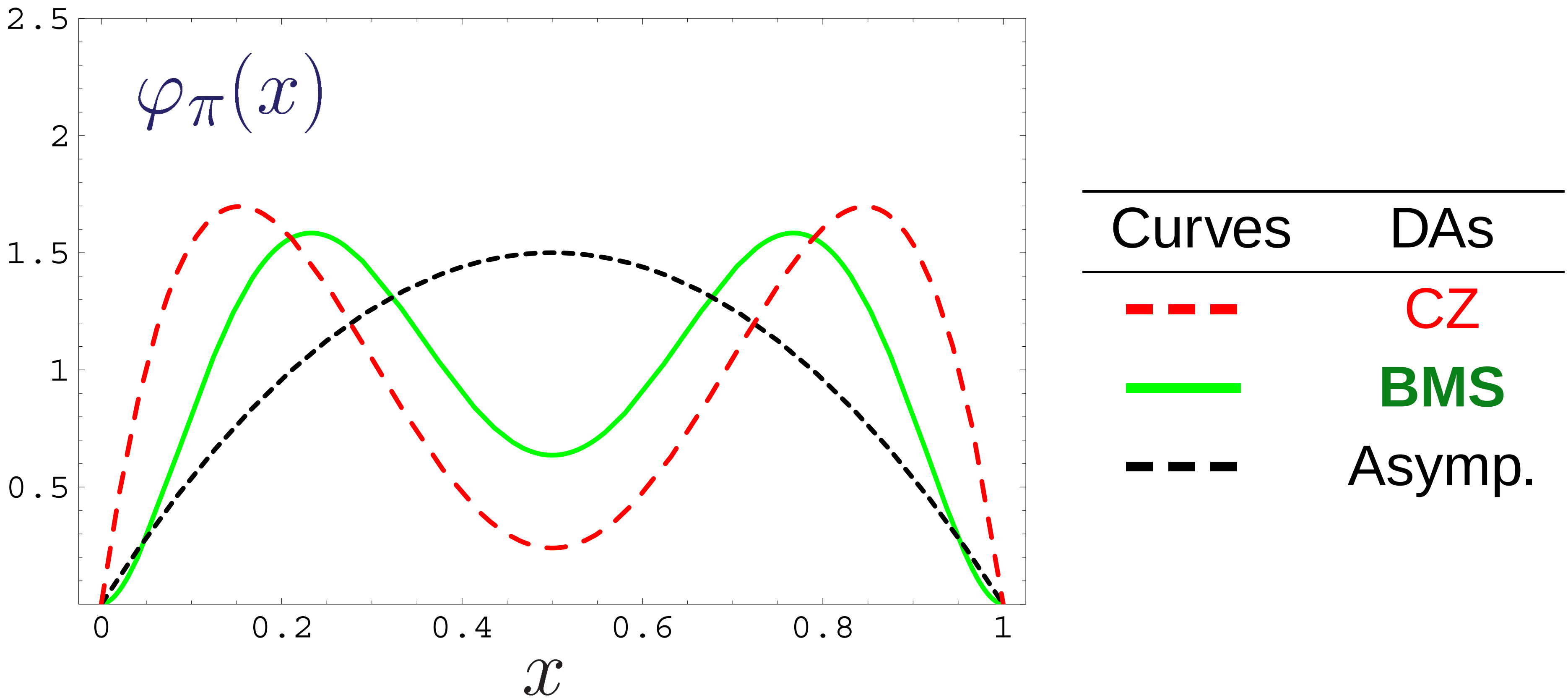
BMS vs CZ distribution amplitude



Curves	DAs
red dashed	CZ
black dashed	Asymp.

CZ DA: end-point enhancement

BMS vs CZ distribution amplitude

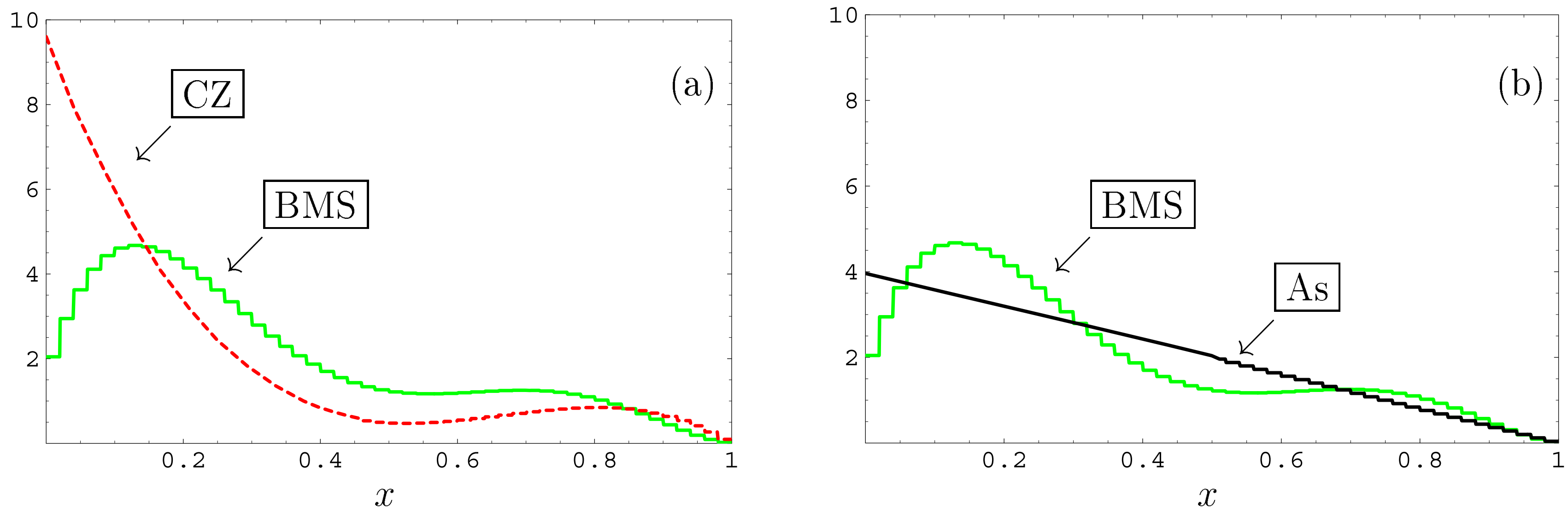


BMS bunch is 2-humped, but end-point suppressed!

Histograms for inverse moment $\langle x^{-1} \rangle_\pi$

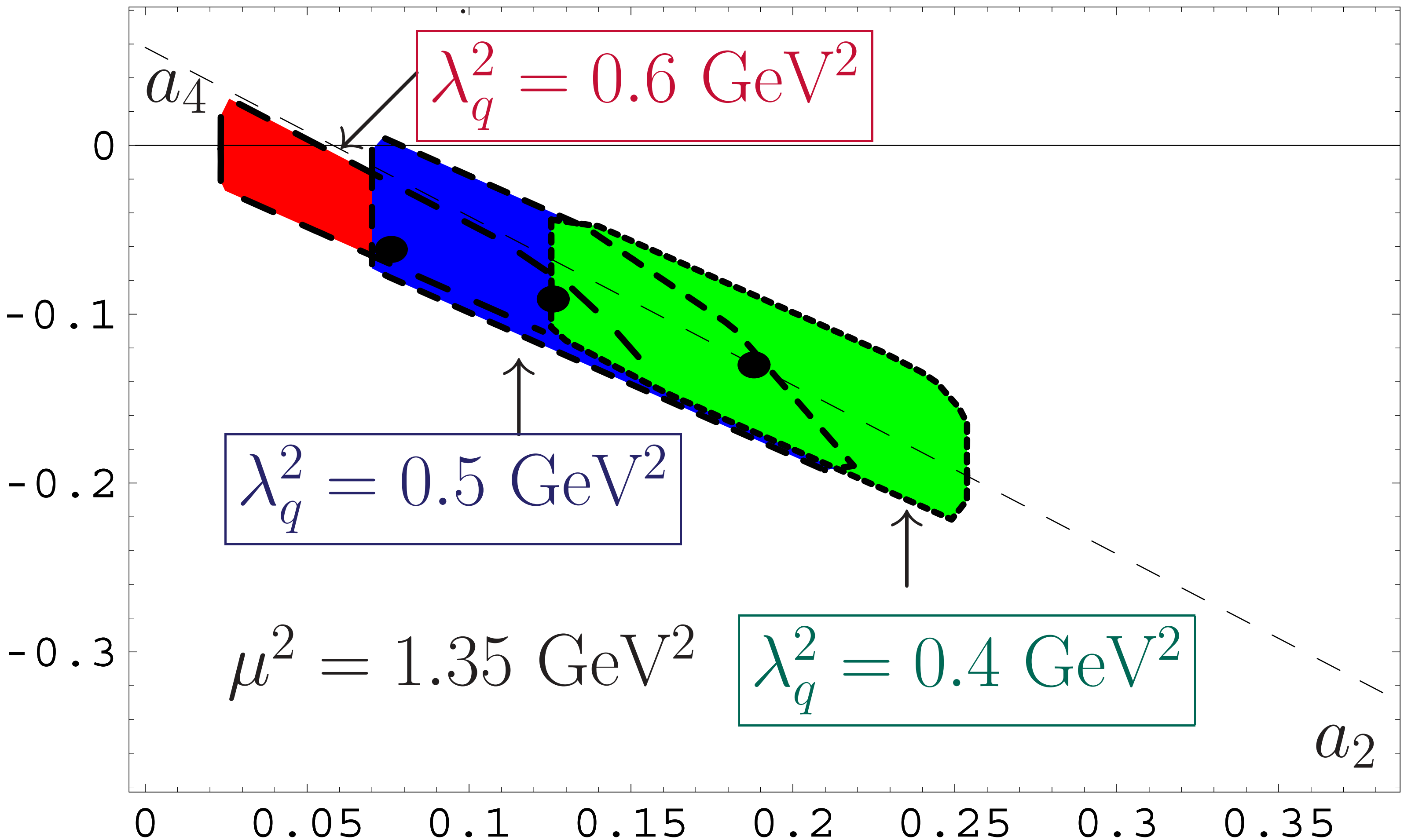
Contributions of different DAs to inverse moment $\langle x^{-1} \rangle_\pi$, calculated as $\int_x^{x+0.02} \phi(x) dx$ and normalized to 100%, for:

(a) **CZ** and **BMS** DAs; (b) Asympt. and **BMS** DAs.



In **BMS** case region $x \leq 0.1$ contributes even less than in Asymptotic DA case.

NLC SR Constraints on a_2, a_4 of Pion DA



NLO Light-Cone SRs $\Rightarrow$

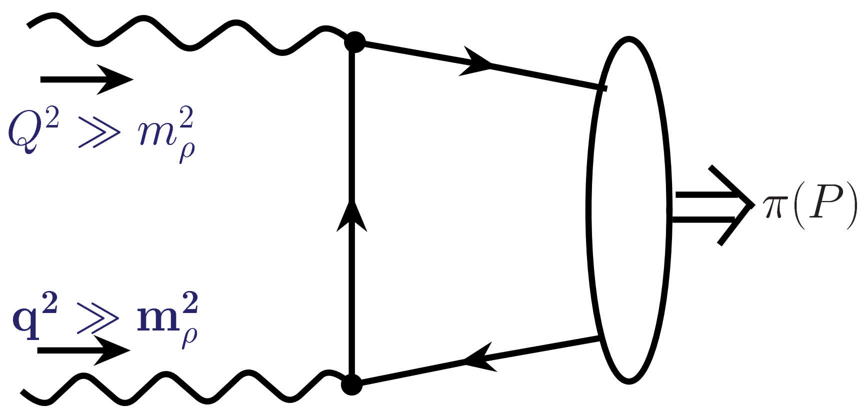
CLEO data on $F_{\gamma\gamma^*\pi}(Q^2) \Rightarrow$

Constraints on Pion DA

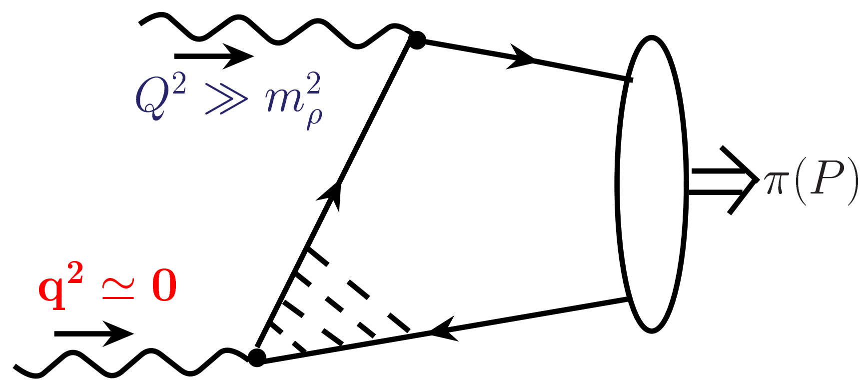
$\gamma^*\gamma \to \pi$: *Why Light-Cone Sum Rules?*

For $Q^2 \gg m_\rho^2,\ \ q^2 \ll m_\rho^2$ pQCD factorization valid only in leading twist and higher twists are of importance **[Radyushkin–Ruskov, NPB (1996)]**.

Reason: if $q^2 \to 0$ one needs to take into account interaction of real photon at long distances $\sim O(1/\sqrt{q^2})$



pQCD is OK



LCSR should be applied

$\gamma^*\gamma \to \pi$: *Why Light-Cone Sum Rules?*

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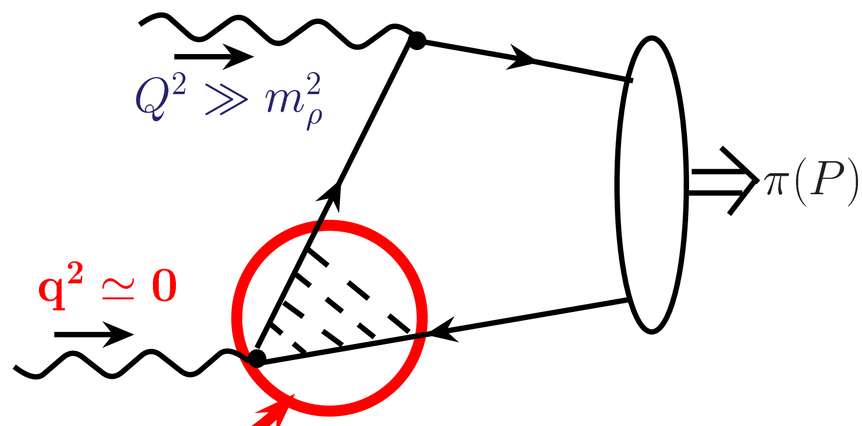

To account for long-distance effects in pQCD one needs to introduce light-cone DA of real photon

$\gamma^*\gamma \to \pi$: *Light-Cone Sum Rules!*

Khodjamirian **[EJPC (1999)]**: LCSR effectively accounts for long-distances effects of real photon using quark-hadron duality in vector channel and dispersion relation in q^2

$$F_{\gamma\gamma^*\pi}(Q^2, q^2) = \frac{1}{\pi}\int_0^{s_0} \frac{\mathrm{Im}F^{\mathrm{PT}}_{\gamma^*\gamma^*\pi}(Q^2, s)}{m_\rho^2 + q^2} e^{(m_\rho^2 - s)/M^2} ds + \frac{1}{\pi}\int_{s_0}^{\infty} \frac{\mathrm{Im}F^{\mathrm{PT}}_{\gamma^*\gamma^*\pi}(Q^2, s)}{s + q^2} ds$$

$s_0 \simeq 1.5$ **GeV^2** – effective threshold in vector channel,
M^2 – Borel parameter ($0.5 - 0.9$ GeV^2).

Real-photon limit $q^2 \to 0$ can be easily done ...

$\gamma^*\gamma \to \pi$: *Light-Cone Sum Rules!*

Khodjamirian **[EJPC (1999)]**: LCSR effectively accounts for long-distances effects of real photon using quark-hadron duality in vector channel and dispersion relation in q^2

$$F_{\gamma\gamma^*\pi}(Q^2,0) = \frac{1}{\pi}\int_0^{s_0} \frac{\text{Im}F^{\text{PT}}_{\gamma^*\gamma^*\pi}(Q^2,s)}{m_\rho^2} e^{(m_\rho^2-s)/M^2} ds + \frac{1}{\pi}\int_{s_0}^{\infty} \frac{\text{Im}F^{\text{PT}}_{\gamma^*\gamma^*\pi}(Q^2,s)}{s} ds$$

$s_0 \simeq 1.5$ GeV^2 – effective threshold in vector channel,
M^2 – Borel parameter ($0.5 - 0.9$ GeV^2).

... as demonstrated here.

Revision of CLEO data analysis

- Accurate NLO evolution for both $\varphi(x, Q^2_{\textsf{exp}})$ and $\alpha_s(Q^2_{\textsf{exp}})$, taking into account quark thresholds;

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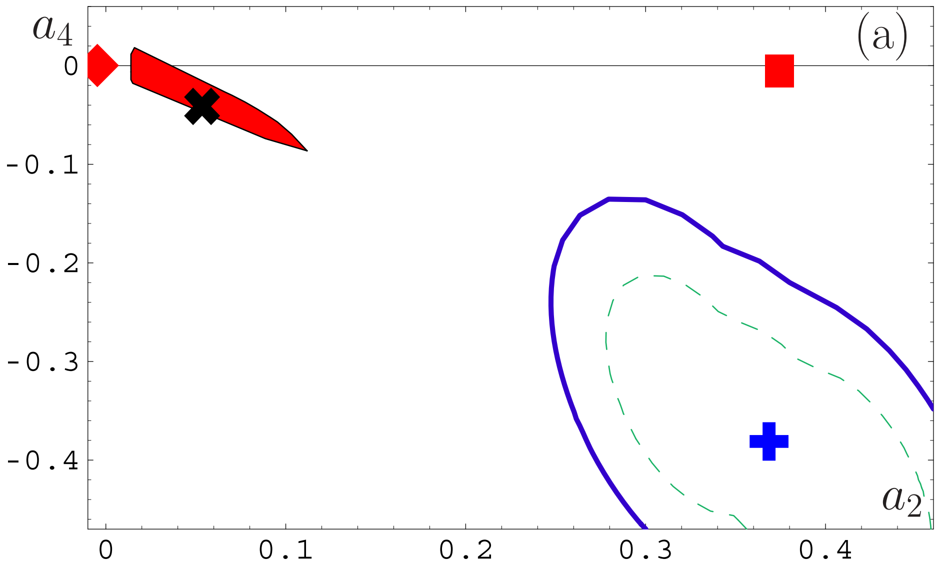
- Constraints on $\langle x^{-1} \rangle_\pi$ from CLEO data.

NLC SR Results vs NLO CLEO Constraints

[BMS, PRD 67 (2003) 074012]



■ $\Leftrightarrow \lambda_q^2 = 0.6$ **GeV**^{2},

$\delta^2_{\text{Tw-4}} = 0.28(3)$ **GeV**^{2}

No agreement with CLEO data for $\lambda_q^2 = 0.6$ GeV^2

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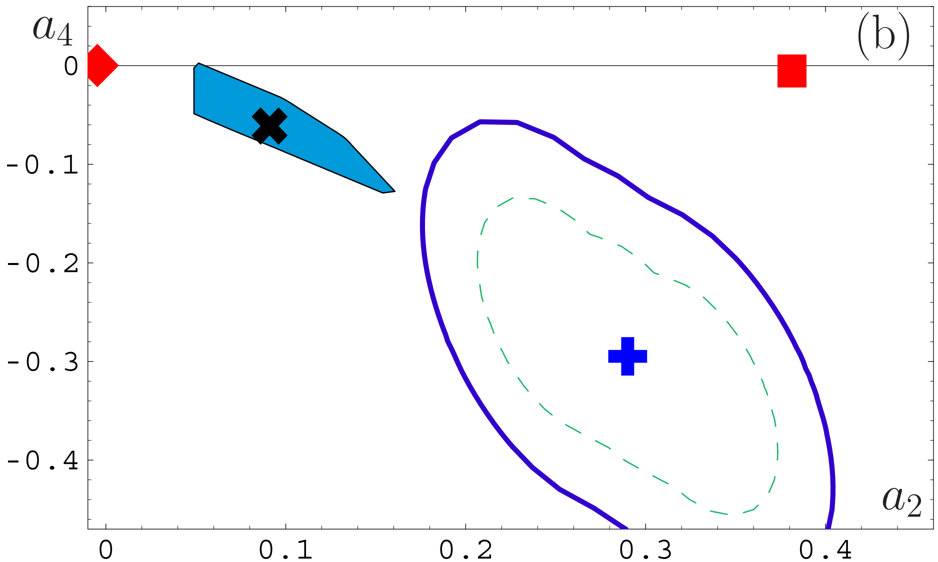
[BMS, PRD 67 (2003) 074012]

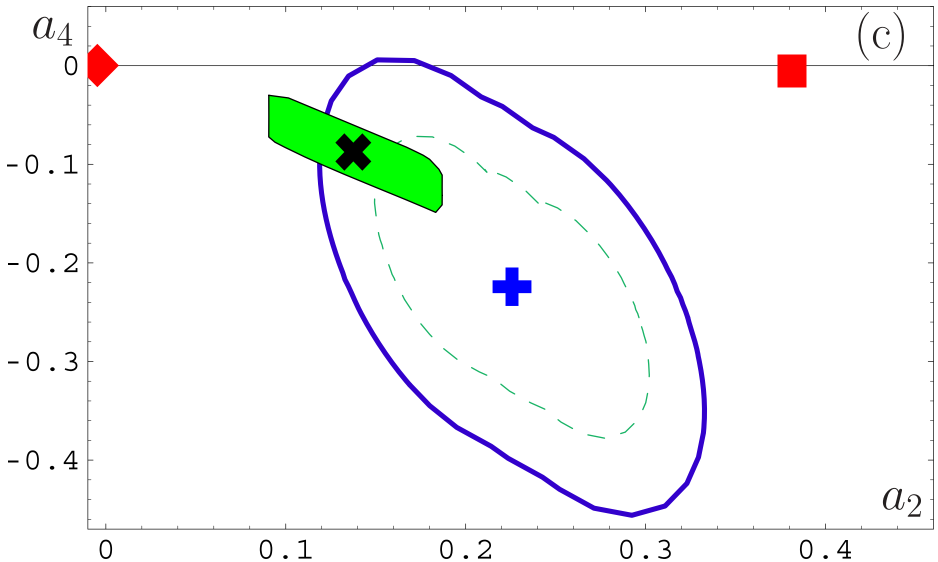


$\Leftrightarrow \lambda_q^2 = 0.5$ **GeV**^{2},
$\delta^2_{\text{Tw-4}} = 0.23(2)$ **GeV**^{2}

Bad agreement with CLEO data for $\lambda_q^2 = 0.5$ GeV^2

NLC SR Results vs NLO CLEO Constraints

[BMS, PRD 67 (2003) 074012]



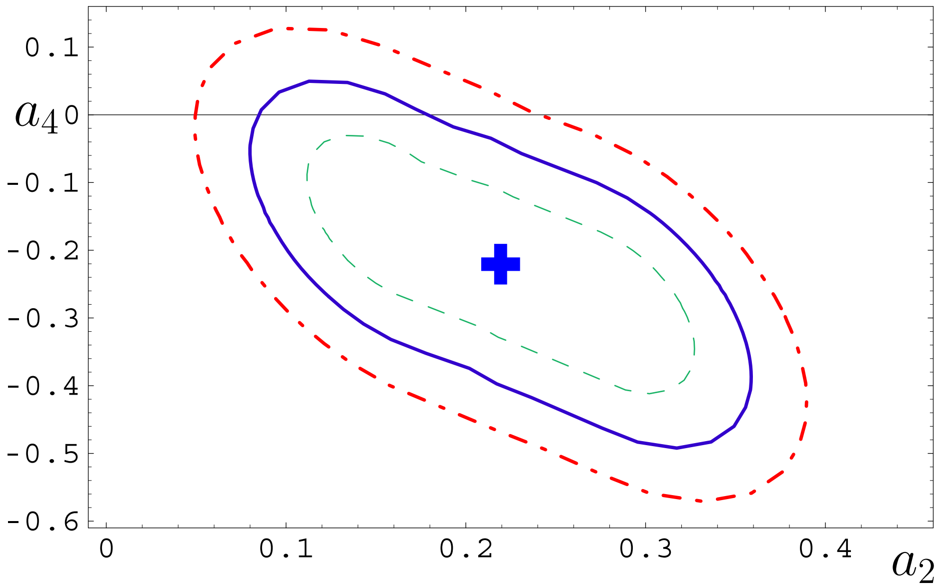
[green band] $\Leftrightarrow \lambda_q^2 = 0.4$ **GeV^2**,

$\delta^2_{\text{Tw-4}} = 0.19(2)$ **GeV^2**

Good agreement with CLEO data for $\lambda_q^2 = 0.4$ GeV^2

NLC SRs vs Revised CLEO Constraints

NLO Light-Cone SR $\oplus$ Twist-4 $\oplus(\mu^2 = Q^2)$
with 20% uncertainty of $\delta^2_{\text{Tw-4}}$
BMS [PLB 578 (2004) 91]: $\lambda_q^2 = 0.4$ **GeV**^{2}, $\delta^2_{\text{Tw-4}} = 0.19(4)$ **Ge**



✚ = best-fit point

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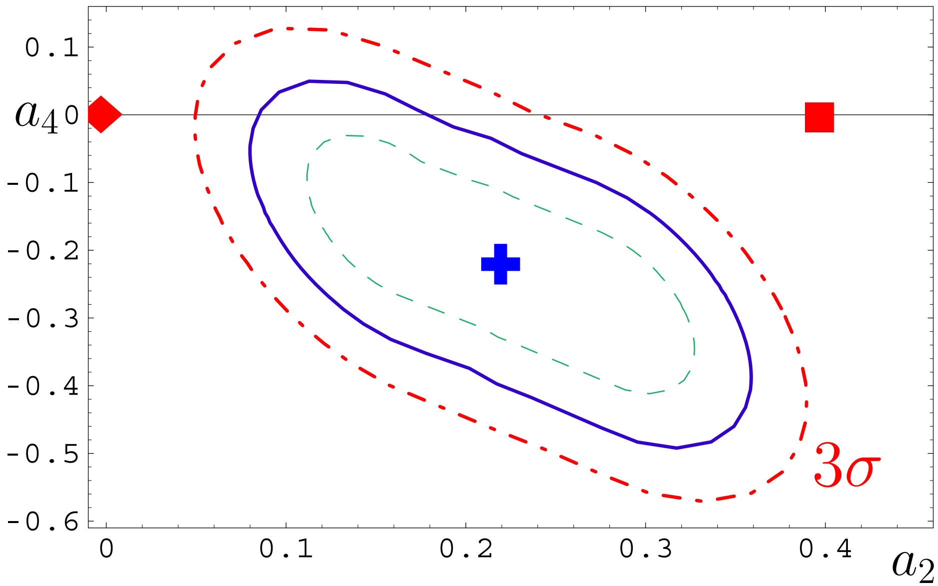

✚ = best-fit point
◆ = Asymptotic DA
■ = CZ DA

Even with 20% uncertainty in twist-4
CZ DA excluded **at least** at 4σ-level! **As** DA — at 3σ-level.

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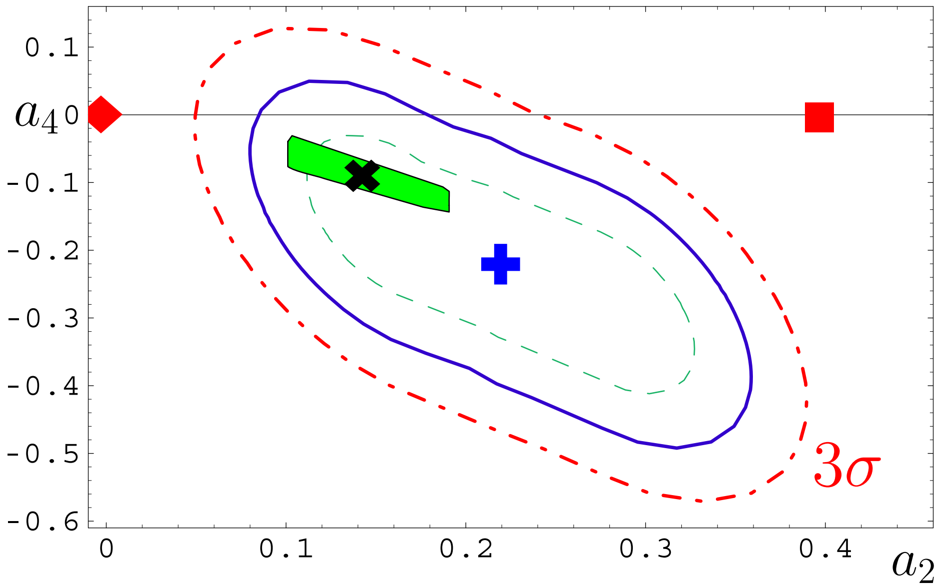

✚ = best-fit point
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BMS DA and most of **BMS bunch** — inside 1σ-domain.

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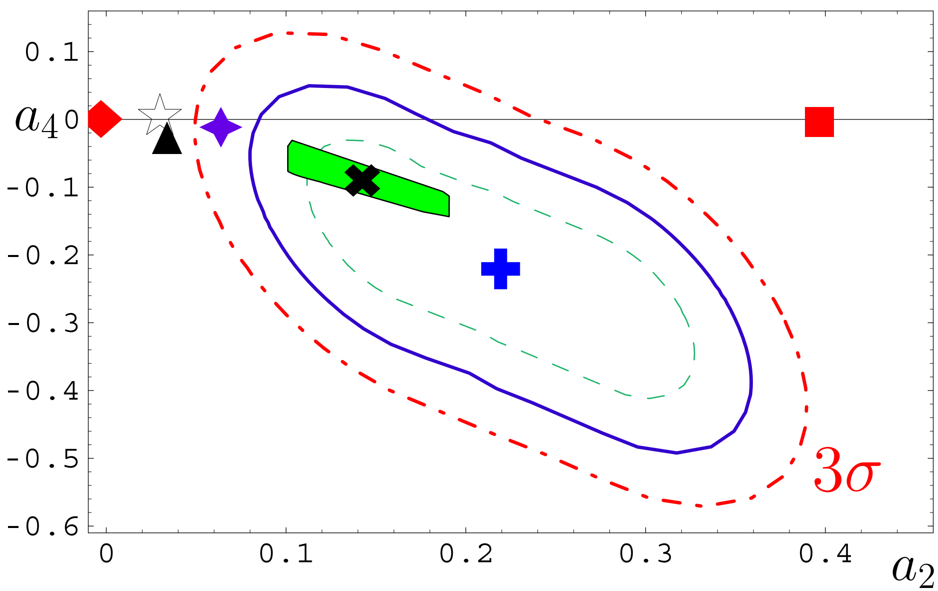

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- ✖ = BMS model
- ☆, ▲ and ✦ = instantons

BMS DA and most of **BMS bunch** — inside 1σ-domain.
Instanton-based models — near 3σ-boundary
(**PR**-model is close to 2σ-boundary).

NLC SRs vs Revised CLEO Constraints

NLO Light-Cone SR $\oplus$ Twist-4 $\oplus(\mu^2 = Q^2)$
with 20% uncertainty of $\delta^2_{\text{Tw-4}}$

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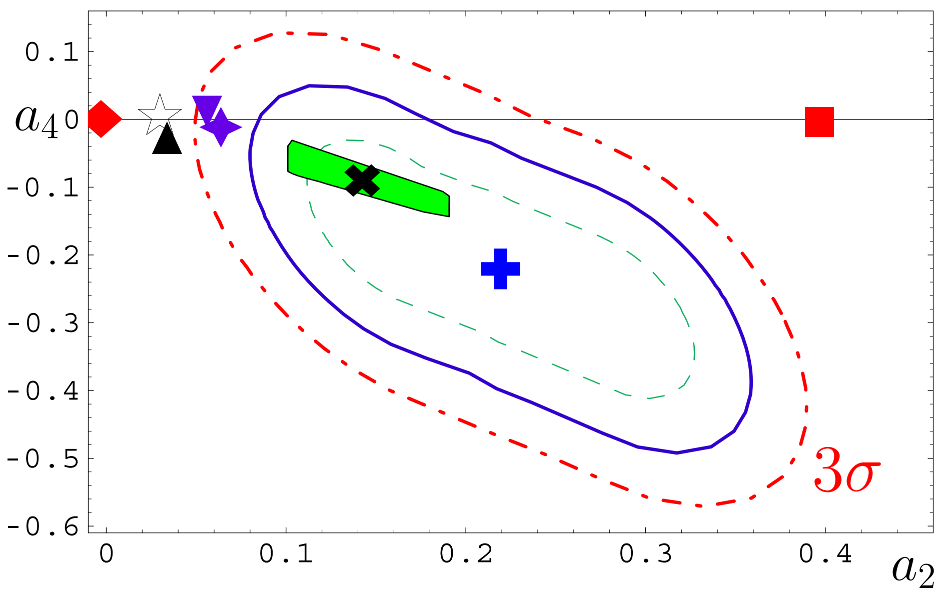

- = best-fit point
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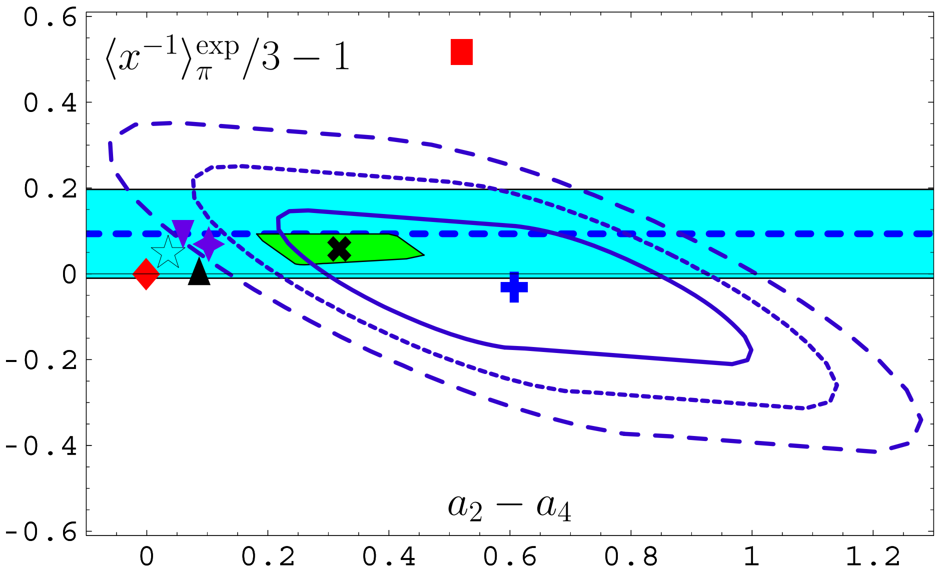
BMS DA and most of **BMS bunch** — inside 1σ-domain.
Transverse lattice model — near 3σ-boundary.

New CLEO data constraints for $\langle x^{-1}\rangle_\pi$

BMS [PLB 578 (2004) 91]: evolution to $\mu^2 = 1$ **GeV^2**



$\lambda_q^2 = 0.4$ **GeV^2**,

$\frac{1}{3}\langle x^{-1}\rangle_\pi^{\rm SR} - 1 = 0.1 \pm 0.1$ ☞

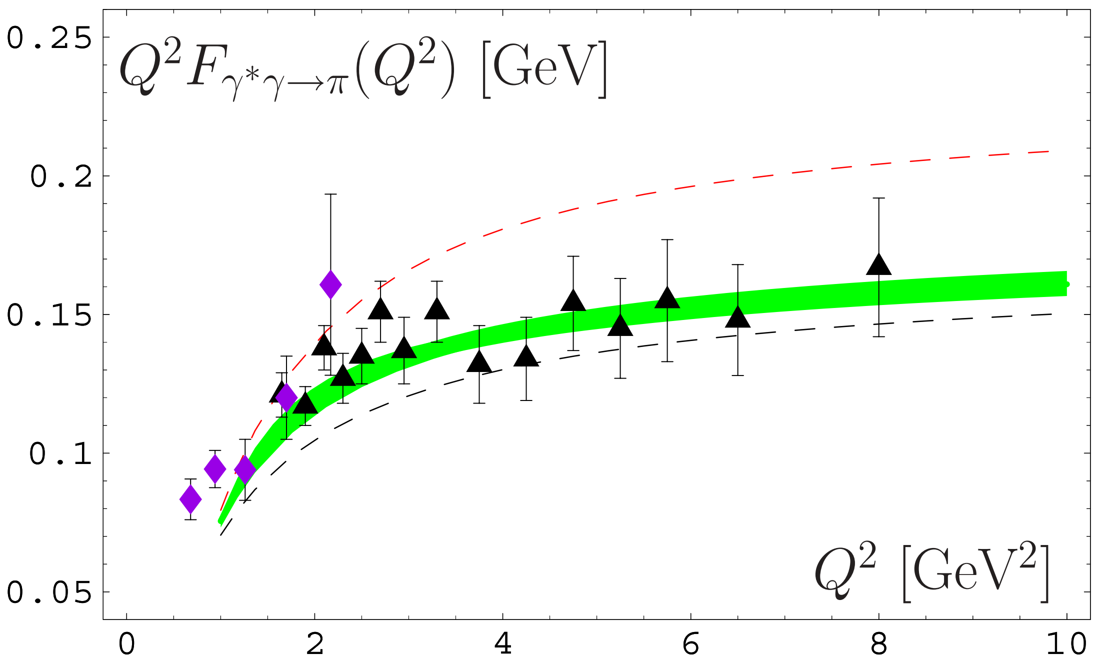
See also **Bijnens&Khodjamirian [EPJC (2002)]**:

$\frac{1}{3}\langle x^{-1}\rangle_\pi - 1 = 0.24 \pm 0.16$

Again:

Good agreement of a theoretical “tool” of different origin with CLEO data

LCSR vs. CELLO (♦) & CLEO (▲) data



curve	DA
	CZ
	BMS bunch
	PR-01
	PPRWG-99
	Asymp.

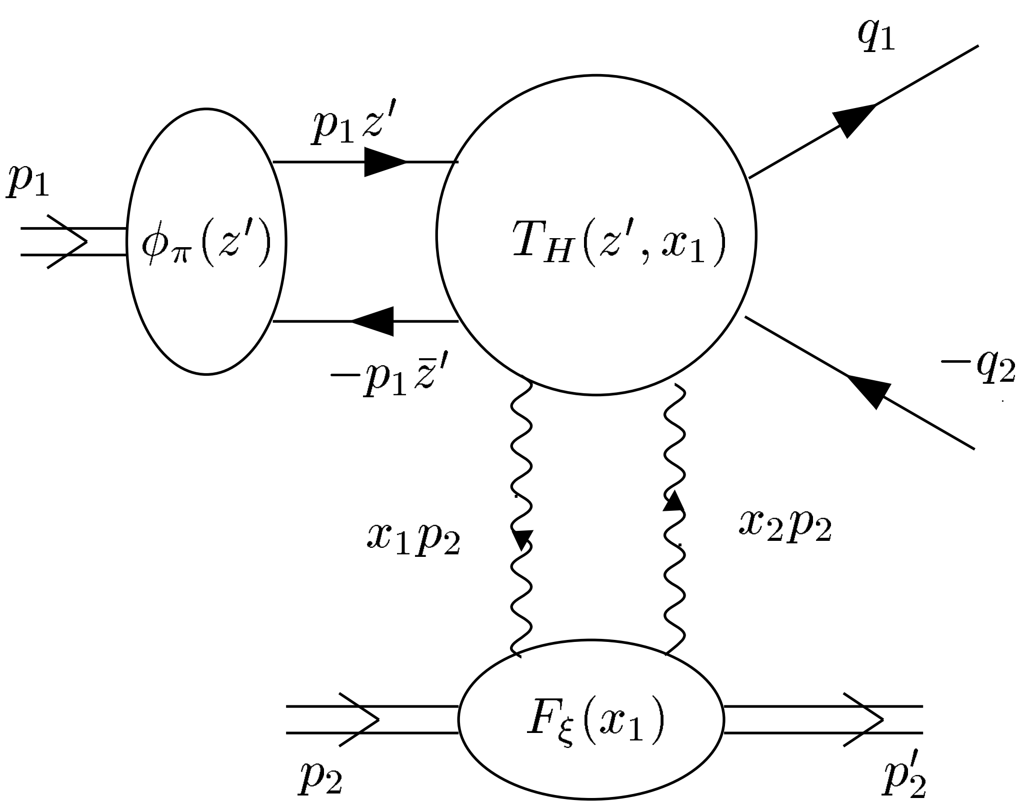
BMS bunch describes rather well all data for $Q^2 \gtrsim 1.5$ **GeV^2**.

Diffractive Dijet Production

What can add
E791 data
(how much time we have?)

E791: Diffractive dijet production

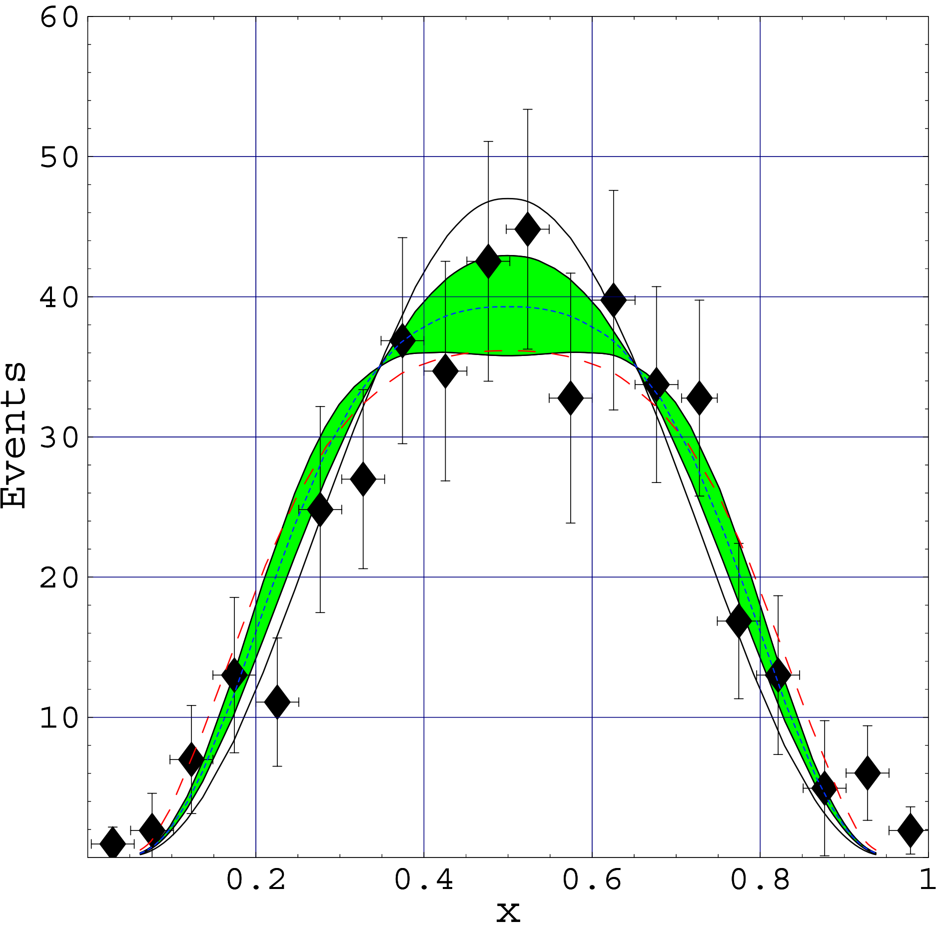
Frankfurt et al. [PLB (1993)]: Rough estimations
Braun et al. [NPB (2002)]: Account for hard GEXs



$q_\perp^2 \simeq 4$ **GeV^2**
$s \simeq 1000$ **GeV^2**

E791: Good agreement with BMS bunch

Following convolution procedure of **Braun et al.**, we found



[PLB 578 (2004) 91]

	DA	χ^2
(solid black line)	Asymp.	**12.56**
(green band)	**BMS bunch**	**10.96**
(red dashed line)	**CZ**	**14.15**

(accounting for 18 data points)

Our bunch of pion DAs has maximum uncertainty in the central region, but **agrees well** with E791 data!

JLab data for $F_\pi(Q^2)$ in Analytic NLO pQCD

Analytic Perturbation Theory

Analyticization means procedure to obtain analyticity of hadronic observables in whole Q^2 region via dispersion relations (**Radyushkin, Krasnikov&Pivovarov, Dokshitzer, Beneke&Braun, Shirkov&Solovtsov**): Analytization combines

- **RG invariance** $\Longrightarrow$ resummation of UV logs and correct QCD asymptotics

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- **RG invariance** $\Longrightarrow$ resummation of UV logs and correct QCD asymptotics
- **Causality** $\Longrightarrow$ spectral representation
  $\Longrightarrow$ **no Landau singularity**

Analytic Perturbation Theory

Analytic Perturbation Theory expresses QCD observables over **non-power sequences** $\{\mathcal{A}_k^{(L)}(Q^2)\}$ in L-loop order **[Shirkov, NPB Proc. 64 (1998) 106]**.
At 1-loop:

$$\mathcal{A}_k^{(1)}(Q^2) = \frac{1}{\pi}\int_0^\infty \frac{\rho_k^{(1)}(\sigma)\, d\sigma}{\sigma + Q^2 - i\epsilon}\ ;\ \rho_k^{(1)}(\sigma) = \mathrm{Im}\left(\frac{4\pi}{b_0\ \ln(-\sigma/\Lambda^2)}\right)^k$$

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with 1-loop explicit expressions

- $\mathcal{A}_1^{(1)}(Q^2) = \frac{4\pi}{b_0}\left[\frac{1}{\ln(Q^2/\Lambda^2)} + \frac{\Lambda^2}{\Lambda^2 - Q^2}\right]$

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- $\mathcal{A}_2^{(1)}(Q^2) = \left(\frac{4\pi}{b_0}\right)^2\left[\frac{1}{\ln^2(Q^2/\Lambda^2)} + \frac{Q^2\Lambda^2}{(\Lambda^2 - Q^2)^2}\right]$

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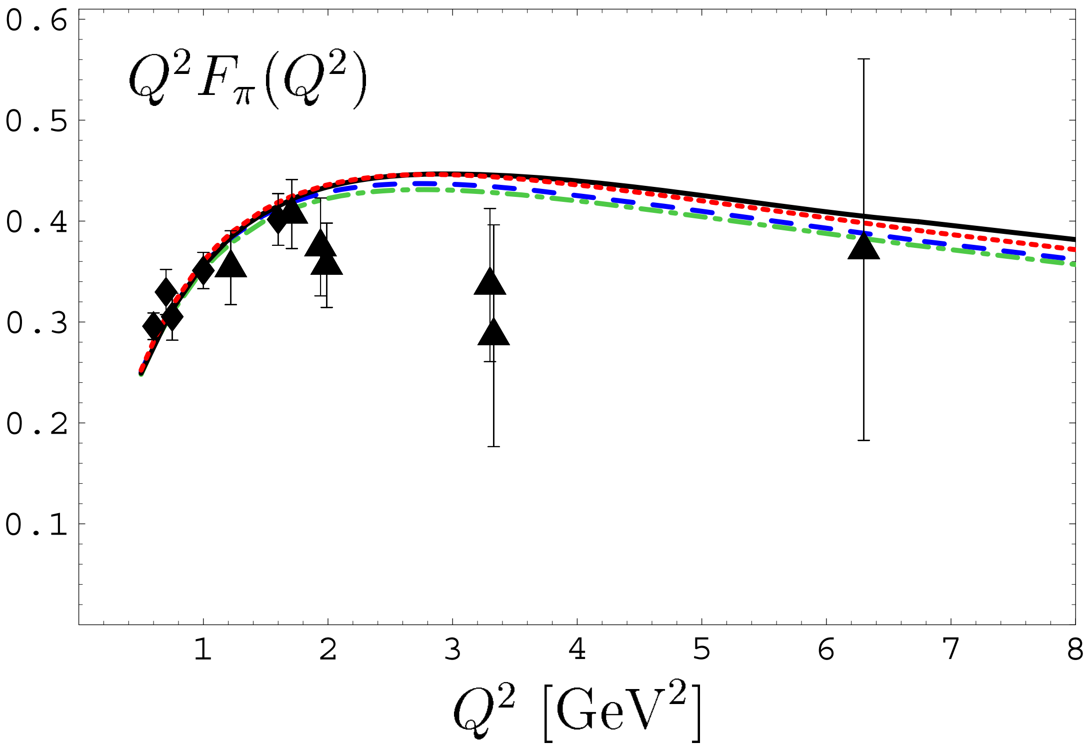
Important: $\mathcal{A}_2(Q^2) \neq \left[\mathcal{A}_1(Q^2)\right]^2$

Pion form factor in analytic NLO pQCD

[AB-Passek-Schroers-Stefanis, PRD 70 (2004) 033014]



Curves	Schemes
solid black	$\mu_R^2 = 1$ GeV^2
dashed blue	$\mu_R^2 = Q^2$
dotted red	BLM scale
dash-dotted green	α_V-scheme

Practical independence on scheme/scale setting!

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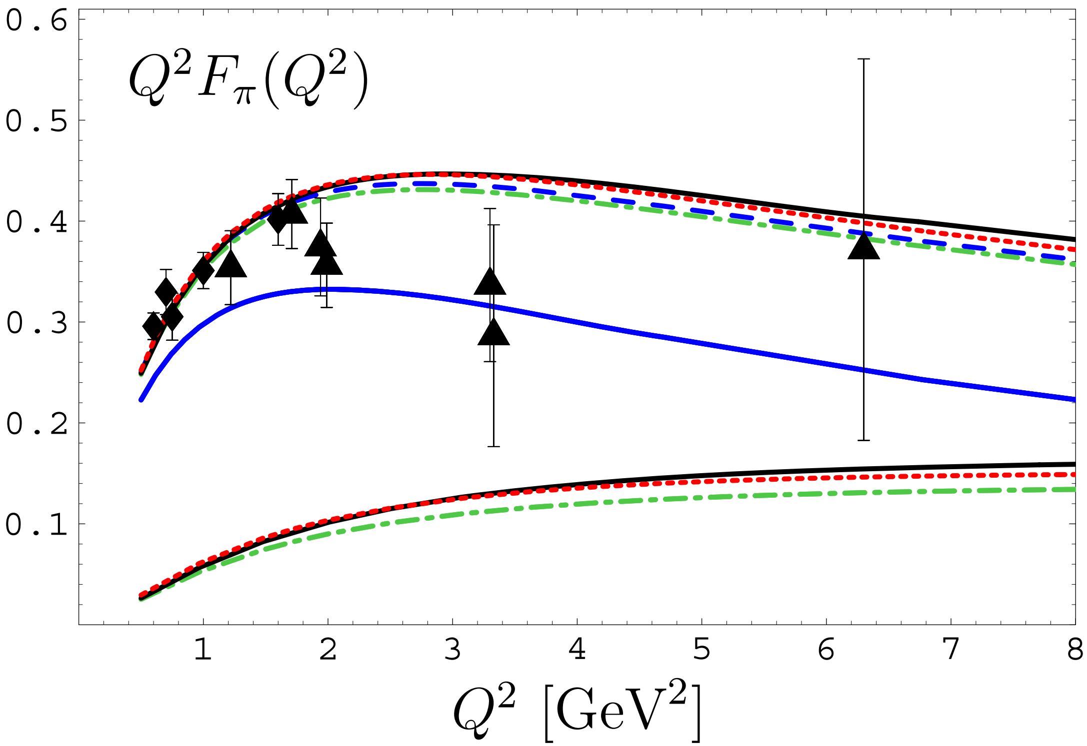
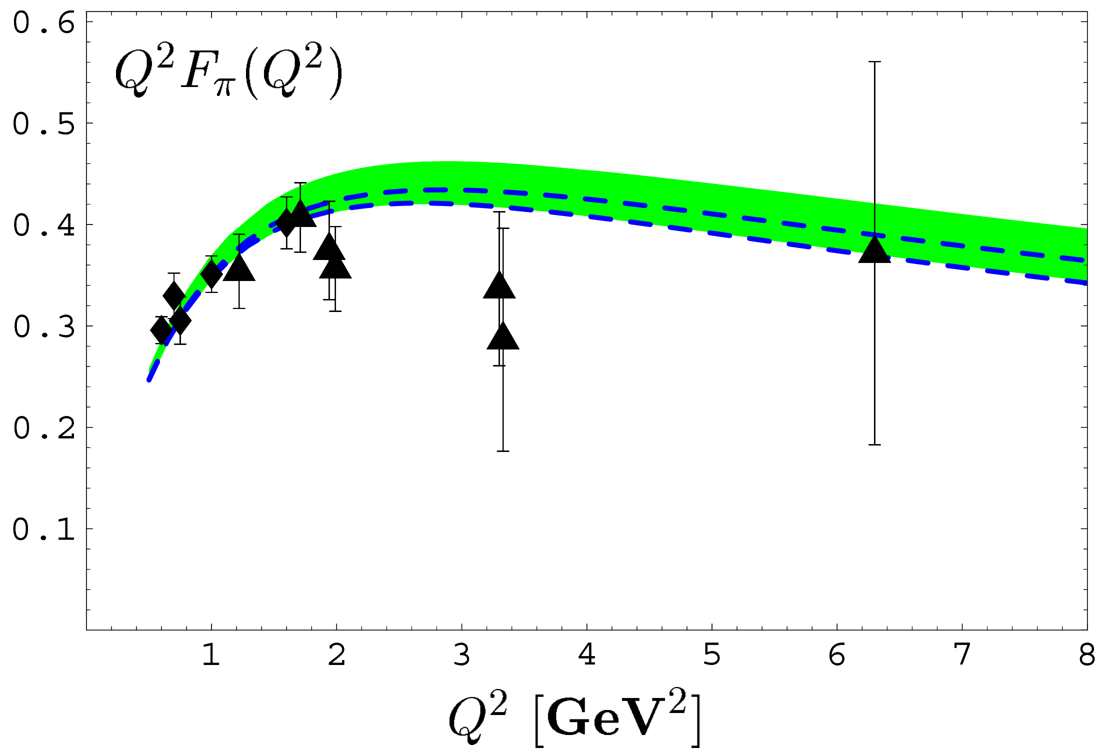


Curves	Schemes
solid black	$\mu_R^2 = 1$ GeV^2
dashed blue	$\mu_R^2 = Q^2$
dotted red	BLM scale
dash-dotted green	α_V-scheme
solid blue	soft part

Practical independence on scheme/scale setting!

Pion FF in analytic NLO pQCD



Green strip includes

- **NLC QCD SRs uncertainties** (**pion DA bunch**);
- **scale-setting ambiguities** at NLO level.

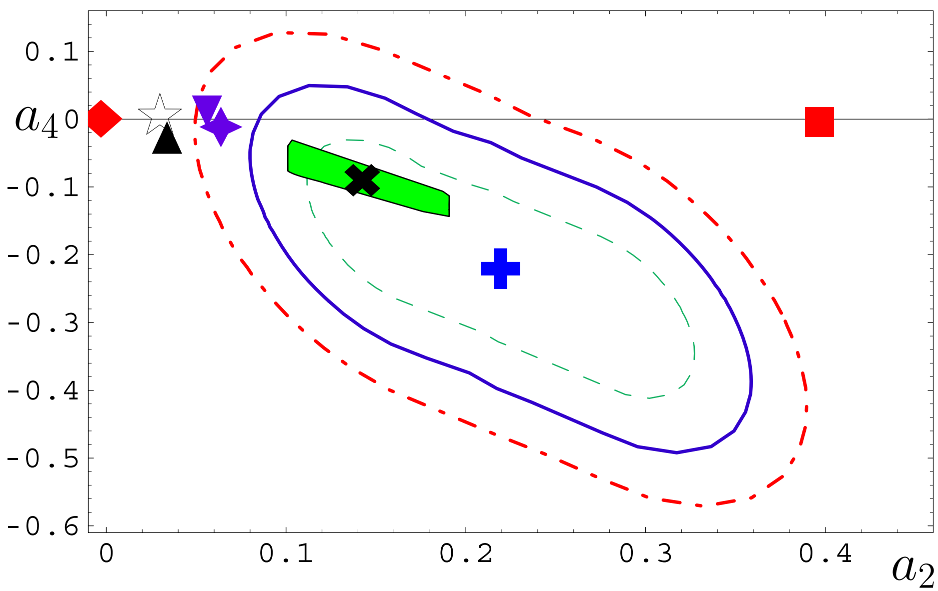
New Lattice Data for pion DA

Revised CLEO Constraints and Lattice Data

NLO Light-Cone SR $\oplus$ Twist-4 $\oplus(\mu^2 = Q^2)$

with 20% uncertainty of $\delta^2_{\text{Tw-4}}$: $\delta^2_{\text{Tw-4}} = 0.19 \pm 0.04$ **GeV**^{2}

BMS [PLB 578 (2004) 91]: $\lambda_q^2 = 0.4$ **GeV**^{2}



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BMS DA and most of **BMS bunch** — inside 1σ-domain.
Transverse lattice model — near 3σ-boundary.

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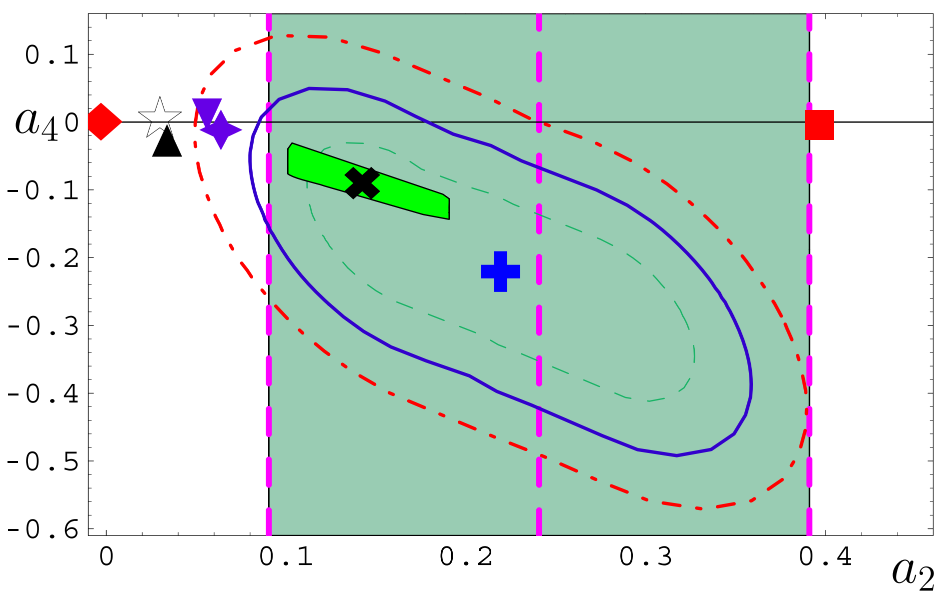

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- **gray strip** = lattice'04 result

BMS DA and most of **BMS bunch** — inside 1σ-domain and inside **2004 lattice strip [PRD 73 (2006) 056002]**.

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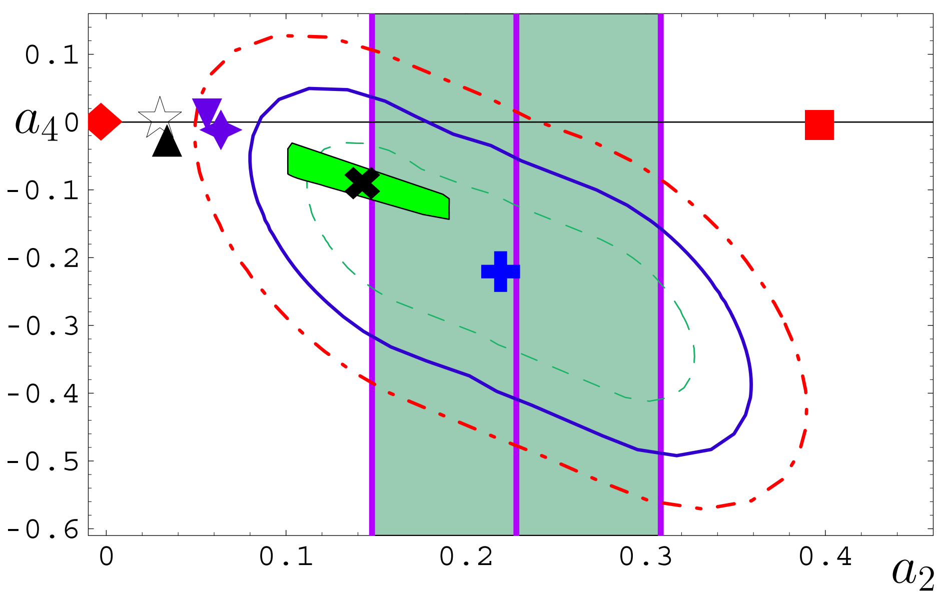

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- **gray strip** = lattice'05 result

BMS DA and most of **BMS bunch** — 1σ-domain and $1/2$ inside **2005 lattice strip [PRD 73 (2006) 056002]**.

Renormalon Model and CLEO Constraints

NLO Light-Cone SR $\oplus$ Twist-4 $\oplus(\mu^2 = Q^2)$
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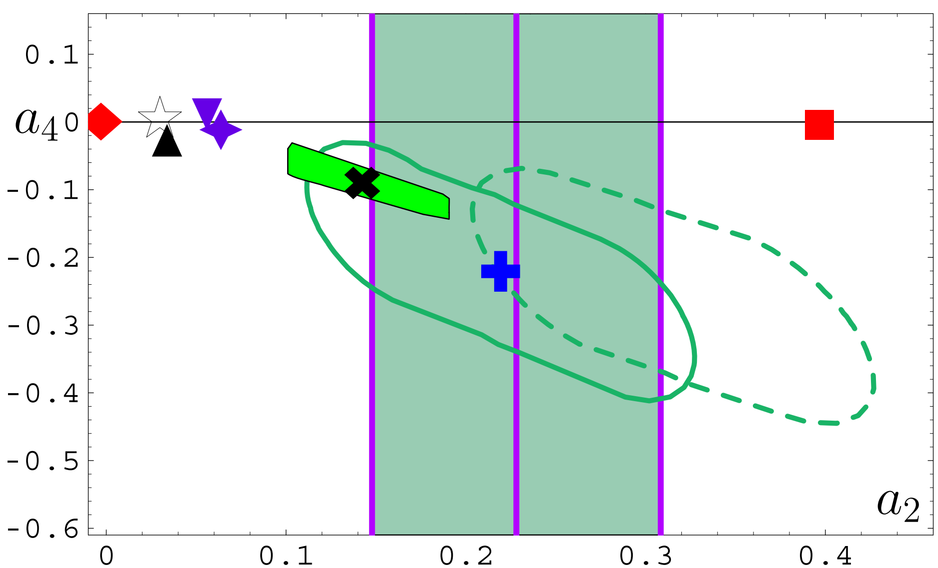
BMS [PRD 73 (2006) 056002]: $\lambda_q^2 = 0.4$ **GeV^2**



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- ▼ = transverse lattice
- **gray strip** = lattice'05 result

BMS DA and most of **BMS bunch** — inside 1σ-domain and $1/2$ inside **2005 lattice strip**. **Dashed contour** = renormalon model estimation of CLEO data.

Improved Model for NLCs
and
Consequences for Pion DA

Non-Local Condensates in QCD SR

Parameterization for scalar and vector condensates:

$$\langle\bar{\psi}(0)\psi(x)\rangle = \langle\bar{\psi}\psi\rangle \int_0^\infty \boxed{f_S(\alpha)}\, e^{\alpha x^2/4}\, d\alpha\,;$$

$$\langle\bar{\psi}(0)\gamma_\mu\psi(x)\rangle = -ix_\mu A_0 \int_0^\infty \boxed{f_V(\alpha)}\, e^{\alpha x^2/4}\, d\alpha\,,$$

where $A_0 = 2\alpha_s\pi\langle\bar{\psi}\psi\rangle^2/81$.

Non-Local Condensates in QCD SR

Convenient to parameterize the 3-local condensate in fixed-point gauge by introduction of three scalar functions:

$$\begin{aligned}
\langle\bar{\psi}(0)\gamma_\mu(-g\widehat{A}_\nu(x))\psi(y)\rangle &= (x_\mu y_\nu - g_{\mu\nu}(xy))\overline{M}_1 \\
&+ (x_\mu x_\nu - g_{\mu\nu}x^2)\overline{M}_2\,; \\
\langle\bar{\psi}(0)\gamma_5\gamma_\mu(-g\widehat{A}_\nu(x))\psi(y)\rangle &= i\varepsilon_{\mu\nu xy}\overline{M}_3\,,
\end{aligned}$$

with

$$\overline{M}_i(y^2, x^2, (x-y)^2) =$$

$$A_i \iiint_0^\infty d\alpha_1\, d\alpha_2\, d\alpha_3 \boxed{f_i(\alpha_1, \alpha_2, \alpha_3)}\, e^{(\alpha_1 y^2 + \alpha_2 x^2 + \alpha_3 (x-y)^2)/4}\,.$$

where $A_i = \{-\frac{3}{2}, 2, \frac{3}{2}\}A_0$ **[Mikhailov&Radyushkin'89]**.

Non-Local Condensates in QCD SR

The minimal Gaussian ansatz:

$$f_S(\alpha) = \delta\,(\alpha - \Lambda)\ ; \quad f_V(\alpha) = \delta'(\alpha - \Lambda)\ ; \quad \Lambda \equiv \lambda_q^2/2\,;$$

$$f_i\,(\alpha_1, \alpha_2, \alpha_3) = \delta\,(\alpha_1 - \Lambda)\ \delta\,(\alpha_2 - \Lambda)\ \delta\,(\alpha_3 - \Lambda)\ .$$

Only one parameter $\lambda_q^2 = 0.35 - 0.55$ **GeV^2**.

Non-Local Condensates in QCD SR

The minimal Gaussian ansatz:

$$f_S(\alpha) = \delta\,(\alpha - \Lambda)\ ; \quad f_V(\alpha) = \delta\,'(\alpha - \Lambda)\ ; \quad \Lambda \equiv \lambda_q^2/2\,;$$

$$f_i\,(\alpha_1, \alpha_2, \alpha_3) = \delta\,(\alpha_1 - \Lambda)\ \delta\,(\alpha_2 - \Lambda)\ \delta\,(\alpha_3 - \Lambda)\ .$$

Only one parameter $\lambda_q^2 = 0.35 - 0.55$ **GeV^2**.

Problems:

- **QCD equations of motion are violated**
- **Vector current correlator is not transverse**
  $\Rightarrow$ gauge invariance is broken

Improved Gaussian model

We modify functions f_i: $f_i^{\text{imp}}(\alpha_1, \alpha_2, \alpha_3) =$

$$(1 + X_i \partial_x + Y_i \partial_y + Z_i \partial_z)\, \delta(\alpha_1 - x\Lambda)\, \delta(\alpha_2 - y\Lambda)\, \delta(\alpha_3 - z\Lambda)$$

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What does it give us?

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What does it give us?

- **If** $12\,(X_2 + Y_2) - 9\,(X_1 + Y_1) = 1\,,\ x + y = 1\,,$ **than QCD equations of motion are satisfied;**

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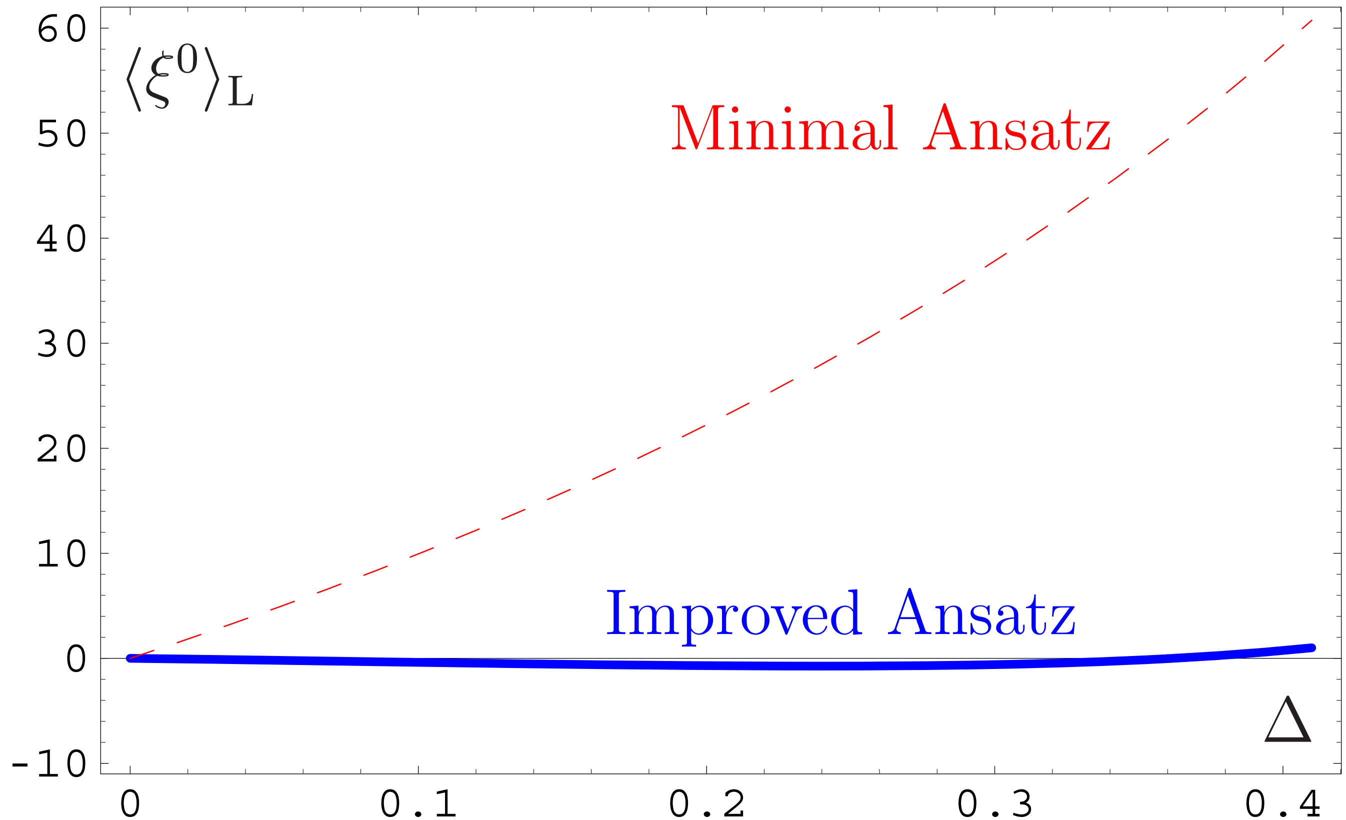
What does it give us?

- **If** $12\,(X_2 + Y_2) - 9\,(X_1 + Y_1) = 1\,,\ x + y = 1\,,$ **than QCD equations of motion are satisfied;**
- **We minimize nontransversity of polarization operator by special choice of model parameters:**

$$\begin{aligned} X_1 &= -0.082\,;\ Y_1 = Z_1 = -2.243\,;\ x = 0.788\,; \\ X_2 &= -1.298\,;\ Y_2 = Z_2 = -0.239\,;\ y = 0.212\,; \\ X_3 &= +1.775\,;\ Y_3 = Z_3 = -3.166\,;\ z = 0.212\,. \end{aligned}$$

Improved Gaussian model



The **improved Gaussian model** makes violation of polarization operator transversity **100 times smaller** as compared with the **minimal model**.

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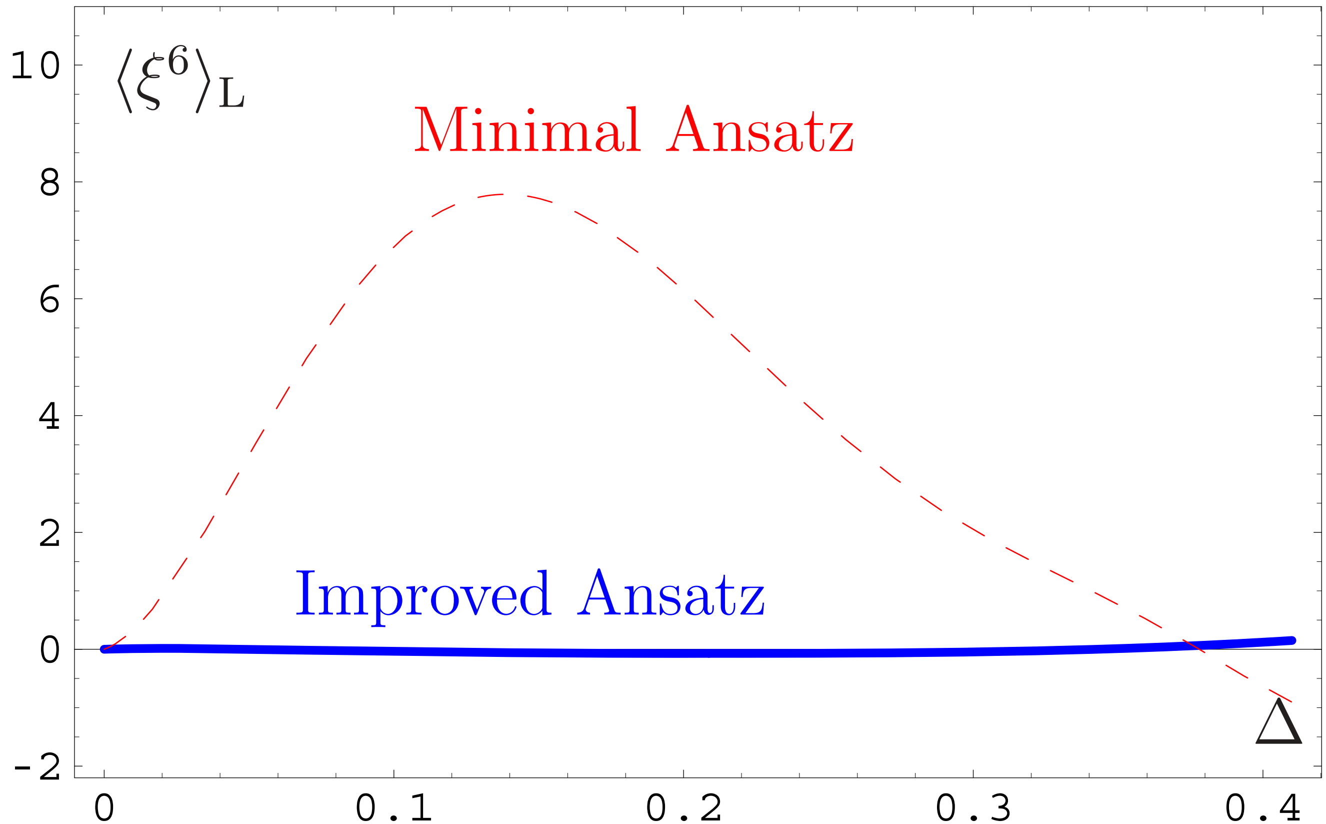

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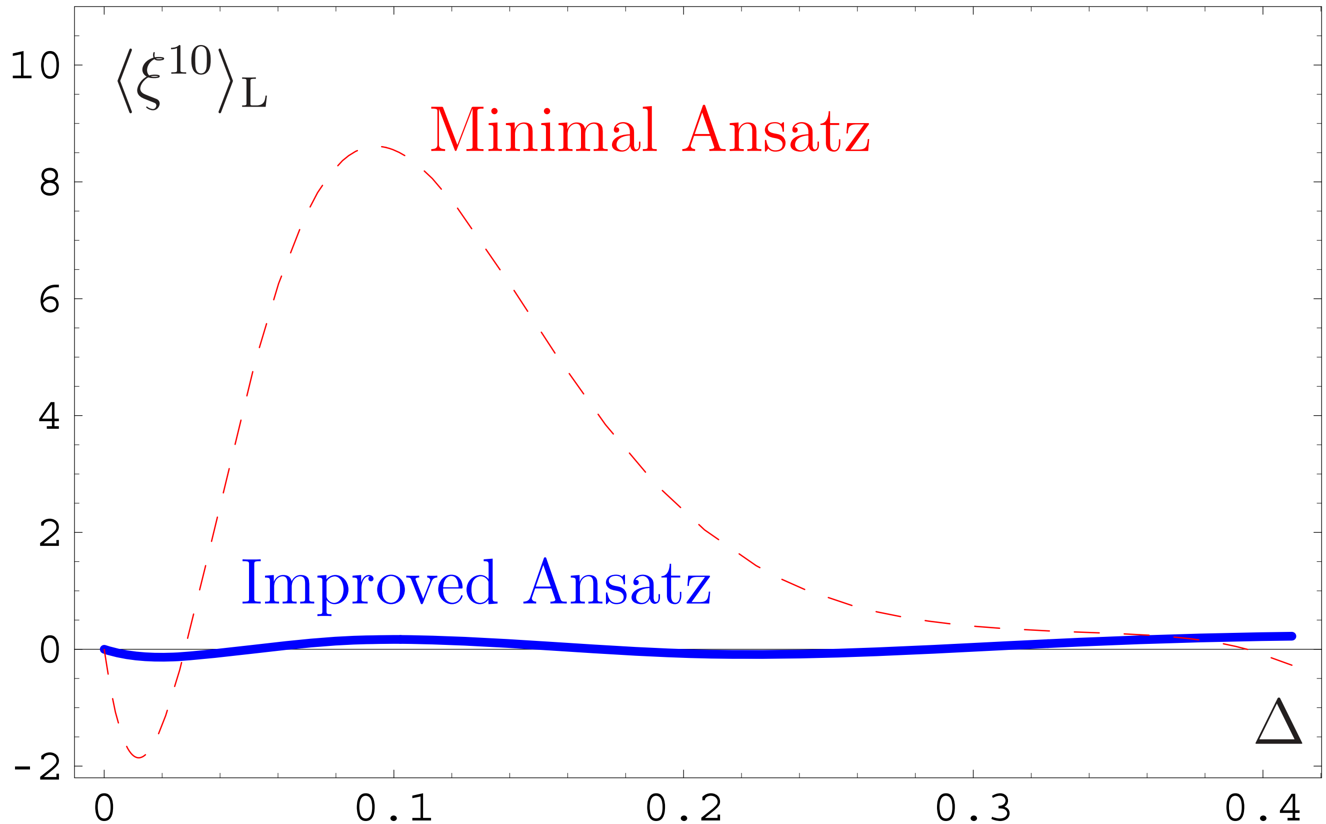

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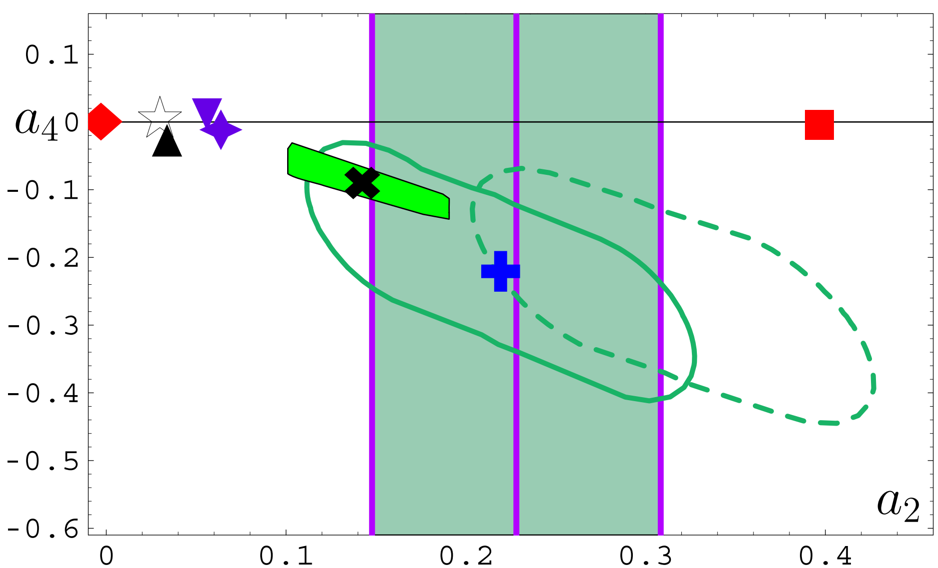
Improved Pion DAs vs. CLEO Constraints

NLO Light-Cone SR $\oplus$ Twist-4 $\oplus(\mu^2 = Q^2)$
with 10% uncertainty of $\delta^2_{\text{Tw-4}}$: $\delta^2_{\text{Tw-4}} = 0.19 \pm 0.02$ **GeV^2**
[PRD 73 (2006) 056002]: $\lambda_q^2 = 0.4$ **GeV^2**



✚ = best-fit point
◆ = Asymptotic DA
■ = CZ DA
✖ = BMS model
☆, ▲ and ✦ = instantons
▼ = transverse lattice
gray strip = lattice'05 result

BMS DA and most of **BMS bunch** — inside 1σ-domain and $1/2$ inside **lattice strip**. **Dashed contour** = renormalon model estimation of CLEO data **[PRD 73 (2006) 056002]**.

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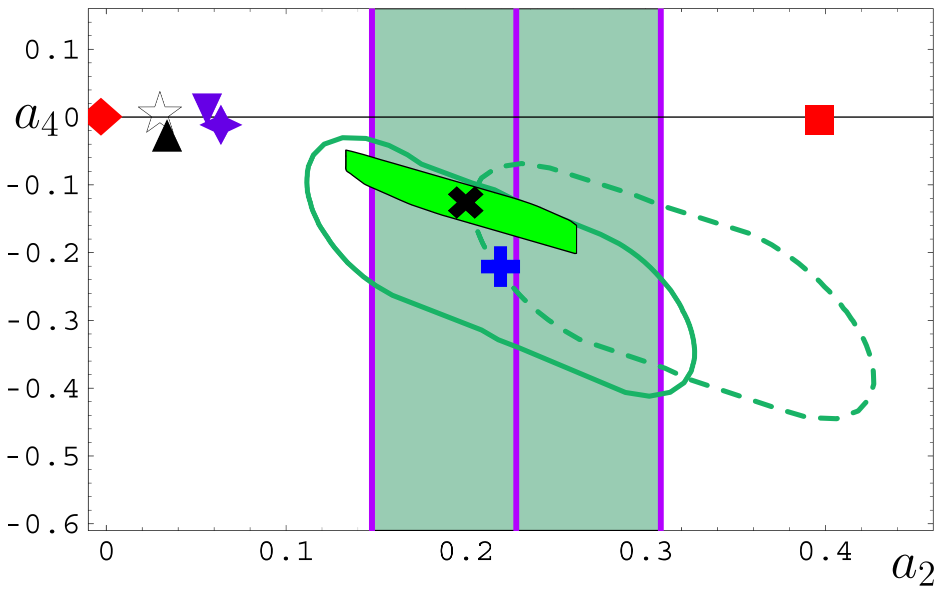

✚ = best-fit point

◆ = Asymptotic DA

■ = CZ DA

✖ = our new model

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▼ = transverse lattice

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Most of **improved BMS bunch** — inside 1σ-domain and inside **lattice strip**. **Dashed contour** = renormalon model estimation of CLEO data.

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