Physics 109 Homework #4

Due Thursday October 18th.

As you've perhaps gathered, I believe understanding the concept of acceleration was the single most important step in making real scientific advance over the Greek and Islamic sciences. Since this is a course based on history, we'll begin with an actual historical document, *admittedly in translation.*

1. The text below is a direct copy from Galileo's own introduction to his work on acceleration, one of the most important concepts in physics. Read it carefully, and then *write an explanation in your own words, including drawing a graph of speed as a function of time for an object falling from rest*. Note: for the word "momenta" he uses, you can just substitute "speed".

THEOREM I, PROPOSITION I

The time in which any space is traversed by a body starting from rest and uniformly accelerated is equal to the time in which that same space would be traversed by the same body moving at a uniform speed whose value is the mean of the highest speed and the speed just before acceleration began.

Let us represent by the line AB (*see figure below*) the time in which the space CD is traversed by a body which starts from rest at C and is uniformly accelerated; let the final and highest value of the speed gained during the interval AB be represented by the line EB, drawn at right angles to AB; draw the line AE, then all lines drawn from equidistant points on AB and parallel to BE will represent the increasing values of the speed, beginning with the instant A. Let the point F bisect the line EB; draw FG parallel to BA, and GA parallel to FB, thus forming a parallelogram AGFB which will be equal in area to the triangle AEB, since the side GF bisects the side AE at the point I; for if the parallel lines in the triangle AEB are extended to GI, then the sum of all the parallels contained in the quadrilateral is equal to the sum of those contained in the triangle AEB; for those in the triangle IEF are equal to those contained in the triangle GIA, while those included in the trapezium AIFB are common.

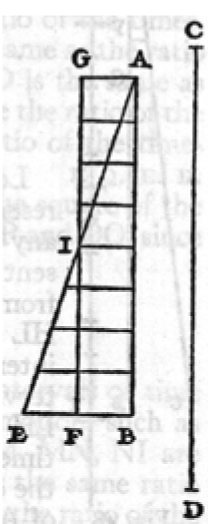


Fig. 47

Since each and every instant of time in the time-interval AB has its corresponding point on the line AB, from which points parallels drawn in and limited by the triangle AEB represent the increasing values of the growing velocity, and since parallels contained within the rectangle represent the values of a speed which is not increasing, but constant, it appears, in like manner, that the momenta [momenta] assumed by the moving body may also be represented, in the case of the accelerated motion, by the increasing parallels of the triangle AEB, and, in the case of the uniform motion, by the parallels of the rectangle GB. For, what the momenta may lack in the first part of the accelerated motion (the deficiency of the momenta being represented by the parallels of the triangle AGI) is made up by the momenta represented by the parallels of the triangle IEF. (*continues on next page*)

Hence it is clear that equal spaces will be traversed in equal times by two bodies, one of which, starting from rest, moves with a uniform acceleration, while the momentum of the other, moving with uniform speed, is one-half its maximum momentum under accelerated motion. Q.E.D.

2. **Measure your reaction time using Galileo's concept of acceleration**

If you hold a ruler against a wall, and let it fall, assuming it's not in contact with the wall, it will pick up speed at a rate of 10 meters per second per second.

(a) Write down how fast it's going after: 0.1 sec., 0.12 sec., 0.15 sec., 0.18 sec., 0.2 sec.

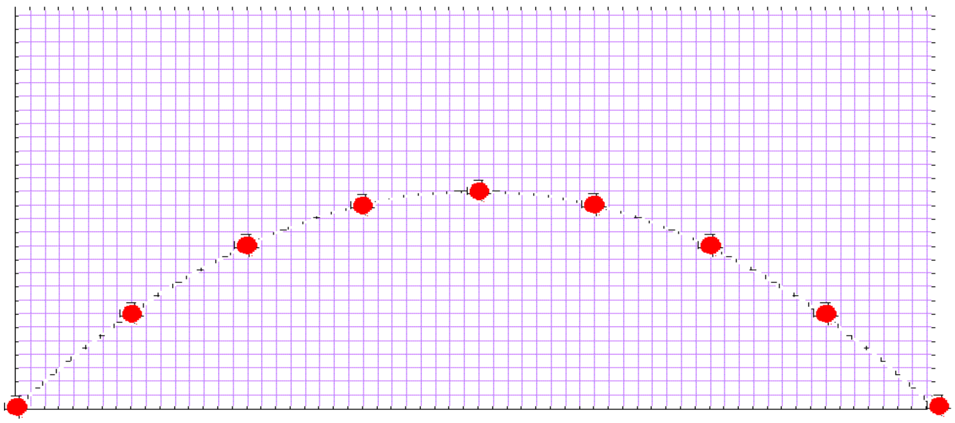
(b) Write down its average speed during the first 0.1 sec., the first 0.12 sec., …, 0.2 sec.

(c) Work out how far it fell after 0.1 sec., 0.12 sec., 0.15 sec., 0.18 sec., 0.2 sec.

(d) Use these results to plot a graph of distance as a function of time from time 0 to time 0.2 sec. (Guess the first 0.1 sec.—remembering it always falls *four* times as far in twice the time!)

(e) Use your graph and a friend to find your reaction time, and state what it is. Have several tries and take the average.

3. **Galileo's Analysis of Projectile Motion**



The above picture represents the flight of a ball thrown into the air (although air resistance effects have been neglected). The background grid is in one-foot squares. The blobs represent the position of the ball *at successive quarter-second intervals.*

(a) By counting lines, find the average *horizontal* velocity of the ball, in feet per second, during the first quarter of a second. (Remember, each square is one foot across (from side to side or top to bottom), and average speed = distance/time.)

(b) By reviewing the whole picture, find how the horizontal velocity (in feet per second) varies throughout the flight.

(c) By counting squares, find the average *vertical* velocity of the ball during the first quarter of a second.

(d) Make a table of the ball's average vertical velocity *for all eight quarter-second periods in the flight*. Count velocity upwards as positive, downwards as negative.

(e) Use your results from part (d) to plot a graph of the ball's vertical velocity (on the *y*-axis) as a function of time (on the *x*-axis) for the whole flight.

(f) Acceleration is *rate of change of velocity*. It is measured here in feet per second per second. Use your graph from (e) to plot a graph of the vertical acceleration as a function of time for the whole flight.

(g)What can you conclude about the acceleration of the ball at the topmost point of the path?

(h)Suppose now instead that the ball had been thrown directly upwards. Would it be accelerating at the topmost point of its flight?

(i) Galileo states in *Two New Sciences* (page 153): *"..so far as I know, no one has yet pointed out that the distances traversed, during equal intervals of time, by a body falling from rest, stand to one another in the same ratio as the odd numbers beginning with unity*". (The odd numbers beginning with unity just means 1, 3, 5, 7, ...). Show how you can verify this assertion using the picture above. What are the *total* distances fallen at the end of each quarter second starting from rest (at the top)?