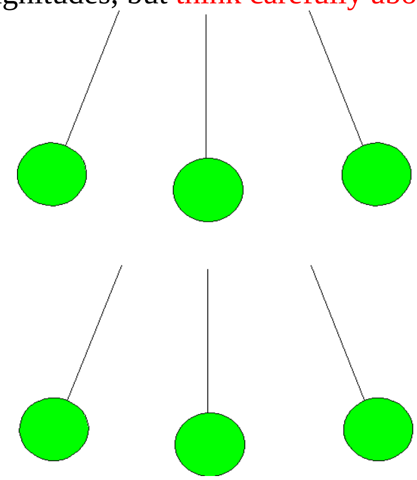
Physics 109 Homework #6

Due Tuesday November 20th

1.(a) The figure represents a pendulum (a bob-a small round weight, on a string) swinging back and forth, assume it is shown at the furthest point of its swing in each direction and in the middle of the swing. On the first set, show the *velocity* vector appropriate to these positions in the swing, on the lower set of pictures show the *acceleration* vectors for the three instants. Don't worry too much about representing the magnitudes, but think carefully about direction.

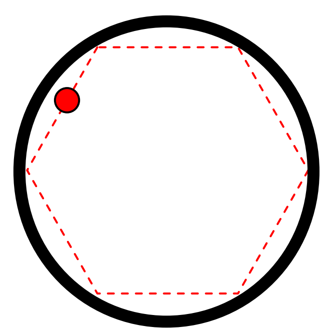


(b) Suppose the string is cut as the pendulum swings through the lowest point. Sketch the path of the pendulum from before the swing is cut until well after. How does the velocity of the pendulum the instant *before* the string is cut compare with that the instant *after*? How does the *acceleration* of the pendulum the instant before the string is cut compare with that the instant after?

(c) Now suppose instead the string is cut when the pendulum has reached its furthest point to the right in the swing. Sketch the path of the pendulum from before the string is cut until well after, and again compare the velocity and acceleration of the pendulum the instant *before* the string is cut with the instant *after*.

2.

Newton's approach to acceleration in steady circular motion. Consider a ball inside a circular container (imagine a pool table with circular walls), moving without friction along a hexagonal path, bouncing off the container with perfect elasticity. He analyzed the motion, then replaced the hexagon with a polygon having many equal sides, then more and more…



Suppose the hexagon has sides 2 meters long, and the ball is moving at 2 meters per second.

(a) Pick one of the bounces, draw vectors representing the velocities before and after the bounce, and draw the vector representing the change in velocity caused by the bounce.

(b) Taking this to be the change in velocity in a one second period (since the ball bounces off the wall once a second), what is the average acceleration during that second (representing the time from the midpoint of the side before the bounce to the midpoint of the side after the bounce)? *Hint*: remember that the hexagon is made up of equilateral triangles—lots of sixty degree angles here!
(c) How does your result relate to the v^2/r formula for acceleration in a circle?

3. **Using Newton's Law of Gravitation to weigh the Sun and the Galaxy**

We've weighed the earth, let's weigh the sun.
We know from Cavendish's experiment, and Newton's Universal Law of Gravitation, that the force of attraction between the earth and the sun is GMm/r^2, where G is the gravitational constant, found by Cavendish to be 6.67 x 10^{-11}, M is the mass of the sun, m the mass of the earth, and r the distance of the earth from the sun, which is 150,000,000 kilometers. (NOTE: in the formula for the force, r must be given in *meters*).
(This force causes the earth to accelerate towards the sun, that is, it deviates from straight line motion into a circle, just like Newton's cannonball. The strategy is to find *how far it falls below a straight line in one second* and figure out from that what its acceleration towards the sun must be. This acceleration is caused by the gravitational attraction force, which depends on the mass of the sun. Newton's Second Law gives the relation between the acceleration and the force, and enables us to find the mass of the sun.)
Using the fact that the earth goes around its orbit completely in one year, find how far the earth travels in *one second*. Now, find how far it "falls" below straight line motion in that one second. (HINT: call the distance it falls x, then write down Pythagoras' theorem for the usual triangle, and argue that x^2 can be safely neglected. Then it's easy to find x.)
Since the time interval we are taking is just *one second*, the distance the earth falls below the straight line in that period must be equal to its *average* velocity in that direction (that is, towards the sun) during the one second. So what is the earth's velocity in that direction at the *end* of the one second period? So what is its acceleration?
Write down Newton's Second Law for this acceleration, which is caused by the sun's gravitational attraction, and from it *deduce the mass of the sun*. Notice that you do not need to know the mass of the earth - why not?

4. *The (almost) ultimate weighing job: estimating the mass of our galaxy*.
The solar system is closer to the edge than the middle of our galaxy (the Milky Way), and is moving in an approximately circular path at about 250 kilometers per second. The radius of the circle—the distance to the center of the galaxy—is about 30,000 light years, where one light year is the distance light travels in one year.
Assume the solar system's circular motion about the center of the galaxy is caused by the gravitational attraction from the other stars in the galaxy. This attraction turns out to be not too different from what it would be if we lumped them all together in the middle in one huge mass $\boldsymbol{M}$. Find this mass by the same method we used to find the earth's mass and the sun's mass, given G = 6.67 x 10^{-11}. Assuming the sun is a typical star, estimate how many stars there are in our galaxy.
(Footnote: it turns out that the gravitational attraction towards the center is greater than can be accounted for from the number of stars. There's something else there that we can't see! There is dark matter, and dark energy… . Nobody understands this very well.)