4. (10 pts) Consider a thick cylinder, inner radius R_1 and outer radius R_2, of height H and total mass M. What is the rotational inertia of this object about the symmetry axis—the axis running down the center of the tube?

(a) ¼ $M(R_2 + R_1)^2$.

(b) ½ $M(R_2^2 - R_1^2)$.

(c) ½ $M(R_2 - R_1)^2$.

**(d) ½ $M(R_2^2 + R_1^2)$.

density $\rho = \frac{M}{\pi(R_2^2 - R_1^2)h}$ ← volume.

shell of with dr at r has

$I_{shell} = m_{shell} \cdot r^2 = \rho \, vol_{shell} \cdot r^2$

$= \rho \cdot (2\pi r \, dr \times h) \times r^2$

$I_{thick\ cyl} = \sum_{R_1 \text{ to } R_2} I_{shell} = \rho \int_{R_1}^{R_2} (2\pi r \, dr \, h) \cdot r^2$

$= \rho \cdot 2\pi h \int_{R_1}^{R_2} r^3 \, dr$

$= \rho \cdot 2\pi h \cdot \frac{1}{4} r^4 \Big|_{R_1}^{R_2}$

$= \rho \cdot 2\pi h \, \frac{1}{4} (R_2^4 - R_1^4)$.

$(R_2^2 - R_1^2)(R_2^2 + R_1^2)$ ← $a^2 - b^2 = (a+b)$

$= \frac{M}{\pi (R_2^2 - R_1^2) h} \cdot 2\pi h \cdot \frac{1}{4} (R_2^2 - R_1^2)(R_2^2 + R_1^2)$

$= \frac{1}{2} M (R_2^2 + R_1^2)$

Note (b) & (c) have bad limits for a thin cylinder.