6. A heavy symmetric wheel, mass m and rotational inertia I, rotates freely with a high angular speed ω about a light central shaft, with one end attached at a frictionless pivot point to a wall. The other end is held by a hand.

shaft is aligned with $\hat{k}$ (z-axis)

A. (6 pts) What force must the hand supply to hold the system fixed?

(a) $-I\omega^2/r\ \hat{k}$.

(b) $+I\omega^2/r\ \hat{k}$.

**(c) $\frac{1}{2} mg\ \hat{\jmath}$.

(d) $mg\ \hat{\jmath}$.

No net torque, accomplished by
& Normal at pivot + Normal from hand $= -m\vec{g}$,
& Hand same distance from wheel as the pivot, hence no net torque if mag. of normal forces are same:
∴ N from hand $= \frac{1}{2} mg$ up.

B. (4 pts) The hand must now raise the shaft by imparting a velocity $v\hat{\jmath}$ (↑ up) to the end. What force must the hand exert on the shaft end?

(a) $mg\ \hat{\jmath} + (I\omega v/r^2)\hat{\imath}$.

(b) $\frac{1}{2} mg\ \hat{\jmath} - (I\omega v/r^2)\hat{k}$.

(c) $(I\omega^2/r)\hat{k} + (I\omega v/r^2)\hat{\imath}$.

**(d) $\frac{1}{2}$ mg $\hat{\jmath} + (I\omega v/r^2)\hat{\imath}$.

The additional force, beyond $\frac{1}{2} mg\hat{\jmath}$, must provide a torque up, so that the change in ω is up. That is a force along $\hat{\imath}$ (because $\vec{r}$ in the torque eq. is along the shaft, $\vec{r} \sim \hat{k}$, and

$\vec{\tau} \sim \vec{r} \times \vec{F} \sim \hat{k} \times \hat{\imath} \sim \hat{\jmath}$.

Hence (d)