Solutions

Print name____________________

ID number____________________

Section in which you are registered (sec 1 = 9 AM lec, sec 2 = 10 AM lec)________

Physics 142E, Test No. 2, test session 2
March 24, 2004, 7:30-9:00 PM

On the bubble sheet, fill in your student id number, and in addition write your name and your section in the appropriate spot. After you have found an answer, fill in the appropriate bubbles on your bubble sheet—note any special instructions on this point in the problem itself. The last problem is a multipart problem whose solution should be written out neatly. You can get partial credit for this problem, so make sure that your answers are written out and contain no ambiguities.

No notes or books are allowed during the exam, nor is any consultation with anyone but me. You should be taking the 5:30 exam if you are in section 1 (9 AM lecture) and the 7:30 exam if you are in section 2 (10 AM lecture).

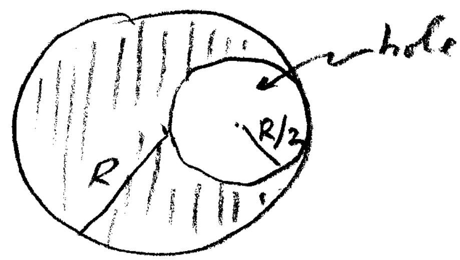
Write out an authorized form of the pledge here, and sign it.

Signed____________________

1. (10) An object of mass M moves through outer space with speed V. It separates into two equal parts such that both pieces move in the same direction as the original object. The speed of one of the pieces is $V/4$. What is the speed of the second?
(a) $5V/3$
(b) $V/2$
(c) $2V$
*(d) $7V/4$
(e) V
Soln (d) If v is the unknown speed, conservation of momentum reads (all velocities are positive)

$$MV = (M/2)(V/4) + (M/2)v$$

with soln $v = 2(V - V/8) = (7/4)V$.



2. (10) A uniform disk of radius R and total mass M has a circular hole of radius $R/2$ cut out of it, the hole running from the center of the original disk to its edge, as in the figure. We'll refer to the disk with the hole as the "holed" disk. What is the rotational inertia of the holed disk about the axis that runs perpendicularly to the original disk and through its (the original disk's) center?
(a) $(19/32)MR^2$
*(b) $(13/32)MR^2$
(c) $(1/2)MR^2$
(d) $(1/4)MR^2$
(e) $(7/16)MR^2$
Soln. (b) First note that the full original disk the rotational inertia is $(1/2)MR^2$ (all I's refer to the original disk axis) and once there is a cutout the answer becomes smaller (mass has been removed at varying radii). Thus you can immediately toss (a) and (c) and improve your chances with a guess.

Now $I_{\text{original disk}} = I_{\text{holed disk}} + I_{\text{cutout piece}}$, and we can invert this:

$$I_{\text{holed disk}} = I_{\text{original disk}} - I_{\text{cutout piece}},$$

where all these rotational inertias refer to the axis of the original disk, what we'll call axis 1. To find $I_{\text{cutout piece}}$ with respect to axis 1, we use the parallel axis theorem:

$$I_{\text{cutout piece about axis 1}} = I_{\text{cutout piece about own axis}} + md^2,$$

where m is the mass of the cutout piece and $d = R/2$ is the distance of the cutout piece axis from axis 1. We have $I_{\text{cutout piece about own axis}} = ½\, m(R/2)^2, = 1/8\, mR^2$, so

$$I_{\text{cutout piece about axis 1}} = 1/8\, mR^2 + m(R/2)^2, = 3/8\, mR^2.$$

We now find m. the original disk's density is $\rho = M/(h\pi R^2)$, where h is the disk thickness, so $m = \rho \times h\pi(R/2)^2 = M/4$. Collecting,

$$I_{\text{holed disk}} = ½\, MR^2 - 3/32\, MR^2 = 13/32\, MR^2.$$

3. (10) Consider an elastic collision between two point masses labeled 1 and 2 constrained to move along a line (i.e. in one dimension). Initially, mass 1 moves to the right while mass 2 is moves to the left. Is it possible that mass 1 continues to move to the right even after the collision?
*(a) Yes.
(b) No.
(c) That cannot be determined from the information given.
Soln. Yes. Imagine mass 1 were a train while mass 2 were a ping-pong ball.

4. (10) Two solid cylinders of the same total mass roll down a ramp starting from rest and from the same initial position. Cylinder 1 has twice the radius of cylinder 2. Which one arrives at the bottom first?
(a) Cylinder 1 arrives first.
(b) You can't tell from the information given.
(c) Cylinder 2 arrives first.
*(d) They arrive at the same time.
Soln. (d) If the mass and radii of cylinder i are m_i and R_i, then the rotational inertia through the axis is of the form $I_i = ½\, m_i R_i^2$. Only the factor ½ in this expression matters, the speed after time t being indep of both m_i and R_i. The cylinders thus arrive at the same time.

5. (10) We described with the plumber's wrench how an extended object could have a net force on it yet no torque. Is it possible that no net force acts on an object while there is a net torque on it?
*(a) Yes.
(b) No.
Answer: Yes, just think of a disk with equal and opposite forces (no net force) in the plane of the disk, one acting along one edge and the other acting along the opposite edge.

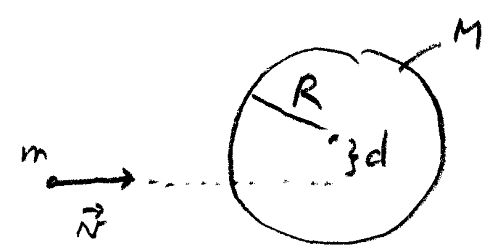
6. (10) A point object of mass $m = 2$ kg moves in one-dimension under the influence of a force for which the potential energy function has the form

$$U = \infty \text{ for } |x| > a.$$
$$U = 0 \text{ for } b < |x| < a.$$
$$U = 25 \text{ J for } |x| < b.$$

What is the minimum speed with which it must move in the region $-a < x < -b$ "the left-hand region" in order to be able to arrive at the region $+b < x < a$ ("the right-hand region")? [*Hint*: It helps to make a sketch of the potential energy function here.]
(a) Cannot be determined from the information given.
*(b) 5 m/s
(c) 3 m/s
(d) 25 m/s
(e) There is no way it can get to the second region.
Soln. (b) Cons of energy applies. In the left hand region $U = 0$ and $½\, mv^2$ is the total energy. The particle can just barely creep across the central region, where $U = 25$ J, and get to the right-hand region, if it has zero kinetic energy in the central region. Thus v_{min} is determined by

$$E \text{ in left hand region} = ½\, mv^2 = E \text{ in central region} = 25 \text{ J}$$
$$½\, mv^2 = 25 \text{ J, or}$$
$$v^2 = (2/\text{m})(25 \text{ J}) = 25 \text{ m}^2/\text{s}^2\text{, or}$$
$$v = 5 \text{ m/s.}$$

7. (10) In the device shown for measuring the speed v of a bullet (here we assume that v is known), the bullet, of mass m, is fired so that it enters the edge of a stationary uniform solid disk as shown and embeds itself within the disk. The disk, which has mass $M >> m$ and radius R, is free to rotate about a fixed central axis, and the disk rotates with angular velocity ω that can be found from the conservation of angular momentum applied to the process. What we have described is a kind of inelastic collision. What, approximately (use $M >> m$), is the ratio of the final energy to initial energy?



(a) $\frac{E_f}{E_i} = 2\frac{d}{MR^2}$

*(b) $\frac{E_f}{E_i} = 2\frac{md^2}{MR^2}$

(c) 1, energy is conserved

(d) $\frac{E_f}{E_i} = 2\frac{MR^2}{md^2}$

(e) ½.

Solution. (b) If we consider the bullet and disk as a single system the forces are all internal and angular momentum is conserved. The initial angular momentum about the axis is due to the bullet alone and has magnitude $L_{initial} = mvd$. The final angular momentum about the axis is due to the rotating disk, and has magnitude given approximately ($M + m \cong M$) by $L_{final} = I\omega$, where I is the rotational inertia of the disk about its axis. We set these equal and solve for ω:

$$\omega = \frac{mvd}{I}$$

The initial energy is the kinetic energy of the bullet: $E_i = ½\, mv^2$.

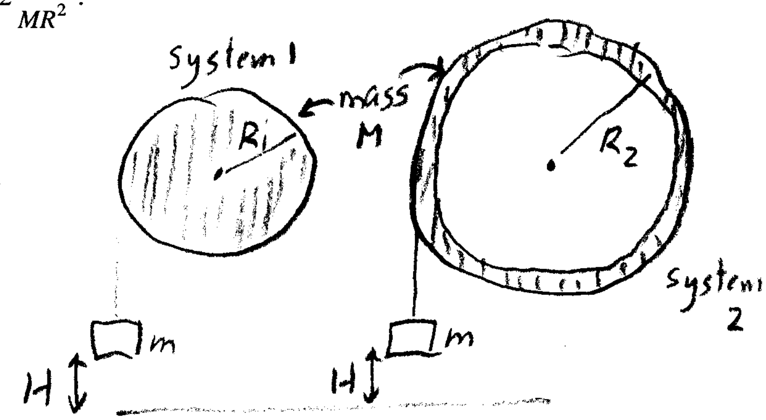
The final energy is approximately ($M >> m$) the KE of the rotating disk: $E_f = ½\, I\omega^2$.

The ratio of these is

$$\frac{E_f}{E_i} = \frac{I\omega^2}{mv^2} = \frac{I\left(\frac{mvd}{I}\right)^2}{mv^2} = \frac{md^2}{I}$$

If we use the fact that the rotational inertia of the solid uniform disk about its axis is $I = ½\, MR^2$, then

$$\frac{E_f}{E_i} = 2\frac{md^2}{MR^2}.$$



8. (30) In class we discussed races between rolling objects. Here we discuss a race between objects suspended from threads that are wrapped around cylinders free to turn about their fixed axes. System 1 consists of a mass m hanging as in the figure from a thread wrapped around a uniform solid cylinder of mass M and radius R_1, while system 2 consists of the same mass m hanging as in the figure from a thread wrapped around a hollow cylinder (thin walls) of the same mass M as the solid cylinder and radius R_2. Each system will start from rest with the masses a height H above the floor.

(a) (6) Make the extended free body diagram(s) necessary to solve for the motion of either of these systems.

Soln draw this →

(b) (8) Write the equations of motion whose solution will allow you to find the time-dependent height of the masses.

Soln: In each case the force eq for the mass is

$T - mg = -ma$ (positive is up, and a is positive as well – we anticipate the fall.);

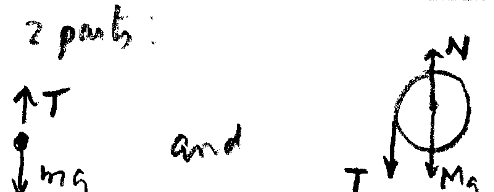
the force equation for the wheel involves N_{axis} and is otherwise not useful;

torque eq on the wheel about the axis is

$$TR = I\alpha.$$

We must add the kinematic relation between α and a: $a = \alpha R$.

Should have 2 parts:



(c) (8) Use energy conservation to find the speed of each mass just before it hits the floor. From this information, which mass reaches the ground first?
Soln: Initial energy = mgH + term for the wheel because it is above the ground.
Final energy = same term for the wheel because it is above the ground + ½ mv^2 + ½ $I\omega^2$.
Set these equal, and use the kinematic relation $v = \omega R$:

$$mgH = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}I\frac{v^2}{R^2}$$

In each case the moment of inertia is of the form $I = CMR^2$, where C = ½ for the solid cylinder and C = 1 for the hollow cylinder. Thus the R^2 term will cancel—the results are all independent of the radius of the rotating cylinder—and

$$mgH = \frac{1}{2}mv^2 + \frac{1}{2}C_iMv^2$$

where i = 1 or 2 according to the case. Solving,

$$v^2 = \frac{2mgH}{m + C_iM}$$

Since C for the hollow cylinder is larger, the velocity of the mass is smaller for that case, and the mass attached to the solid cylinder arrives at the ground first.

(d) (8) Solve the dynamical equations of motion to find the time it takes for each mass to hit the ground.
Soln we go back to part (b) and actually solve to find the acceleration of the mass. The force equation for the mass gives the tension T: $T = m(g - a)$. Plug this into torque eq:

$$TR = I\alpha, \text{ or } m(g - a)R = I(a/R).$$

This can be solved for a: $a(I/R + mR) = mgR$, or dividing by R,

$$a = \frac{mg}{\frac{I}{R^2} + m} = \frac{mg}{CM + m}$$

Once we have a we know that $y = H - \frac{1}{2}at^2 = 0$ for ground level. Thus the time to reach the ground is

$$t = \sqrt{\frac{2H}{a}} = \sqrt{\frac{2H(CM + m)}{mg}}$$

Again the smallest time is for the smallest C-value, namely for the solid cylinder.