Name_____Date_____Partners_____

Lab 9:
Rotational Dynamics

OBJECTIVES

- To study angular motion including angular velocity and angular acceleration.
- To relate rotational inertia to angular motion.
- To determine kinetic energy as the sum of translational and rotational components.
- To determine whether angular momentum is conserved.

OVERVIEW

We want to study the rotation of a rigid body about a fixed axis. In this motion the distance traveled by a point on the body depends on its distance from the axis of rotation. However, the angle of rotation θ, (also called the angular displacement), the angular velocity ω, and the angular acceleration α, are each the same for every point. For this reason, the latter parameters are better suited to describe rotational motion. The unit of angular displacement that is commonly used is the radian. By definition, θ is given in radians by the relation $\theta = \dfrac{s}{r}$,

where s is the arc length and r is the radius as shown in Fig. 1. One radian is the angle, measured at the center of a circle, whose legs subtend on the periphery an arc equal in length to the radius. An angle of 90° thus equals π/2 radians, a full turn 2π radians, etc. The angular velocity is the rate of change of the angular displacement with time. It is equal to the angle through which the body rotates per unit time and is measured in radians per second. The angular acceleration is the rate of change of the angular velocity with time and is measured in radians per second squared. In the limit of very small times, the angular velocity is the derivative of the angular displacement with respect to time and the angular acceleration is the derivative of the angular velocity with respect to time:
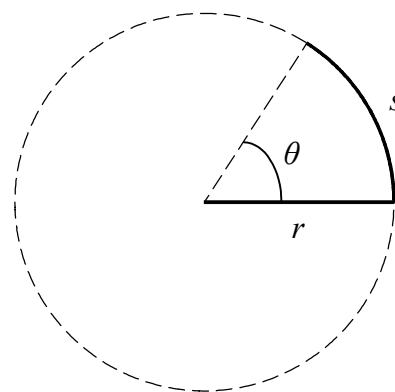

Fig. 1

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta \theta}{\Delta t} = \frac{d\theta}{dt}$$

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta \omega}{\Delta t} = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

(1)

In linear motion the position, velocity, and acceleration are described by vectors. Rotational quantities can also be described by (axial) vectors. In these experiments, however, you will only have to make use of the magnitudes and signs of these quantities. There will be no explicit reference to their vector character.

Sometimes one needs the parameters of the linear motion of some point on the rotating rigid body. They are related very simply to the corresponding angular quantities. Let s be the distance a point moves on a circle of radius r around the axis; let v be the linear velocity of that point and a its linear acceleration.

Then s, v, and a are related to θ, ω, and α by

$$s = r\theta$$
$$v = r\omega \tag{2}$$
$$a = r\alpha$$

Let us now imagine a rigid body of mass m rotating with angular speed ω about an axis that is fixed in a particular inertial frame. Each particle of mass m_i in such a rotating body has a certain amount of kinetic energy $\frac{1}{2}m_i v_i^2 = \frac{1}{2}m_i r_i^2 \omega^2$. The total kinetic energy of the body is the sum of the kinetic energies of its particles.

If the body is rigid, as we assume in this section, ω is the same for all particles. However, the radius r may be different for different particles. Hence, the total kinetic energy K of the rotating body can be written as

$$K = \frac{1}{2}\left(m_1 r_1^2 + m_2 r_2^2 + \ldots\right)\omega^2 = \frac{1}{2}\left(\sum m_i r_i^2\right)\omega^2 . \tag{3}$$

The term $\sum m_i r_i^2$ is the sum over i of the products of the masses of the particles by the squares of their respective distances from the axis of rotation. We denote this quantity by the symbol I. I is called the *rotational inertia*, or *moment of inertia*, of the body with respect to the particular axis of rotation. I has dimensions of [ML^2] and is usually expressed in $\mathrm{kg \cdot m^2}$. For extended bodies, the sum must be replaced by an integral:

$$I = \sum m_i r_i^2 \Rightarrow \int r^2 dm = \int \rho r^2 dV \tag{4}$$

If the body has a uniform density, as in this experiment, the integral can be rewritten as $I = \rho \int r^2 dV$.
This integral can be easily calculated only for bodies with a simple shape that is symmetrical around the axis of rotation. For example, some simple shapes are given in Fig. 2. Note that the rotational inertia of a body depends on the *particular axis about which it is rotating* as well as on the *shape of the body* and the *manner in which its mass is distributed*.

In terms of rotational inertia, we can now write the kinetic energy of the rotating rigid body as:

$$K_{rot} = \frac{1}{2}I\omega^2 . \tag{5}$$

This is analogous to the expression for the kinetic energy of translation of a body, $K_{tran} = \frac{1}{2}mv^2$. We have already seen that the angular speed ω is analogous to the linear speed v. Now we see that the rotational inertia I is analogous to the translational inertial mass m. In rotational dynamics, Newton's second law ($F = ma$ where F is the force, m is the mass and a is the acceleration) becomes:

$$\tau = I\alpha$$

where τ is the *torque* in Newton-meters about the axis of rotation. Another way of determining the torque is:

torque = (*distance*, $r_\perp$, from the axis of rotation) $\times$ (*force*).   (7)

In Table 1, we compare the translational motion of a rigid body along a straight line with the rotational motion of a rigid body about a fixed axis.

Hoop about cylinder axis
$$I = mR^2$$

Annular cylinder (or ring) about cylinder axis
$$\frac{m}{2}\left(R_1^2 + R_2^2\right)$$

Solid cylinder about cylinder axis
$$I = \frac{1}{2}mR^2$$

Solid cylinder (or disk) about a central diameter
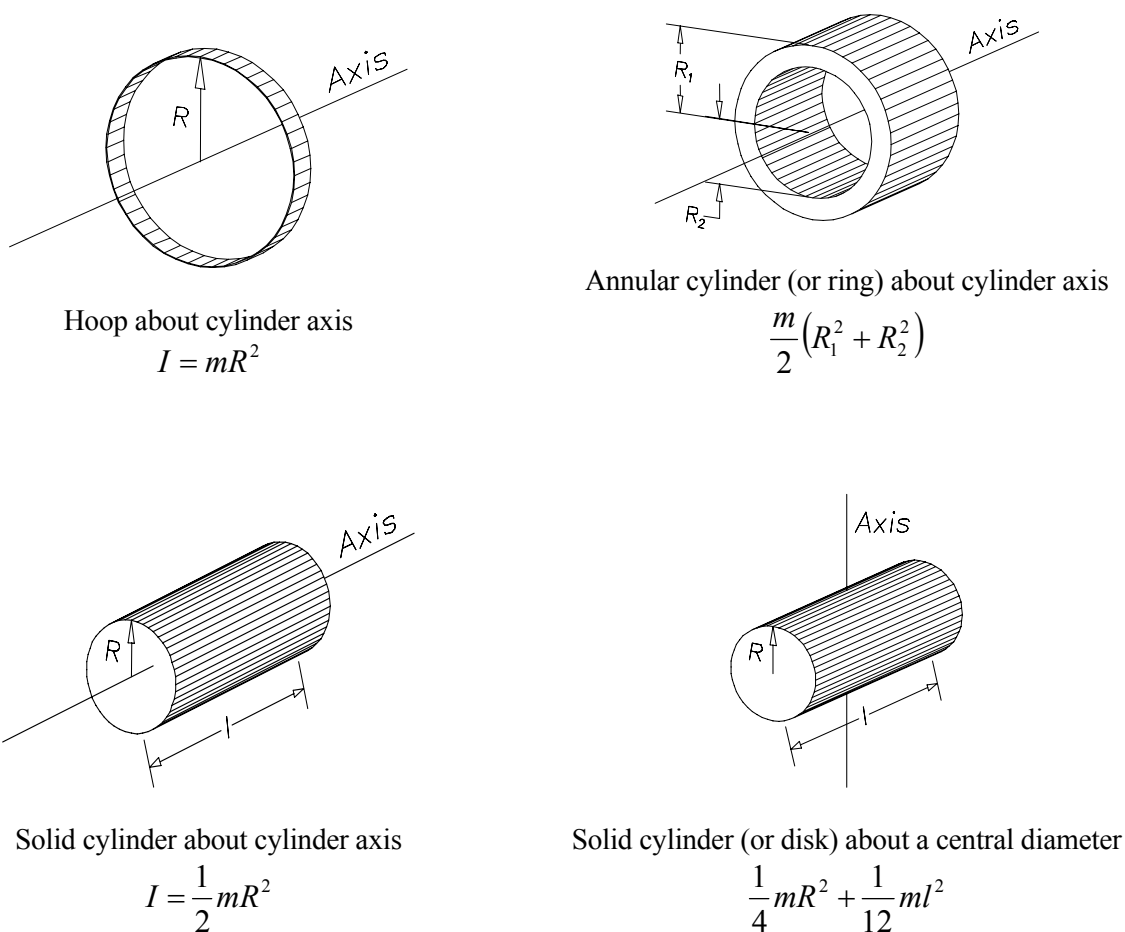$$\frac{1}{4}mR^2 + \frac{1}{12}ml^2$$

Fig. 2. Rotational Inertia for some simple geometries.

Recall that in the absence of external forces, linear momentum is conserved. Similarly, in the absence of external torques, angular momentum is conserved. Finally, if no non-conservative forces (such as friction) or torques act, then mechanical energy is conserved. In summary, you will test the following conservation principles in this experiment:

1. $E_{potential} + E_{linear\ kinetic} + E_{rotational\ kinetic}$ = constant

2. The sum of the angular momentum = constant

Table 1. Rectilinear and Rotational Quantities

Rectilinear Motion		Rotation About a Fixed Axis	
Displacement	x	Angular displacement	θ
Velocity	$v = \dfrac{dx}{dt}$	Angular velocity	$\omega = \dfrac{d\theta}{dt}$
Acceleration	$a = \dfrac{dv}{dt}$	Angular acceleration	$\alpha = \dfrac{d\omega}{dt}$
Mass (translational inertia)	$m = \int \rho dV$	Rotational inertia	$I = \int r^2 \rho dV$
Force	$F = ma$	Torque	$\tau = I\alpha$
Work	$W = \int F\, dx$	Work	$W = \int \tau\, d\theta$
Kinetic energy	$K = \frac{1}{2}mv^2$	Kinetic energy	$K = \frac{1}{2}I\omega^2$
Power	$P = Fv$	Power	$P = \tau\omega$
Linear momentum	$p = mv$	Angular momentum	$L = I\omega$

You will need to have the following apparatus for these experiments:

- PASCO Rotational Dynamics kit
- photogate timer and stand
- thread
- meter stick

- vernier calipers
- air supply
- digital extension cable

INVESTIGATION 1: EXPERIMENTAL SET UP

WARNING:
DO NOT MOVE THE ROTATING DISKS UNLESS THE AIR SUPPLY IS ON.
THE TA MUST ASSIST WHENEVER DISKS ARE CHANGED.

Activity 1-1: Apparatus leveling

1. The clearance between the spindle and disks is only 0.001", and any nicks will damage the low friction support. The PASCO apparatus is connected to an air supply with a pressure regulator. The pressure should be preset at about 10 psi (with air flowing) for operation.

Question 1-1: Why is it important for the rotational apparatus to be level? What errors might result if it were not level?
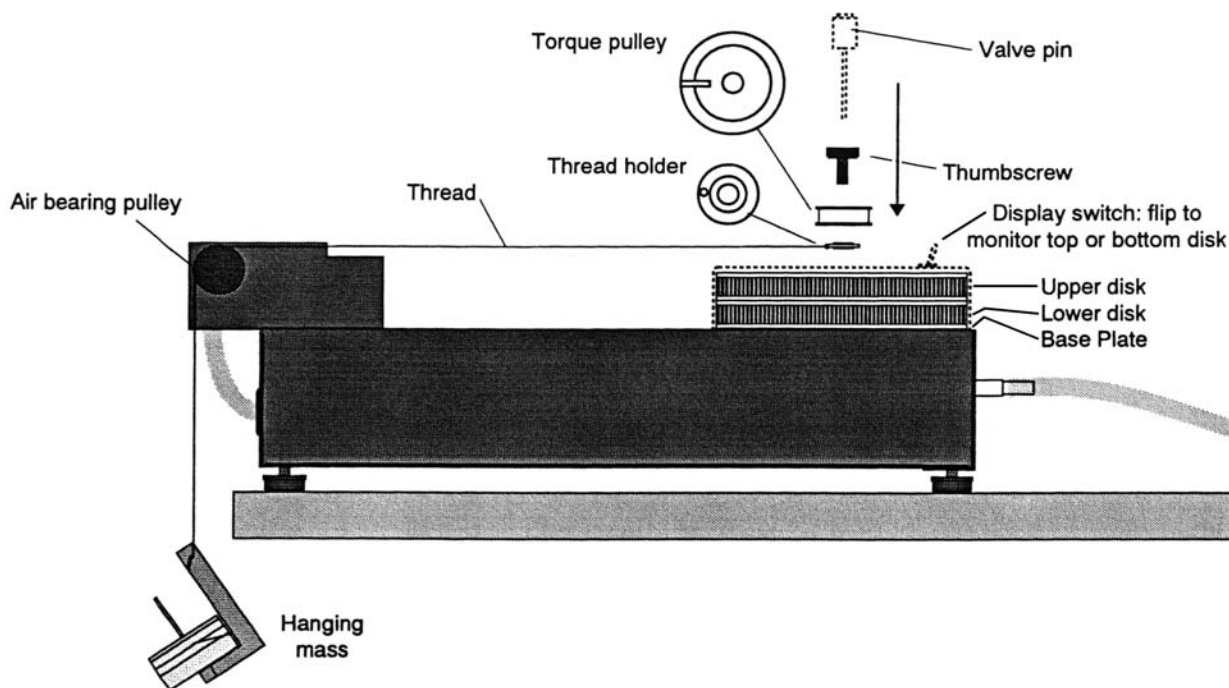
Fig. 3. Side view of rotational dynamics apparatus

To ensure that the disk rotates with uniform velocity or acceleration, even with an eccentric load, the base must be leveled accurately. Arrange the Rotational Dynamics apparatus (see Fig. 3) so the air-bearing pulley extends over the edge of your lab table. Check with your Instructor if you are unsure.

2. Turn on the air with the top aluminum disk on the spindle. If the disk on the apparatus is not made of aluminum, ask your TA to put it on.

3. Place bubble level on top of the aluminum disk and adjust the three leveling feet until the base is level.

Activity 1-2: Disk rotation

1. The two disks can spin independently or together, or the upper disk can spin while the lower disk does not (see Fig. 3). These options are controlled using the two valve pins that are provided with the unit. When not in use, these pins can be stored in the valve pin storage holes on the top of the base (see Fig. 4).

2. Place one valve pin in the bottom disk valve, located next to the valve pin storage holes. Give the upper disk a spin. Notice that it lies firmly on the lower disk so the two disks spin together. Remove the valve pin and notice how both disks drop onto the base plate.

3. Replace the valve pin in the bottom disk valve and then place the remaining valve pin into the hole in the middle of the upper disk, as in the figure. Now spin the disks in opposite directions. Notice that the two disks now spin independently.

4. Pull the valve pin from the center of the upper disk. The upper disk drops onto the lower disk so that the disks now spin together, as a single rotating body. This is the rotational equivalent of an inelastic collision. You may find that the aluminum disk tends to float, because it is lighter. If so, reduce the air pressure slightly.
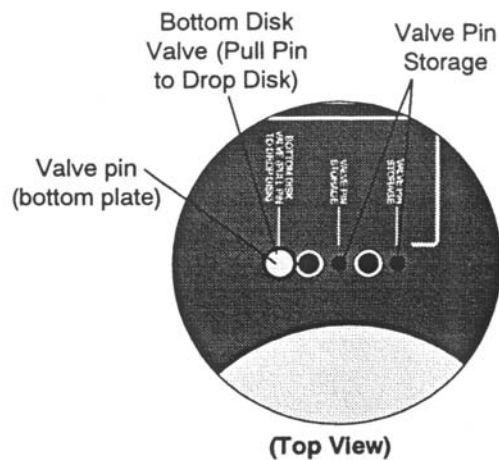
(Top View)

Fig. 3. View of the pin storage area.

The theoretical rotational inertia of the rotating disks (anngular cylinder) is given by

$$ I \; = \; \frac{1}{2} M \; (r_1^{\,2} \; + \; r_2^{\,2}) \; , \tag{8} $$

where M is the mass of the disk, and r_1 and r_2 are the disk's outer and inner radii, respectively. The masses are

    Bottom stainless steel: 1,344 grams
    Top stainless steel: 1,357 grams
    Top aluminum: 464 grams

    The geometric shape of the disks are not identical; however, within the accuracy of your measurement you can use for all three disks.

$$ r_1 = 6.32 \text{ cm} $$

$$ r_2 = 0.79 \text{ cm} $$

5.   Calculate (if you have not already done so) the rotational inertia of disks.

    rotational inertia I _____ kg·m^2 (top stainless steel)

    rotational inertia I _____ kg·m^2 (bottom stainless steel)

    rotational inertia I _____ kg·m^2 (aluminum)

INVESTIGATION 2: ROTATIONAL KINEMATICS AND TORQUE

Activity 2-1   How Does Torque Affect An Object's Rotational Motion?

We want to verify the rotational analogue of the relationship between force and acceleration as given by Equation (6). We will apply a constant torque to an aluminum disk by attaching a mass to a string wrapped around the disk and then hang the mass over a frictionless pulley. The constant force due to gravity acting on the mass ($F_{\text{gravity}} = mg$) is transferred to the disk by the string hanging over the pulley as a constant tension.

Prediction 2-1: Explain what will happen to the disk when the mass hanging down over the pulley is released. Does this represent a constant torque?

Question 2-1:   If you have the means to measure the angular velocity of the disk as a function of time, how can you determine if the angular acceleration is constant?

The tension in the string causes a torque on the disk since the string is attached to the disk via a small pulley with a radius of 1.27 cm. When the disk is released, it will start to rotate with constant angular acceleration as the mass falls with constant linear acceleration. We will measure this angular acceleration by plotting the angular velocity of the disk over time. Because $\omega = \alpha\, t$, if we fit a straight line to the angular velocity data, the slope of the line will give us the angular acceleration of the disk. We will then see if this angular acceleration agrees with the angular acceleration calculated by solving the force equations for the disk and the mass.

1. Remove the valve pin for the bottom disk so that the bottom disk does not rotate. **Make the measurements using the lighter aluminum top disk**. If the disk on the apparatus is <u>not</u> made of aluminum, ask your TA to put it on. **Do not replace the disks yourself!**

2. Cut a piece of thread about 135 cm in length.

3. Referring to Fig. 3, tie one end of the thread to the hole in the thread holder. Place the thread holder in the recess of the small torque pulley, with the thread passing through the slot in the pulley. Then use the thumbscrew to attach the pulley to the top of the rotating disk, with the flat side of the pulley facing up, so the thread holder is underneath the pulley. Tighten the thumbscrew so the pulley is secure.

    Mass the 20 g mass: _____ g

4. Attach the 20 g mass to the other end of the thread. When the thread is fully extended, the mass should almost touch the floor.

5. Open the experimental file **Angular Velocity L9.2-1.** This will set up the computer to graph the angular velocity of the disk in deg/sec.

6. Rotate the disk until the mass is approximately 90 cm above the floor. Start the computer and immediately release the disk. Stop the computer when the mass has fallen at least 80 cm.

7. As the mass falls, the disk will rotate with constant angular acceleration and the angular velocity on the graph should increase linearly. Using the mouse, select a region in the middle of the data where you are sure the mass was falling freely. Click on the **Fit** icon on the graph toolbar and select Linear Fit. Record the slope of the fit (m) here:

Slope _____ deg/sec^2.

8. Because the disk is rotating on a nearly frictionless layer of air, the only torque acting on it is from the tension in the string. We can write the force and torque equations for the mass and the disk as:

$$\text{Forces on Mass}: \quad mg - T = ma$$

$$\text{Torques on disk}: \quad \tau = I\alpha$$

where T is the tension in the string, m is the mass, and a is the linear acceleration of the mass. Solving the force equation for T gives $T = mg - ma$. Using Equations (2) and (7) (because the mass is attached to the edge of the torque pulley, the magnitude of the linear acceleration of the mass is the same as the magnitude of the linear acceleration of the outer edge of the torque pulley) we can rewrite the torque equation above as:

$$\tau = Tr = (mg - m\alpha r)r = I\alpha$$

where τ is the torque on the disk, T is the tension in the string, r is the radius of the torque pulley, and α is the angular acceleration of the mass. Solving the above equation for α gives:

$$\alpha = \frac{mgr}{\left(I + mr^2\right)}$$

You calculated the rotational inertia previously, so calculate α using the expression above and compare with the value of the angular acceleration of the disk you obtained from your data.

α = _____ rad/s^2 (calculated)

α = _____ rad/s^2 (experimental)

Difference in α _____ %

Question 2-2: How did your results compare? If instead of angular velocity data you were given angular position data, what type of fit would you need to perform the find the angular acceleration of the disk?

INVESTIGATION 3: CONSERVATION OF ENERGY

Activity 3-1 Does Potential Energy Lost Equal Kinetic Energy Gained?

We want to demonstrate that mechanical energy is conserved in the absence of non-conservative forces. We will apply a torque to a pulley attached to the aluminum disk via a thread attached to a small hanging mass and see if the potential energy lost by the mass as it falls is equal to the gain in the linear kinetic

energy of the mass and the rotational kinetic energy of the disk. The weight of the mass supplies a constant torque that accelerates the rotating disk. [See the discussion in step 8 to see that we do get a constant torque.]   When the disk is released, the mass descends, moving with constant linear acceleration and causing the disks and axle to rotate with constant angular acceleration.

1. Open the experimental file **Conservation of Energy L9.3-1.** Two tables will be visible. Table 1 shows the state of the Accessory Photogate and Table 2 shows the Angular Position of the upper disk. Data are recorded from the photogate whenever the gate changes state from unblocked to blocked or from blocked to unblocked. To get a feel for this, **start the computer** and move your hand through the photogate a couple of times. Notice that each time you block or unblock the photogate, the time of the event is recorded in Table 1. Delete the data you just recorded.

2. Turn on the air and rotate the disk so that the thread wraps around the torque pulley. Place and adjust the position of the photogate on the floor so that the mass will pass through the photogate when released. **Start the computer**, release the disk, and let the mass drop. **Stop the computer** just after the mass has gone through the photogate. There should be two data points in Table 1, one when the bottom of the mass blocked the gate and one when the top of the mass unblocked the gate as it went through. In Table 2, you should see the angular position slowly increasing with time as the mass accelerated downward. Delete all data and leave the photogate where it is as you will be using this setup for the remainder of this Investigation.

3. Because mechanical energy is conserved as the mass falls, the initial potential energy of the hanging mass m is converted to kinetic energy. After falling a distance s, the mass loses an amount of potential energy, mgs. The mass has a translational kinetic energy due to its velocity v and rotational kinetic energy due to the upper disk rotating with an angular velocity ω. For energy to be conserved, the following relation must hold:

$$mgs = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \tag{9}$$

Now, $v = \omega\, r$, where r is the distance from the axis where the torque is applied, i.e. the radius of the *torque pulley*. You should be using the pulley with a radius of 1.27 cm.

Using the equations relating angular quantities to linear quantities, we can rewrite Equation (9) as

$$mgs = \frac{1}{2}mv^2 + \frac{Iv^2}{2r^2}$$
$$mgs = \frac{1}{2}mv^2(1+\frac{I}{mr^2}) \tag{10}$$

where I is found using Eq. (8). We will now perform an experiment to verify that mechanical energy is conserved by calculating and comparing the left hand and right hand sides of Eq. (10).

4. In order to verify Eq. (10), we need to find v and s for a given run. To find v, we could simply measure the length of the mass and divide it by the time it took to pass through the photogate. But because of the finite size of the photogate optics, the mass appears to have a different length as it passes through. We need to measure this effective length before we can perform the experiment. **Start the computer** and allow the mass to drop from a height of about 15 cm above the photogate. Make sure to **stop the computer** just after the mass goes through the gate. Repeat this procedure 3 times, keeping the data for each run.

5. We can use the data we have just taken to find the effective length of the mass as seen by the photogate. The position of the mass at any given time can be determined by using the data of the angular position of the disk. Arrange Tables 1 and 2 on the screen so that they both show the data from run #1. Find the angular position of the disk at the exact time that the bottom of the mass blocked the photogate (this should be the first time point in Table 1 on the computer). Interpolate

between time points in Table 2 to obtain a better reading of the angle. Now find the position at the time that the top of the mass left the photogate. Enter these data in Table 3-1 below. Do the same for Runs #2 and #3

Question 3-1: It is important to know how to convert the measured angle difference to distance. How do you do this? Write down a conversion equation that allows you to convert $\Delta\theta$ (rad) to length that the mass has fallen.

Table 3-1.   Determination of Mass Effective Length through Photogate

Trial	Upper Position (deg)	Lower Position (deg)	Difference (deg)
1			
2			
3			
Average	XXXXXXXXXXXX	XXXXXXXXXXXX	

Average angle difference θ _____ rad (convert average from deg to rad)

Arc length $r\theta$ _____ m (see Question 3-1)

Mass effective length L _____ m

Question 3-2:   Why do you think the measurement of the effective mass length is so critical? Use the vernier calipers and measure the actual length of the mass. Write it down. How does this compare to the effective length you just measured? How will this effect your eventual velocity and kinetic energy determination?

Actual mass length _____ m

6. You are now ready to perform the experiment to verify equation (10). Delete all the data from the previous runs. Raise the mass to a height of approximately 80 cm above the photogate (it doesn't matter exactly how high yet, this information will be extracted from the data). **Start the computer** and quickly let go of the disk so the mass will fall. You can now determine the average speed of the falling mass as it passes through the photogate by determining how long it takes the mass to move through the photogate (T) using the photogate data and the effective length (L) of the mass that you just determined. Enter this information below:

Time mass blocked photogate: $T =$ _____ s

Effective Length: $L =$ _____ m

Velocity: $v = L/T$: _____ m/s

7. We need to determine the height (distance) through which the mass falls to the nearest millimeter. (Technically, you need to measure the height to the point where the local velocity *equals* the average velocity while the mass passes through the photogate. Remember that the mass continues to accelerate through the photogate.) You can determine this height by finding the angular position at the point when the mass is halfway through the photogate with respect to time. Find this time by taking the average of the two time points in the photogate state table on the computer. Look up the angular position, which corresponds to this time making sure to interpolate between time points. Note that the initial angular position was zero when you let go of the disk. Use the conversion you derived earlier to turn the final angular position into the final position of the mass. This will be the height that the mass fell. Enter this value below.

Height mass falls s = _____ m

Question 3-3: Why is it not necessary to measure the distance that the mass drops to the nearest 0.1 mm? Hint: consider the comparison of how well you know KE and PE. How is this different from determining the effective length of the mass?

8. Verify that mechanical energy is conserved by calculating and comparing the left hand and right hand sides of Eq. (10). Enter your results in Table 3-1. Note that by differentiating both sides of Eq. (10) with respect to time we can see that the linear acceleration of the mass $(a = dv/dt = d^2s/dt^2)$ is constant in this situation and therefore the angular acceleration of the disk $(\alpha = ra)$ and the torque applied to the disk $(\tau = I\alpha)$ are also constant.

Table 3-1: Comparing Potential Energy Lost and Kinetic Energy Gained

Final Velocity (v)	Height (s)	Potential Energy Lost (mgs)	Kinetic Energy Gained $(\frac{1}{2}mv^2\left(1+\frac{I}{mr^2}\right))$

Question 3-4: What are your final results? How well do kinetic energy and potential energy agree? Explain any differences that you found? What are possible errors?

INVESTIGATION 4: CONSERVATION OF ANGULAR MOMENTUM

Activity 4-1 Is Angular Momentum Conserved?

Angular momentum, $L = I\omega$, is conserved whenever there are no *external* torques. In the case of rotating disks that engage each other, all torques are *internal* and we expect to have conservation of angular momentum.

In this measurement, you will use the optical reader on the PASCO apparatus, which counts the black bars on the disks as they pass. Each LED comes on when the corresponding optical reader senses a black bar and goes off when it detects a white bar. Using this data, the computer will display a graph of angular velocity vs. time.

1. Remove the torque pulley and mass. Ask your TA to remove the top aluminum disk and replace it with the top stainless steel disk. Replace valve stems so both bottom and top disks rotate separately.

2. Open experiment file **Angular Momentum L9.4-1**. You will see a graph showing the angular velocity of each disk vs. time. Ch 1 should show data from the upper disk and Ch 2 should show data from the lower disk. Verify this by starting the computer and spin one of the disks while holding the other one still. What happens when you reverse the direction of the spin?

3. Perform the following four experiments. Then calculate the initial and final angular momentum and determine whether it is conserved. Enter your data and calculations in Table 4-1. **Note: completing Table 4-1 is time consuming. It is crucial for at least one team member to be working on these calculations throughout the experiments.**

    a. **Top disk spinning; bottom disk stationary:** Start the computer, hold the bottom disk stationary, and give the top disk a spin, so that its angular velocity is between 600 and 800 deg/sec. Wait for a couple of seconds then pull the valve pin from the top disk so that the top disk falls onto the bottom disk. Wait for *two full* seconds and then stop the computer. Record the angular velocity of each disk just before and after releasing the valve. You will get a better reading by finding a range of data points for each measurement and using the statistics function to find the mean of these values. Enter your data in the table below.

Question 4-1: To what percentage accuracy do the initial and final angular momentum agree? Is this good enough agreement? What are possible sources of errors?

    b. **Top and bottom disks spinning in the same direction but at different rates:** Perform the same procedure as in part a, but this time spin both disks in the same direction but at different rates, at least 200 deg/sec apart. Enter your data into Table 4-1.

Question 4-2: To what percentage accuracy do the initial and final angular momentum agree? Is this good enough agreement? What are possible sources of errors?

    c. **Top and bottom disks spinning in opposite directions at different rates:** Spin both disks in opposite directions and at different rates. Make sure to record the direction in which each disk is spinning, i.e. clockwise or counter-clockwise. Since the sensors have no way of knowing which direction the disks are spinning, the angular velocity of each disk will be positive on the graph even though they are spinning in opposite directions. Remember that this will make one disk's angular momentum negative relative to the other's. Perform the same procedure as in part a and enter your data into Table 4-1.

Question 4-3: To what percentage accuracy do the initial and final angular momentum agree? Is this good enough agreement? What are possible sources of errors?

      d. Top and bottom disks spinning in opposite directions at the same rate: Try to spin the two disks at the same rate but in opposite directions. Follow the same procedures as in part a and enter your data into Table 4-1.

Question 4-4: What happened when you removed the pin? To what percentage accuracy do the initial and final angular momentum agree? Is this good enough agreement? What are possible sources of errors?

Table 4-1 Conservation of Angular Momentum

For purposes of this calculation let $I_{\text{stainless steel}}$ (bottom) = $I_{\text{stainless steel}}$ (top). Remember that the angular velocity ω can be negative. You need to keep track of its sign. Data Studio always indicates a positive number.

	Part a	Part b	Part c	Part d
ω Initial Top				
ω Initial Bottom				
ω Final				
L Initial Total				
L Final				
L difference (%)				

Please clean up your lab area.

**Turn off the Rotational Dynamics apparatus,
the timers, and the AIRFLOW.**