PHYS 232
Homework #4 Solutions

25.60 The positive plate by itself creates a field $E = \dfrac{\sigma}{2\epsilon_0} = \dfrac{36.0 \times 10^{-9}\ \text{C/m}^2}{2(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = 2.03\dfrac{\text{kN}}{\text{C}}$

away from the + plate. The negative plate by itself creates the same size field and between the plates it is in the same direction. Together the plates create a uniform field 4.07 kN/C in the space between.

(a) Take $V = 0$ at the negative plate. The potential at the positive plate is then

$$V - 0 = -\int_0^{12.0\ \text{cm}} (-4.07\ \text{kN/C})\,dx$$

The potential difference between the plates is $V = (4.07 \times 10^3\ \text{N/C})(0.120\ \text{m}) = \boxed{488\ \text{V}}$

(b) $\left(\frac{1}{2}mv^2 + qV\right)_i = \left(\frac{1}{2}mv^2 + qV\right)_f$

$$qV = (1.60 \times 10^{-19}\ \text{C})(488\ \text{V}) = \frac{1}{2}mv_f^2 = \boxed{7.81 \times 10^{-17}\ \text{J}}$$

(c) $v_f = \boxed{306\ \text{km/s}}$

26.4 (a) $C = 4\pi e_0 R$

$$R = \frac{C}{4\pi e_0} = k_e C = (8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.00 \times 10^{-12}\ \text{F}) = \boxed{8.99\ \text{mm}}$$

(b) $C = 4\pi e_0 R = \dfrac{4\pi(8.85 \times 10^{-12}\ \text{C}^2)(2.00 \times 10^{-3}\ \text{m})}{\text{N}\cdot\text{m}^2} = \boxed{0.222\ \text{pF}}$

(c) $Q = CV = (2.22 \times 10^{-13}\ \text{F})(100\ \text{V}) = \boxed{2.22 \times 10^{-11}\ \text{C}}$

26.8

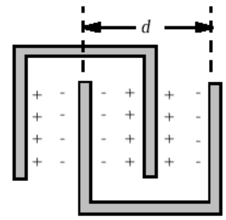
$$C = \frac{\kappa \epsilon_0 A}{d} = 60.0 \times 10^{-15}\text{ F}$$

$$d = \frac{\kappa \epsilon_0 A}{C} = \frac{(1)(8.85 \times 10^{-12})(21.0 \times 10^{-12})}{60.0 \times 10^{-15}}$$

$$d = 3.10 \times 10^{-9}\text{ m} = \boxed{3.10\text{ nm}}$$

26.10

With $\theta = \pi$, the plates are out of mesh and the overlap area is zero. With $\theta = 0$, the overlap area is that of a semi-circle, $\pi R^2/2$. By proportion, the effective area of a single sheet of charge is $(\pi - \theta)R^2/2$.

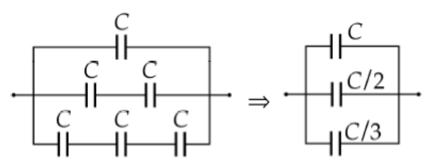


When there are two plates in each comb, the number of adjoining sheets of positive and negative charge is 3, as shown in the sketch. When there are N plates on each comb, the number of parallel capacitors is $2N-1$ and the total capacitance is

$$C = (2N-1)\frac{\epsilon_0 A_{\text{effective}}}{\text{distance}} = \frac{(2N-1)\epsilon_0(\pi-\theta)R^2/2}{d/2} = \boxed{\frac{(2N-1)\,\epsilon_0\,(\pi-\theta)R^2}{d}}$$

26.22

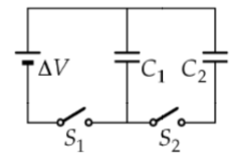
The circuit reduces first according to the rule for capacitors in series, as shown in the figure, then according to the rule for capacitors in parallel, shown below.



$$C_{\text{eq}} = C\left(1 + \frac{1}{2} + \frac{1}{3}\right) = \frac{11}{6}C = \boxed{1.83C}$$

26.23 $C = \frac{Q}{\Delta V}$ so $6.00 \times 10^{-6} = \frac{Q}{20.0}$ and $Q = \boxed{120\ \mu C}$

$Q_1 = 120\ \mu C - Q_2$ and $\Delta V = \frac{Q}{C}$



$\frac{120 - Q_2}{C_1} = \frac{Q_2}{C_2}$ or $\frac{120 - Q_2}{6.00} = \frac{Q_2}{3.00}$

$(3.00)(120 - Q_2) = (6.00)Q_2$

$Q_2 = \frac{360}{9.00} = \boxed{40.0\ \mu C}$

$Q_1 = 120\ \mu C - 40.0\ \mu C = \boxed{80.0\ \mu C}$

***26.45** (a) With air between the plates, we find $C_0 = \frac{Q}{\Delta V} = \frac{48.0\ \mu C}{12.0\ V} = \boxed{4.00\ \mu F}$

(b) When Teflon is inserted, the charge remains the same (48.0 μC) because the plates are isolated. However, the capacitance, and hence the voltage, changes. The new capacitance is

$C' = \kappa C_0 = 2.10(4.00\ \mu F) = \boxed{8.40\ \mu F}$

(c) The voltage on the capacitor now is $\Delta V' = \frac{Q}{C'} = \frac{48.0\ \mu C}{8.40\ \mu F} = \boxed{5.71\ V}$

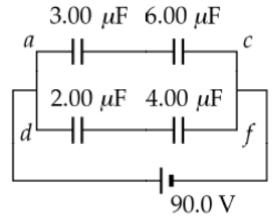
and the charge is $\boxed{48.0\ \mu C}$

26.47

$$\frac{1}{C} = \frac{1}{\left(\frac{\kappa_1 ab}{k_e(b-a)}\right)} + \frac{1}{\left(\frac{\kappa_2 bc}{k_e(c-b)}\right)} = \frac{k_e(b-a)}{\kappa_1 ab} + \frac{k_e(c-b)}{\kappa_2 bc}$$

$$C = \frac{1}{\frac{k_e(b-a)}{\kappa_1 ab} + \frac{k_e(c-b)}{\kappa_2 bc}} = \frac{\kappa_1\kappa_2\, abc}{k_e\kappa_2(bc-ac) + k_e\kappa_1(ac-ab)} = \boxed{\frac{4\pi\kappa_1\kappa_2\, abc\epsilon_0}{\kappa_2 bc - \kappa_1 ab + (\kappa_1 - \kappa_2)ac}}$$

26.54 (a) $C = \left[\frac{1}{3.00} + \frac{1}{6.00}\right]^{-1} + \left[\frac{1}{2.00} + \frac{1}{4.00}\right]^{-1} = \boxed{3.33\ \mu\text{F}}$



(c) $Q_{ac} = C_{ac}\,(\Delta V_{ac}) = (2.00\ \mu\text{F})(90.0\ \text{V}) = 180\ \mu\text{C}$

Therefore, $Q_3 = Q_6 = \boxed{180\ \mu\text{C}}$

$Q_{df} = C_{df}\left(\Delta V_{df}\right) = (1.33\ \mu F)(90.0\ \text{V}) = \boxed{120\ \mu\text{C}}$

(b) $\Delta V_3 = \frac{Q_3}{C_3} = \frac{180\ \mu\text{C}}{3.00\ \mu\text{F}} = \boxed{60.0\ \text{V}}$

$\Delta V_6 = \frac{Q_6}{C_6} = \frac{180\ \mu\text{C}}{6.00\ \mu\text{F}} = \boxed{30.0\ \text{V}}$

$\Delta V_2 = \frac{Q_2}{C_2} = \frac{120\ \mu\text{C}}{2.00\ \mu\text{F}} = \boxed{60.0\ \text{V}}$

$\Delta V_4 = \frac{Q_4}{C_4} = \frac{120\ \mu\text{C}}{4.00\ \mu\text{F}} = \boxed{30.0\ \text{V}}$

(d) $U_T = \frac{1}{2}\,C_{\text{eq}}\,(\Delta V)^2 = \frac{1}{2}(3.33 \times 10^{-6})(90.0\ \text{V})^2 = \boxed{13.4\ \text{mJ}}$

26.61 Note that the potential difference between the plates is held constant at ΔV_i by the battery.

$C_i = \frac{q_0}{\Delta V_i}$ and $C_f = \frac{q_f}{\Delta V_i} = \frac{q_0 + q}{\Delta V_i}$

But $C_f = \kappa C_i$, so $\frac{q_0 + q}{\Delta V_i} = \kappa\left(\frac{q_0}{\Delta V_i}\right)$

Thus, $\kappa = \frac{q_0 + q}{q_0}$ or $\kappa = \boxed{1 + \frac{q}{q_0}}$

Homework #5 Solutions

27.9 (a) $J = \frac{I}{A} = \frac{5.00\text{ A}}{\pi(4.00 \times 10^{-3}\text{ m})^2} = \boxed{99.5\text{ kA/m}^2}$

(b) $J_2 = \frac{1}{4}J_1$; $\frac{I}{A_2} = \frac{1}{4}\frac{I}{A_1}$

$A_1 = \frac{1}{4}A_2$ so $\pi(4.00 \times 10^{-3})^2 = \frac{1}{4}\pi r_2^2$

$r_2 = 2(4.00 \times 10^{-3}) = 8.00 \times 10^{-3}\text{ m} = \boxed{8.00\text{ mm}}$

27.14 (a) Applying its definition, we find the resistance of the rod,

$$R = \frac{\Delta V}{I} = \frac{15.0\text{ V}}{4.00 \times 10^{-3}\text{ A}} = 3750\ \Omega = \boxed{3.75\text{ k}\Omega}$$

(b) The length of the rod is determined from Equation 27.11: $R = \rho\ell / A$. Solving for ℓ and substituting numerical values for R, A, and the values of ρ given for carbon in Table 27.1, we obtain

$$\ell = \frac{RA}{\rho} = \frac{(3.75 \times 10^3\ \Omega)(5.00 \times 10^{-6}\text{ m}^2)}{(3.50 \times 10^{-5}\ \Omega \cdot \text{m})} = \boxed{536\text{ m}}$$

27.24

$$R = \frac{\rho_1 l_1}{A_1} + \frac{\rho_2 l_2}{A_2} = (\rho_1 l_1 + \rho_2 l_2) / d^2$$

$$R = \frac{(4.00 \times 10^{-3}\ \Omega \cdot \text{m})(0.250\text{ m}) + (6.00 \times 10^{-3}\ \Omega \cdot \text{m})(0.400\text{ m})}{(3.00 \times 10^{-3}\text{ m})^2} = \boxed{378\ \Omega}$$

27.49 At operating temperature,

(a) $\mathcal{P} = I(\Delta V) = (1.53\text{ A})(120\text{ V}) = \boxed{184\text{ W}}$

(b) Use the change in resistance to find the final operating temperature of the toaster.

$$R = R_0(1 + \alpha\,\Delta T)$$

$$\frac{120}{1.53} = \frac{120}{1.80}\left[1 + (0.400 \times 10^{-3})\Delta T\right]$$

$$\Delta T = 441°\text{C}$$

$$T = 20.0°\text{C} + 441°\text{C} = \boxed{461°\text{C}}$$

***27.52** (a) $I = \frac{\Delta V}{R}$ so $\mathcal{P} = (\Delta V)I = \frac{(\Delta V)^2}{R}$

$R = \frac{(\Delta V)^2}{\mathcal{P}} = \frac{(120\text{ V})^2}{25.0\text{ W}} = \boxed{576\ \Omega}$ and $R = \frac{(\Delta V)^2}{\mathcal{P}} = \frac{(120\text{ V})^2}{100\text{ W}} = \boxed{144\ \Omega}$

(b) $I = \frac{\mathcal{P}}{\Delta V} = \frac{25.0\text{ W}}{120\text{ V}} = 0.208\text{ A} = \frac{Q}{t} = \frac{1.00\text{ C}}{t}$

$t = \frac{1.00\text{ C}}{0.208\text{ A}} = \boxed{4.80\text{ s}}$ The charge has $\boxed{\text{lower potential energy}}$.

(c) $\mathcal{P} = 25.0\text{ W} = \frac{\Delta U}{t} = \frac{1.00\text{ J}}{t}$

$t = \frac{1.00\text{ J}}{25.0\text{ W}} = \boxed{0.0400\text{ s}}$ The energy $\boxed{\text{changes from electrical to heat and light}}$.

(d) $\Delta U = \mathcal{P}t = (25.0\text{ J/s})(86400\text{ s/d})(30.0\text{ d}) = 64.8 \times 10^6\text{ J}$

The energy company sells $\boxed{\text{energy}}$.

$$\text{Cost} = 64.8 \times 10^6\text{ J}\left(\frac{\$0.0700}{\text{kWh}}\right)\left(\frac{\text{k}}{1000}\right)\left(\frac{\text{W}\cdot\text{s}}{\text{J}}\right)\left(\frac{\text{h}}{3600\text{ s}}\right) = \boxed{\$1.26}$$

$$\text{Cost per joule} = \frac{\$0.0700}{\text{kWh}}\left(\frac{\text{kWh}}{3.60 \times 10^6\text{ J}}\right) = \boxed{\$1.94 \times 10^{-8}/\text{J}}$$

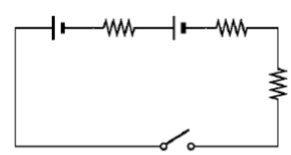
27.66 (a) $R = \frac{\rho \ell}{A} = \boxed{\frac{\rho L}{\pi\left(r_b^2 - r_a^2\right)}}$

(b) $R = \frac{\left(3.50\times 10^5\ \Omega\cdot\text{m}\right)(0.0400\ \text{m})}{\pi\left[(0.0120\ \text{m})^2 - (0.00500\ \text{m})^2\right]} = 3.74\times 10^7\ \Omega = \boxed{37.4\ \text{M}\Omega}$

(c) $dR = \frac{\rho\, d\ell}{A} = \frac{\rho\, dr}{(2\pi r)L} = \left(\frac{\rho}{2\pi L}\right)\frac{dr}{r}$, so $R = \frac{\rho}{2\pi L}\int_{r_a}^{r_b}\frac{dr}{r} = \boxed{\frac{\rho}{2\pi L}\ln\left(\frac{r_b}{r_a}\right)}$

(d) $R = \frac{\left(3.50\times 10^5\ \Omega\cdot\text{m}\right)}{2\pi(0.0400\ \text{m})}\ln\left(\frac{1.20}{0.500}\right) = 1.22\times 10^6\ \Omega = \boxed{1.22\ \text{M}\Omega}$

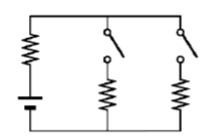
28.3 The total resistance is $R = \frac{3.00\ \text{V}}{0.600\ \text{A}} = 5.00\ \Omega$



(a) $R_{\text{lamp}} = R - r_{\text{batteries}} = 5.00\ \Omega - 0.408\ \Omega = \boxed{4.59\ \Omega}$

(b) $\frac{\mathcal{P}_{\text{batteries}}}{\mathcal{P}_{\text{total}}} = \frac{(0.408\ \Omega)I^2}{(5.00\ \Omega)I^2} = 0.0816 = \boxed{8.16\%}$

28.4 (a) Here $\mathcal{E} = I(R + r)$, so $I = \frac{\mathcal{E}}{R + r} = \frac{12.6\ \text{V}}{(5.00\ \Omega + 0.0800\ \Omega)} = 2.48\ \text{A}$



Then, $\Delta V = IR = (2.48\ \text{A})(5.00\ \Omega) = \boxed{12.4\ \text{V}}$

(b) Let I_1 and I_2 be the currents flowing through the battery and the headlights, respectively.

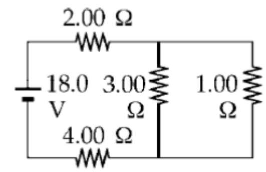
Then, $I_1 = I_2 + 35.0\ \text{A}$, and $\mathcal{E} - I_1 r - I_2 R = 0$

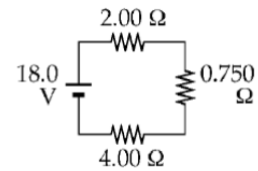
so $\mathcal{E} = (I_2 + 35.0\ \text{A})(0.0800\ \Omega) + I_2(5.00\ \Omega) = 12.6\ \text{V}$

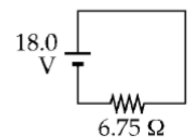
giving $I_2 = 1.93\ \text{A}$

Thus, $\Delta V_2 = (1.93\ \text{A})(5.00\ \Omega) = \boxed{9.65\ \text{V}}$

28.15







$$R_p = \left(\frac{1}{3.00} + \frac{1}{1.00}\right)^{-1} = 0.750\ \Omega$$

$$R_s = (2.00 + 0.750 + 4.00)\ \Omega = 6.75\ \Omega$$

$$I_{\text{battery}} = \frac{\Delta V}{R_s} = \frac{18.0\ \text{V}}{6.75\ \Omega} = 2.67\ \text{A}$$

$$P = I^2 R: \quad P_2 = (2.67\ \text{A})^2 (2.00\ \Omega)$$

$$P_2 = \boxed{14.2\ \text{W}} \text{ in } 2.00\ \Omega$$

$$P_4 = (2.67\ \text{A})^2 (4.00\ \Omega) = \boxed{28.4\ \text{W}} \text{ in } 4.00\ \Omega$$

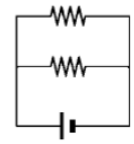
$$\Delta V_2 = (2.67\ \text{A})(2.00\ \Omega) = 5.33\ \text{V}, \quad \Delta V_4 = (2.67\ \text{A})(4.00\ \Omega) = 10.67\ \text{V}$$

$$\Delta V_p = 18.0\ \text{V} - \Delta V_2 - \Delta V_4 = 2.00\ \text{V} \quad (= \Delta V_3 = \Delta V_1)$$

$$P_3 = \frac{(\Delta V_3)^2}{R_3} = \frac{(2.00\ \text{V})^2}{3.00\ \Omega} = \boxed{1.33\ \text{W}} \text{ in } 3.00\ \Omega$$

$$P_1 = \frac{(\Delta V_1)^2}{R_1} = \frac{(2.00\ \text{V})^2}{1.00\ \Omega} = \boxed{4.00\ \text{W}} \text{ in } 1.00\ \Omega$$

28.17 (a) $\Delta V = IR$: $33.0\text{ V} = I_1(11.0\ \Omega)$ $33.0\text{ V} = I_2(22.0\ \Omega)$



$I_1 = 3.00\text{ A}$ $I_2 = 1.50\text{ A}$

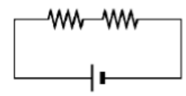
$\mathcal{P} = I^2R$: $\mathcal{P}_1 = (3.00\text{ A})^2(11.0\ \Omega)$ $\mathcal{P}_2 = (1.50\text{ A})^2(22.0\ \Omega)$

$\mathcal{P}_1 = 99.0\text{ W}$ $\mathcal{P}_2 = 49.5\text{ W}$

The 11.0-Ω resistor uses more power.

(b) $\mathcal{P}_1 + \mathcal{P}_2 = \boxed{148\text{ W}}$ $\mathcal{P} = I(\Delta V) = (4.50)(33.0) = \boxed{148\text{ W}}$

(c) $R_s = R_1 + R_2 = 11.0\ \Omega + 22.0\ \Omega = 33.0\ \Omega$



$\Delta V = IR$: $33.0\text{ V} = I(33.0\ \Omega)$, so $I = 1.00\text{ A}$

$\mathcal{P} = I^2R$: $\mathcal{P}_1 = (1.00\text{ A})^2(11.0\ \Omega)$ $\mathcal{P}_2 = (1.00\text{ A})^2(22.0\ \Omega)$

$\mathcal{P}_1 = 11.0\text{ W}$ $\mathcal{P}_2 = 22.0\text{ W}$

The 22.0-Ω resistor uses more power.

(d) $\mathcal{P}_1 + \mathcal{P}_2 = I^2(R_1 + R_2) = (1.00\text{ A})^2(33.0\ \Omega) = \boxed{33.0\text{ W}}$

$\mathcal{P} = I(\Delta V) = (1.00\text{ A})(33.0\text{ V}) = \boxed{33.0\text{ W}}$

(e) The parallel configuration uses more power.