Combination of Capacitors

Parallel combination

- The individual potential differences across capacitors connected in parallel are all the same and are equal to the potential difference applied across the combination.
- For a parallel combination total charge Q,

$$Q = Q_1 + Q_2 \tag{30}$$

- Because the voltages are the same:

$$Q_1 = C_1 \Delta V \quad Q_2 = C_2 \Delta V \tag{31}$$

- For a capacitor equivalent to the combination;

$$Q = C_{eq} \Delta V \tag{32}$$

- Substituting:

$$C_{eq} \Delta V = C_1 \Delta V + C_2 \Delta V \tag{33}$$

$$C_{eq} = C_1 + C_2 \tag{34}$$

For a parallel combination of many capacitors:

$$C_{eq} = C_1 + C_2 + C_3 + C_4 \ldots\ldots\ldots\ldots\ldots\ldots \tag{35}$$

The equivalent capacitance of a parallel combination of capacitors is greater than any of the individual capacitances.

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Figure 26.8

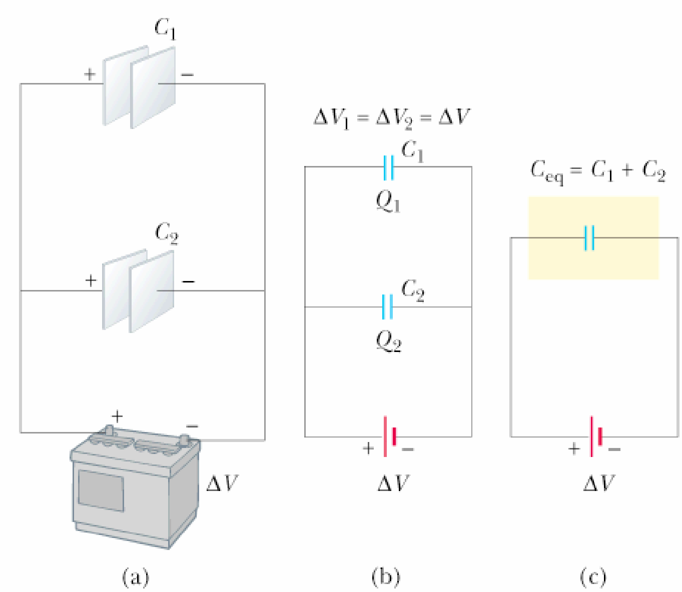

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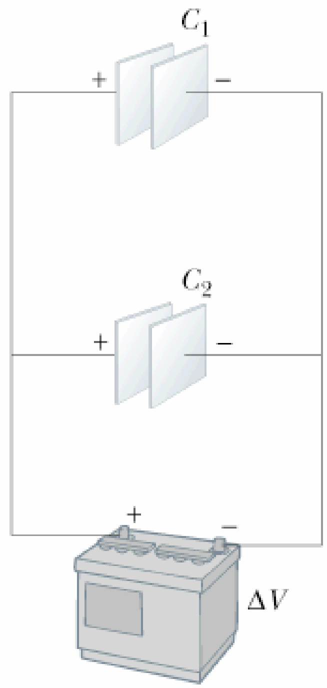
Figure 26.8



(a)

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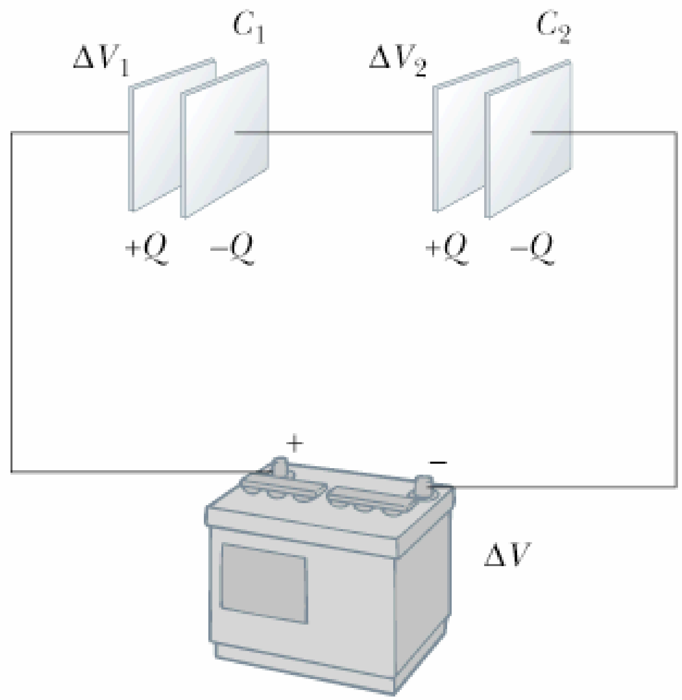
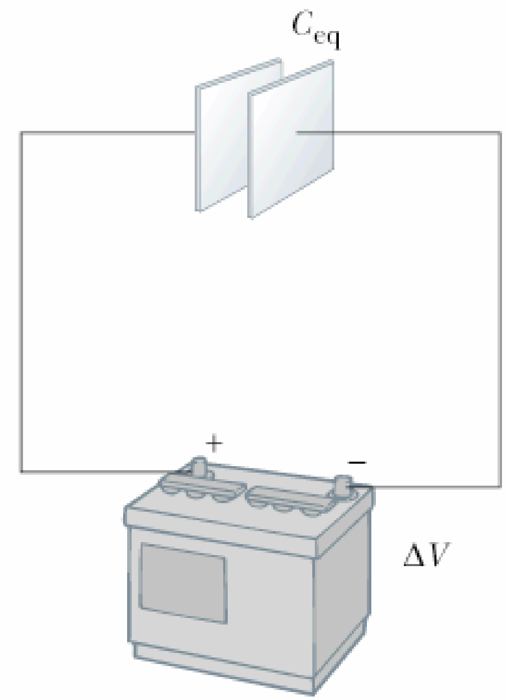
Figure 26.9



(a)



(b)

Series combination

- **The charges on capacitors connected in series are the same**
- For a series combination total voltage ΔV,

$$\Delta V = \Delta V_1 + \Delta V_2 \tag{36}$$

- Because the charges are the same:

$$\Delta V_1 = \frac{Q}{C_1} \quad \Delta V_2 = \frac{Q}{C_2} \tag{37}$$

- For a capacitor equivalent to the combination;

$$\Delta V = \frac{Q}{C_{eq}} \tag{38}$$

- Substituting:

$$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2} \tag{39}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \tag{40}$$

- For a series combination of many capacitors:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4} \ldots\ldots\ldots\ldots \tag{41}$$

- **The equivalent capacitance of a series combination of capacitors is always less than any individual capacitance in the combination.**

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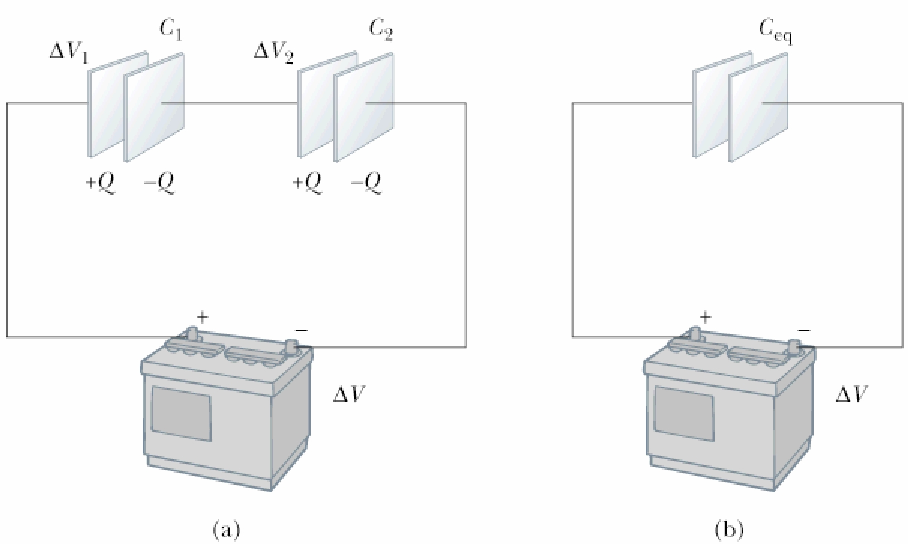
Figure 26.9



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Example 26.4

Find the equivalent capacitance between a and b for the combination shown.

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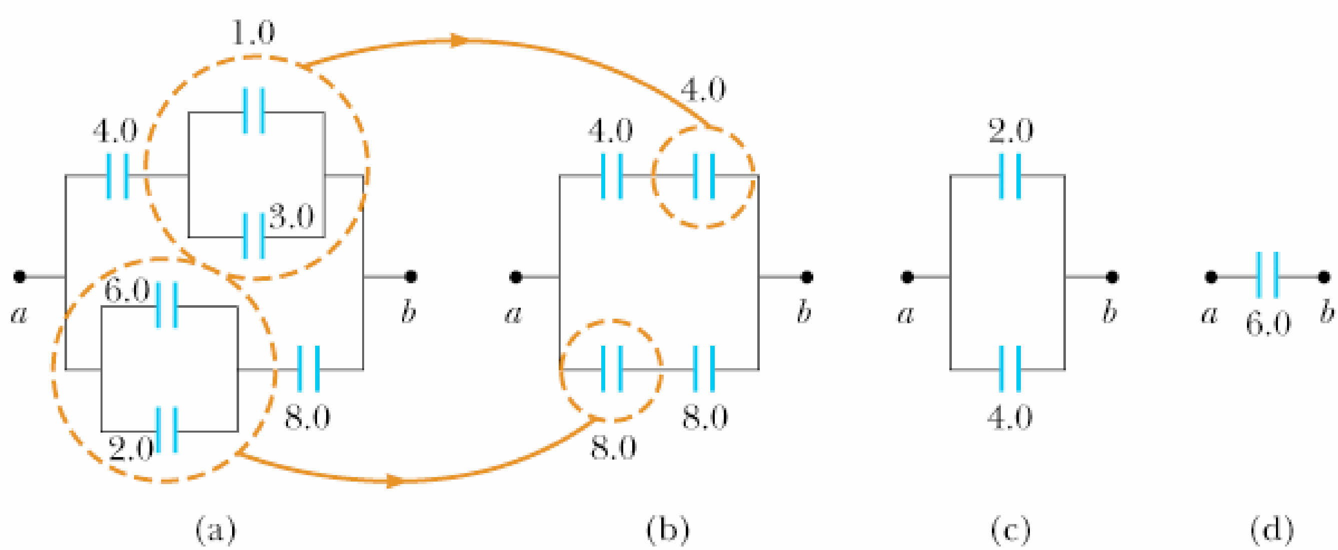
Figure 26.10



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Energy stored in a capacitor

- We have to spend energy to charge the capacitor (energy from the battery), this energy is stored in the electric field of the charged capacitor as potential energy.
- Suppose that we charge the capacitor by moving small amounts of charge dq from one plate the other.
- Assume that at any given time the plates have charges $-q$ and q.
- Potential difference between the plates

$$\Delta V = \frac{q}{C} \tag{42}$$

- Amount of work required to move dq between the plates.

$$dW = dq\Delta V = \frac{q}{C}dq \tag{43}$$

- Amount of work required to charge the capacitor from $q = 0$ to $q = Q$:

$$W = \int_0^Q dW \tag{44}$$

$$= \int_0^Q \frac{q}{C}dq \tag{45}$$

$$= \frac{1}{2C}\left[q^2\right]_0^Q \tag{46}$$

$$W = \frac{Q^2}{2C} \tag{47}$$

- Potential energy U stored in the capacitor:

$$U = W = \frac{Q^2}{2C} \tag{48}$$

- Substituting from $C = \frac{Q}{\Delta V}$:

$$U = \frac{Q^2}{2C} = \frac{1}{2}Q\Delta V = \frac{1}{2}C(\Delta V)^2 \tag{49}$$

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Figure 26.11

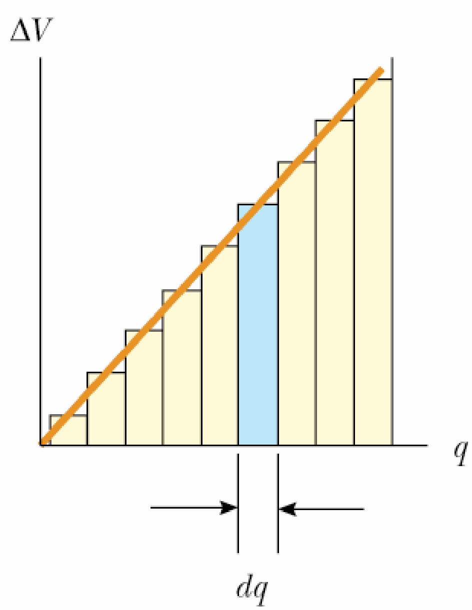

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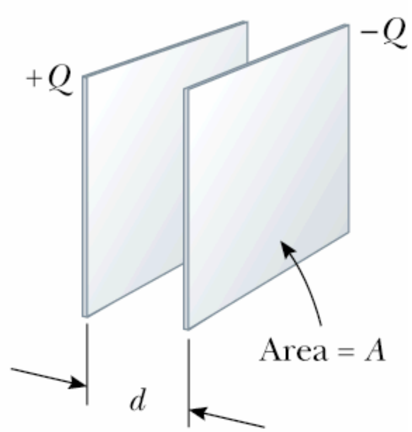
Figure 26.2



Energy stored in a capacitor

- Energy is stored in the electric field of the charged capacitor. Consider a parallel plate capacitor.
- Using $\Delta V = Ed$, and $C = \frac{\epsilon_0 A}{d}$ (for a parallel plate capacitor)

$$U = \frac{1}{2}\frac{\epsilon_0 A}{d}(Ed)^2 = \frac{1}{2}(\epsilon_0 Ad)E^2 \qquad (50)$$

- But Ad is the volume occupied by the electric field. Hence **Energy per unit volume of the electric field** or *energy density* of the electric field is:

$$u_E = \frac{1}{2}(\epsilon_0 Ad)E^2/Ad \qquad (51)$$

$$u_E = \frac{1}{2}\epsilon_0 E^2 \qquad (52)$$

- This is a general result: **energy density of the electric field is proportional to the square of the magnitude of the electric field at a given point**

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Figure 26.14

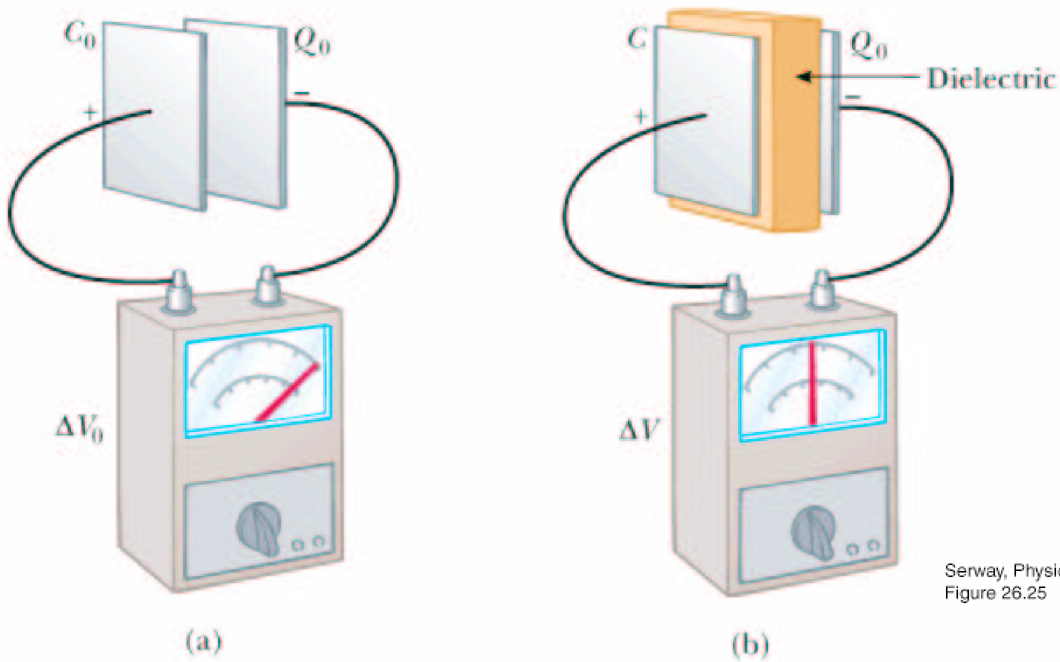

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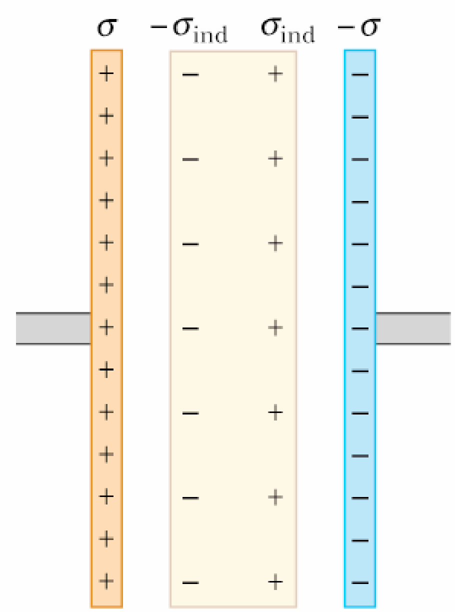
Figure 26.25



Capacitors with Dielectrics

- A dielectric in a non-conducting material that lowers the electric field, hence the voltage of a capacitor for a given charge Q.

$$\Delta V = \frac{\Delta V_0}{\kappa} \tag{53}$$

- κ is the **dielectric constant** of the material and is larger than 1. ($\kappa = 1$ for vacuum)

- If the capacitance, charge and the voltage of the capacitor without dielectric are C_0, Q_0, and ΔV_0, capacitance C of the capacitor with the dielectric:

$$C = \frac{Q_0}{\Delta V} \tag{54}$$

$$= \frac{Q_0}{\Delta V_0/\kappa} \tag{55}$$

$$= \kappa \frac{Q_0}{\Delta V_0} \tag{56}$$

$$C = \kappa C_0 \tag{57}$$

- **The capacitance increases by factor κ when the dielectric completely fills the region between the plates.**

- For a parallel plate capacitor with a dielectric in:

$$C = \kappa \frac{\epsilon_0 A}{d} \tag{58}$$

- In addition to increasing the capacitance of a capacitor, dielectrics also increase the maximum electric field that can be applied on a capacitor without causing a discharge. This maximum electric field is known as the **dielectric strength**

Example 26.7

A parallel plate capacitor is charged with a battery to a charge Q_0. Battery is then removed and a slab of material with dielectric constant κ is inserted between the plates. Find the energy stored in the capacitor before and after the dielectric is inserted.

$$U = \frac{Q^2}{2C} \tag{59}$$

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Figure 26.17

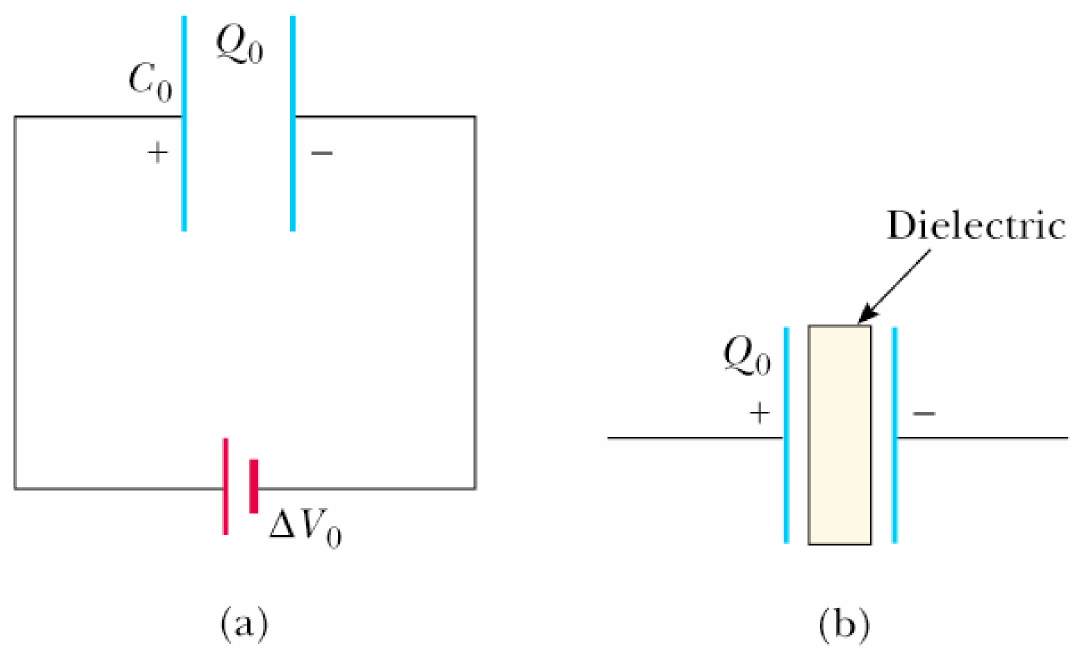

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Figure 26.15