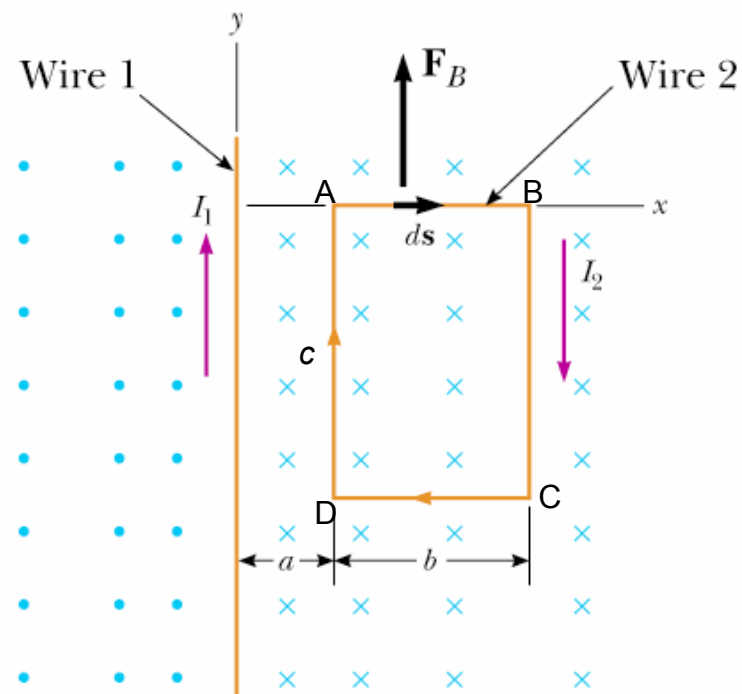


### Example 30.7 (and more)

If the rectangular loop has a width  $b$  and a length  $c$ ,  
find the Net magnetic force exerted by wire 1 on on  
the loop.

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Figure 30.15



Field due to wire 1, at a distance  $x$  from wire 1: Into the paper.

$$B = \frac{\mu_0 I_1}{2\pi x}$$

First find the force on each segment of the loop.

- segment AB: Force  $\mathbf{F}_{AB}$  is in  $+y$  direction

$$d\mathbf{F} = I_2 d\mathbf{s} \times \mathbf{B} \quad (11)$$

$$dF = I_2 \frac{\mu_0 I_1}{2\pi x} dx$$

$$F_{AB} = \frac{\mu_0 I_1 I_2}{2\pi} \int_a^{a+b} \frac{dx}{x}$$

$$F_{AB} = \frac{\mu_0 I_1 I_2}{2\pi} \ln \left( 1 + \frac{b}{a} \right)$$

- segment CD: Force  $\mathbf{F}_{CD}$  is in  $-y$  direction, and has same magnitude as  $F_{AB}$
- segment BC: Two parallel wires with currents in opposite directions (repel each other):  $\mathbf{F}_{BC}$  is in  $+x$ :

$$F_{BC} = \frac{\mu_0 I_1 I_2}{2\pi(a+b)} c$$

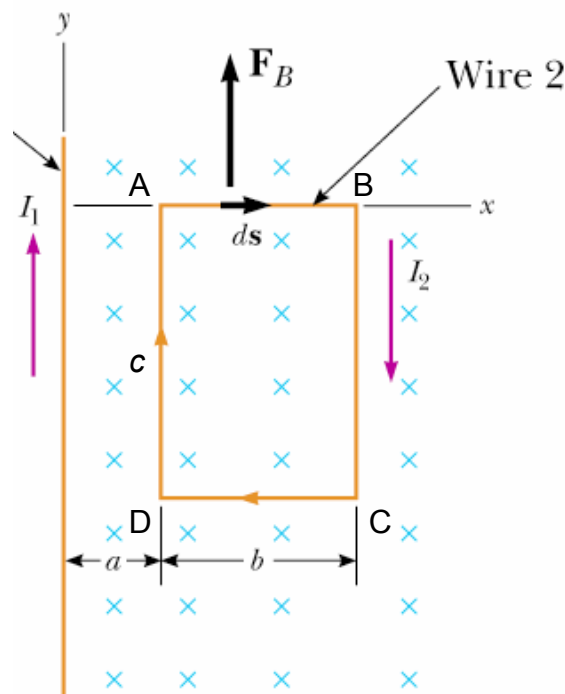
- segment DA: Two parallel wires with currents in the same direction (attract each other):  $\mathbf{F}_{DA}$  is in  $-x$ :

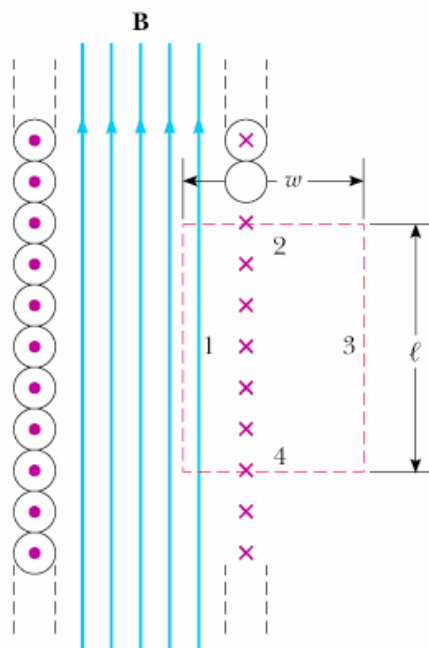
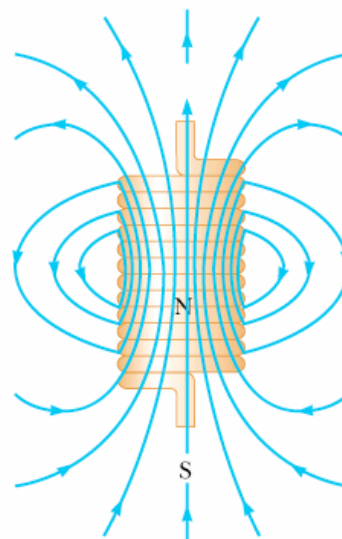
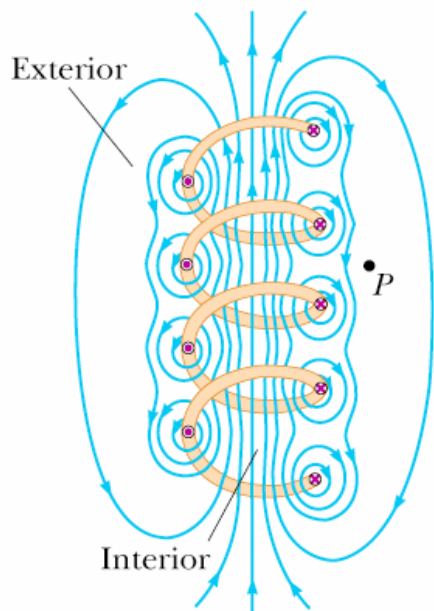
$$F_{DA} = \frac{\mu_0 I_1 I_2}{2\pi a} c$$

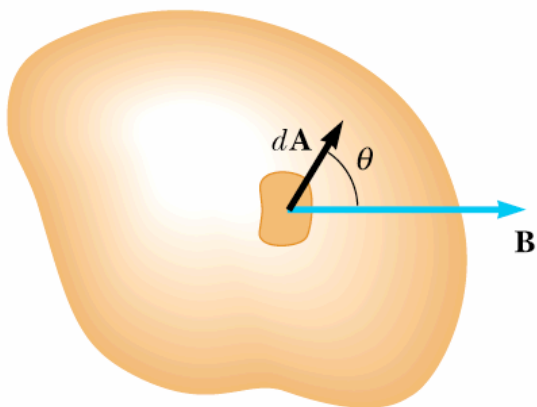
- $\mathbf{F}_{AB}$  and  $\mathbf{F}_{CD}$  cancel each other; but  $\mathbf{F}_{DA} > \mathbf{F}_{BC}$ . So there is a net force on the loop in  $-y$  direction:

$$\begin{aligned} F &= F_{DA} - F_{BC} \\ &= \frac{\mu_0 I_1 I_2}{2\pi a} c - \frac{\mu_0 I_1 I_2}{2\pi(a+b)} c \end{aligned} \quad (12)$$

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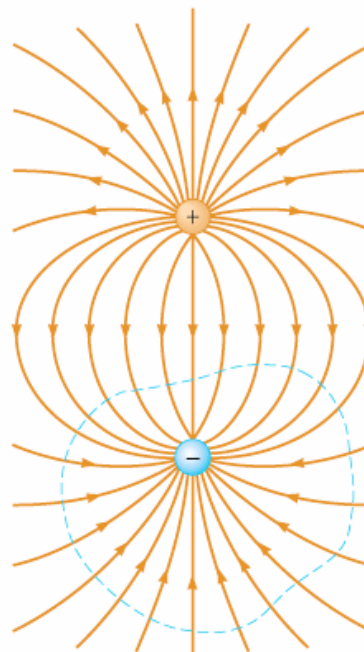
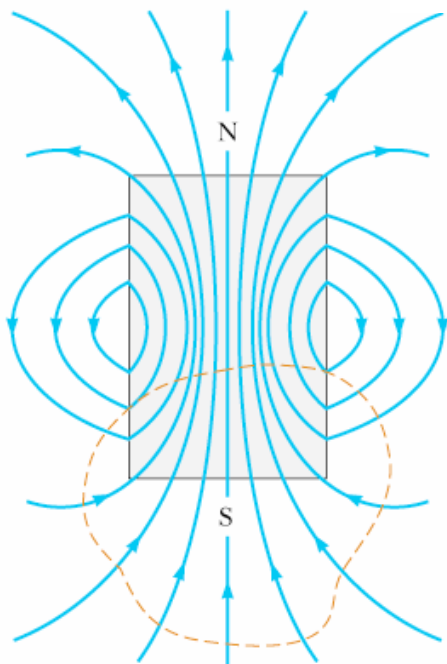
## Magnetic Flux

$$\Phi_B = \int \mathbf{B} \cdot d\mathbf{A}$$

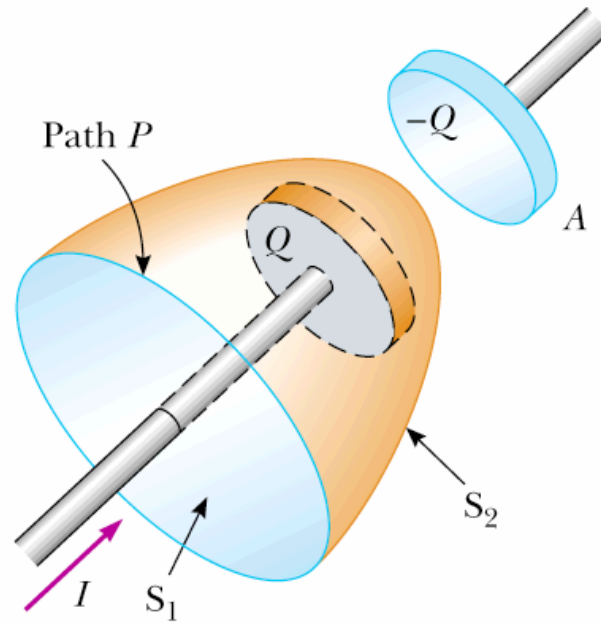
## Gauss' Law in Magnetism

No isolated magnetic monopoles:

$$\oint \mathbf{B} \cdot d\mathbf{A} = 0$$



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Figure 30.24



# Ampere, Maxwell and Displacement Current

Electric flux in the region between the plates of a capacitor:

$$\Phi_E = \int \mathbf{E} \cdot d\mathbf{A} = EA = \frac{\sigma}{\epsilon_0} A = \frac{Q}{\epsilon_0}$$

We can define the **displacement current**  $I_d$  in the region of the electric field as:

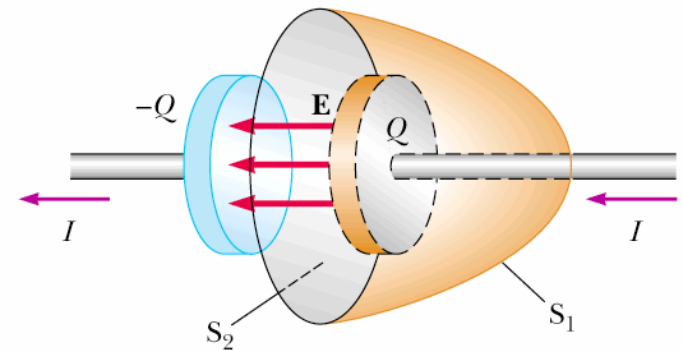
$$I_d = \frac{dQ}{dt} = \epsilon_0 \frac{d\Phi_E}{dt}$$

A time varying electric field behaves similar to a current of magnitude  $\epsilon_0 \frac{d\Phi_E}{dt}$

## Ampere-Maxwell Law

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0(I + I_d) = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

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Figure 30.25



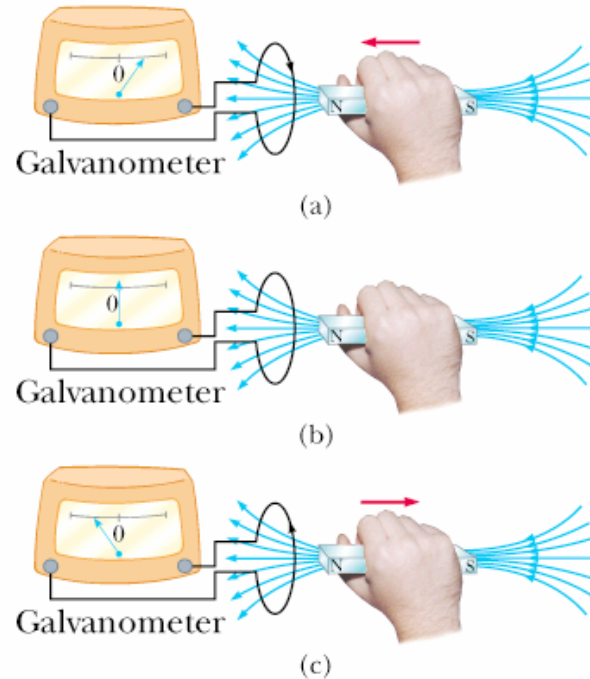
# Faraday's law

- An **emf** is induced in a circuit when the magnetic flux through that circuit is changing:

$$\varepsilon = - \frac{d\Phi_B}{dt} \quad (1)$$

$$\varepsilon = - \frac{d}{dt} (B A \cos \theta)$$

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Figure 31.1



## Motional *EMF*

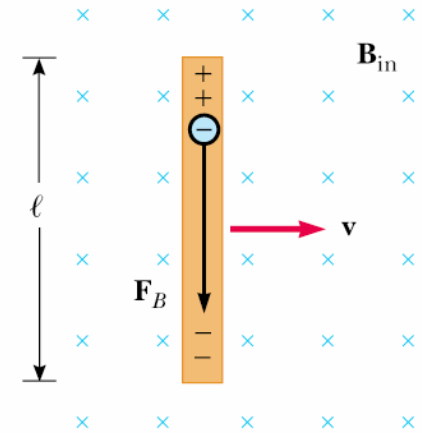
- The magnetic flux through the circuit:

$$\Phi_B = Blx$$

- The induced motional *emf*

$$\begin{aligned} \varepsilon &= -\frac{d\Phi_B}{dt} \\ &= -\frac{d}{dt}(Blx) = -Bl\frac{dx}{dt} \\ &= -Blv \end{aligned}$$

(2)



Serway, Figure 31.8

Harcourt, Inc.

- The current in the circuit:

$$\begin{aligned} I &= \frac{|\varepsilon|}{R} \\ &= \frac{BLV}{R} \end{aligned}$$

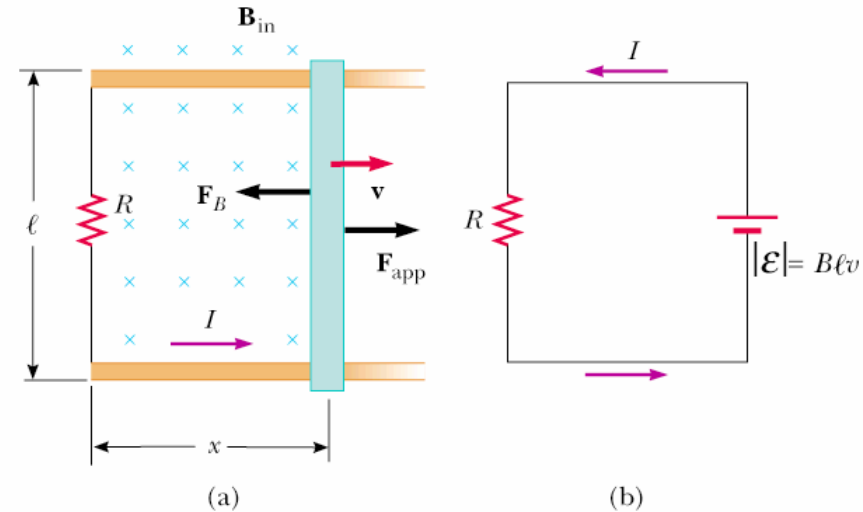
- Due to this current in the moving bar, it experiences a magnetic force ( $F_B$ ) that opposes the motion; need to apply an equal and opposite force ( $F_{app}$ ) to maintain the velocity:

$$F = IlB$$

- Work required to move the bar (and the associated power)

$$W = (F_{app})x = (IlB)x$$

$$P = \frac{d}{dt}W = (F_{app})v = (IlB)v = \frac{B^2 l^2 v^2}{R} = \frac{\varepsilon^2}{R}$$

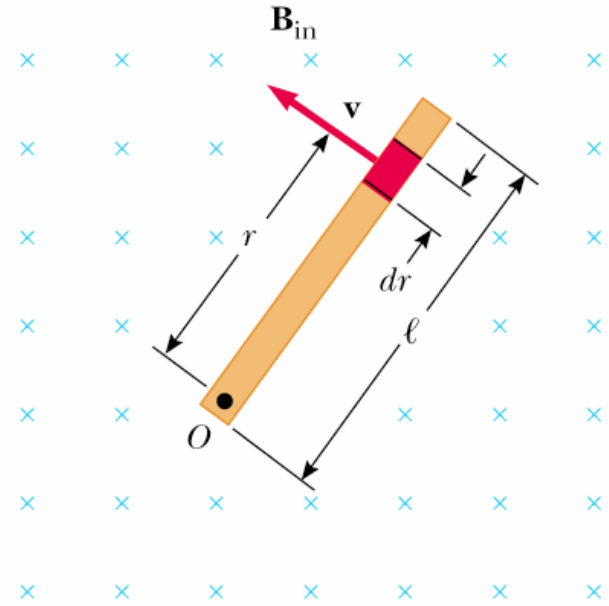


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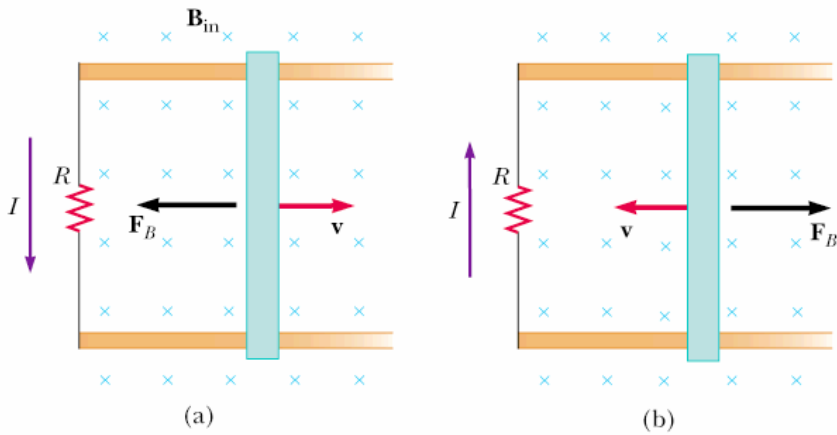
**Example 31.4:**

Find the motional *emf* between the ends of the bar

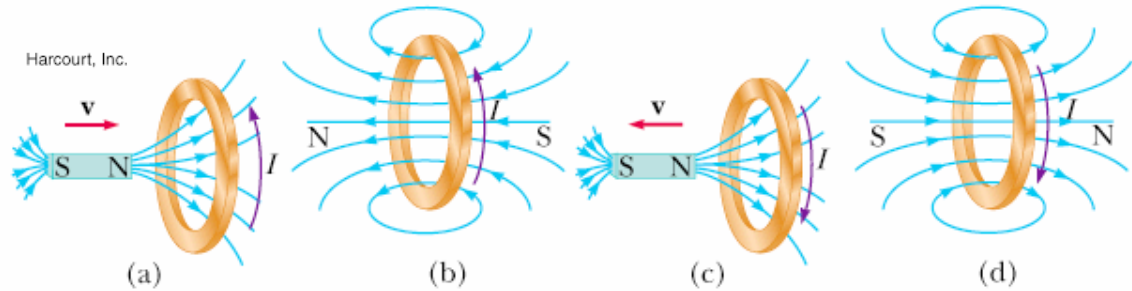


# Lenz's law

The polarity of the induced *emf* is such that it tends to produce a magnetic flux to oppose the change in magnetic flux through the area enclosed by the current loop



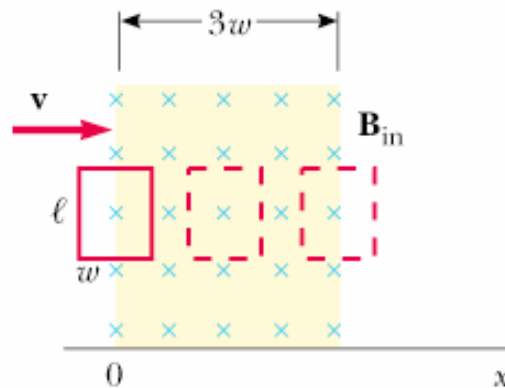
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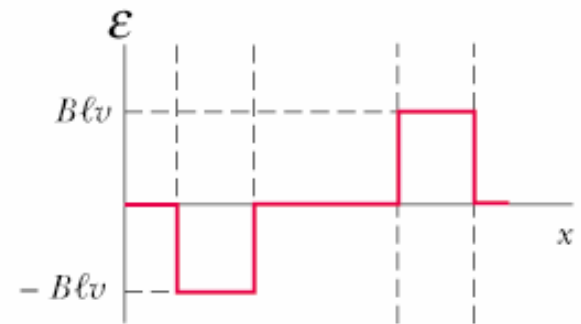
## Example 31.7: A loop moving through a magnetic field;

A rectangular loop of dimensions  $l$  and  $w$  and resistance  $R$  moves with constant speed  $v$  to the right, passing through a uniform magnetic field  $\mathbf{B}$ .  $x$  is defined as the position of the front leg of the loop. Plot as a function of  $x$ :

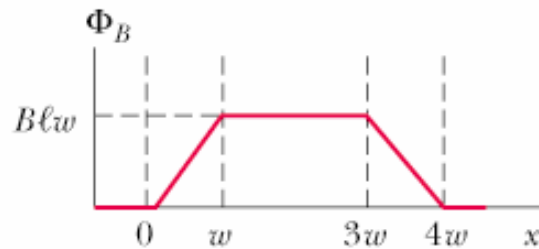
- Magnetic flux through the loop
- The induced motional *emf*
- The external force required to keep  $v$  constant.



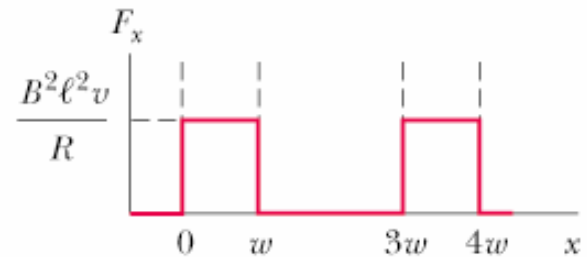
(a)



(c)



(b)



(d)