Conductors in Electrostatic equilibrium

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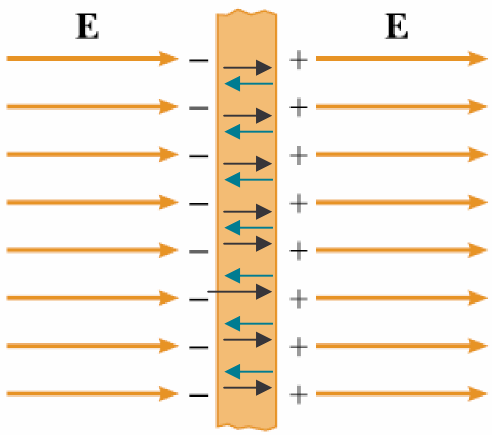
Figure 24.16



1. The electric field is zero everywhere inside the conductor

2. If an isolated conductor carries a charge, the charge Resides on the conductor’s surface.

3. The electric field just outside the conductor is perpendicular to its surface.

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Figure 24.17

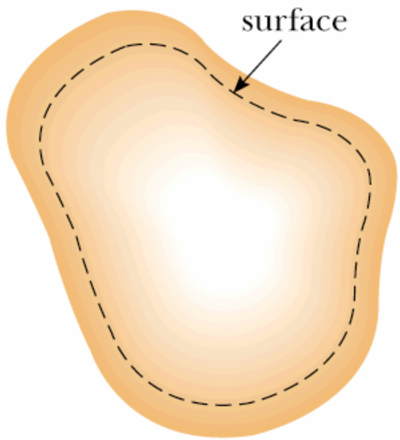

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Figure 25.22

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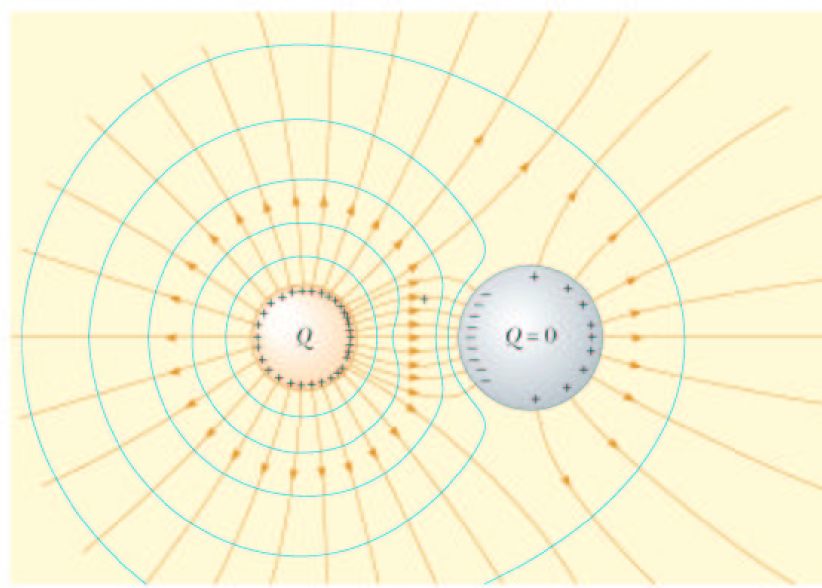

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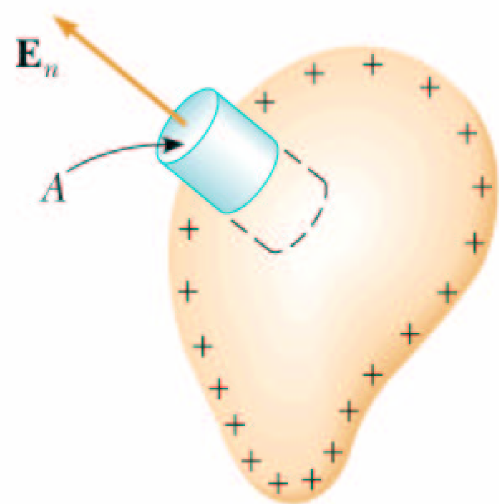
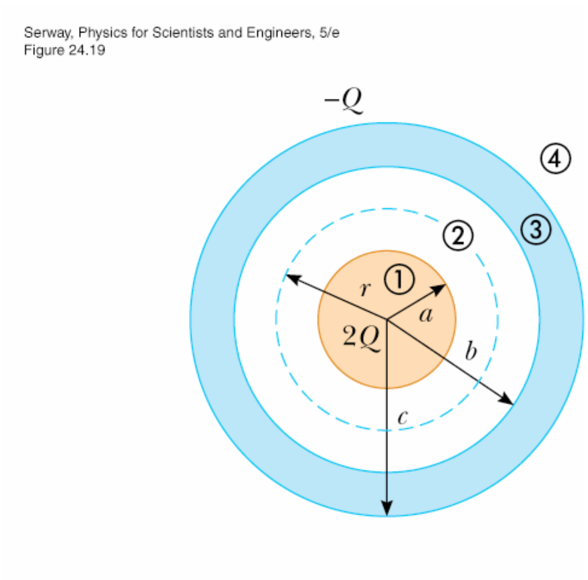
Figure 24.18



$$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = \oint E \; dA = \frac{q_{in}}{\epsilon_0} = \frac{\sigma A}{\epsilon_0}$$

Example 24.10



A solid conducting sphere of radius a carries a net positive charge $2Q$. A conducting spherical shell concentric with the sphere carries a net charge $-Q$. find the electric field in the regions labeled 2,3 and 4. Find the charge distribution of the shell when the entire system is in electrostatic equilibrium.

- **REGION 2: Construct a Gaussian surface with radius r, $a < r < b$**

$$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = \frac{q_{in}}{\epsilon_0} \tag{1}$$

$$\oint E \, dA = E \oint dA = E(4\pi r^2) = \frac{2Q}{\epsilon_0} \tag{2}$$

$$E = \frac{2Q}{4\pi\epsilon_0 r^2} = 2k_e \frac{Q}{r^2} \tag{3}$$

- **REGION 4: Construct a Gaussian surface with radius r, $r > c$**

$$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = \frac{q_{in}}{\epsilon_0} \tag{4}$$

The net charge inside this Gaussian sphere is: $2Q + (-Q) = Q$

$$\oint E \, dA = E \oint dA = E(4\pi r^2) = \frac{Q}{\epsilon_0} \tag{5}$$

$$E = \frac{Q}{4\pi\epsilon_0 r^2} = k_e \frac{Q}{r^2} \tag{6}$$

- **REGION 3: Construct a Gaussian surface with radius r, $b < r < c$**

$$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = \frac{q_{in}}{\epsilon_0} \tag{7}$$

But we know that $E = 0$ inside the conductor.

$\Rightarrow q_{in} = 0$ for this case.
$\Rightarrow$ Inner surface of the shell (at $r = b$) must contain a charge $-2Q$ to neutralize the charge $2Q$ on the inner sphere.
$\Rightarrow$ Outer surface of the shell (at $r = c$) must contain a charge $+Q$, so that the shell has its net charge $-Q$.

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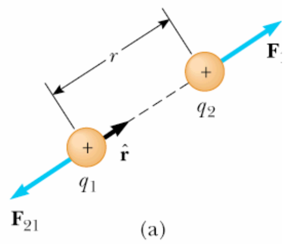
Figure 23.6



Electric Potential

Imagine that we try to bring a positive charge q_0, closer to another positive charge q by a distance $d\mathbf{s}$:

Work done by the electric field of q on our charge q_0:

$$\mathbf{F} \cdot d\mathbf{s} = q_0 \mathbf{E} \cdot d\mathbf{s} \tag{8}$$

Amount of work we had to do against the field of charge q:

$$-\mathbf{F} \cdot d\mathbf{s} = -q_0 \mathbf{E} \cdot d\mathbf{s} \tag{9}$$

This amount of energy is stored on our charge as **electric potential energy**

Now we move our charge from point A to point B:

Change in the potential energy:

$$\Delta U = -q_0 \int_A^B \mathbf{E} \cdot d\mathbf{s} \tag{10}$$

Electric force is a conservative force:

$\Rightarrow$ **Energy required to move a charge from A to B (and hence the difference in potential energy between the two points) does not depend on the path taken.**

- **Electric Potential** V

  Potential energy per unit charge

$$V = \frac{U}{q_0}$$

- **Potential Difference** ΔV

  Potential energy difference per unit charge

$$\Delta V = \frac{\Delta U}{q_0} = -\int_A^B \mathbf{E} \cdot d\mathbf{s}$$

- Electric potential is a relative quantity: only **differences** in potential are meaningful.

- Electric field at infinity is defined to be zero.

- **Electric potential of any point is defined as the energy required to bring a unit positive charge from infinity to that point.**

$$V = -\int_{\infty}^{B} \mathbf{E} \cdot d\mathbf{s} \tag{13}$$

- Electric potential is a scaler characteristic of an electric field. It is a **scaler field**.

- Units of potential: **volt** (V):

$$1V = 1\frac{J}{C} \tag{14}$$

- **Electron Volt** (eV): defined as **the amount of energy an electron gains by moving through a potential difference of 1V**

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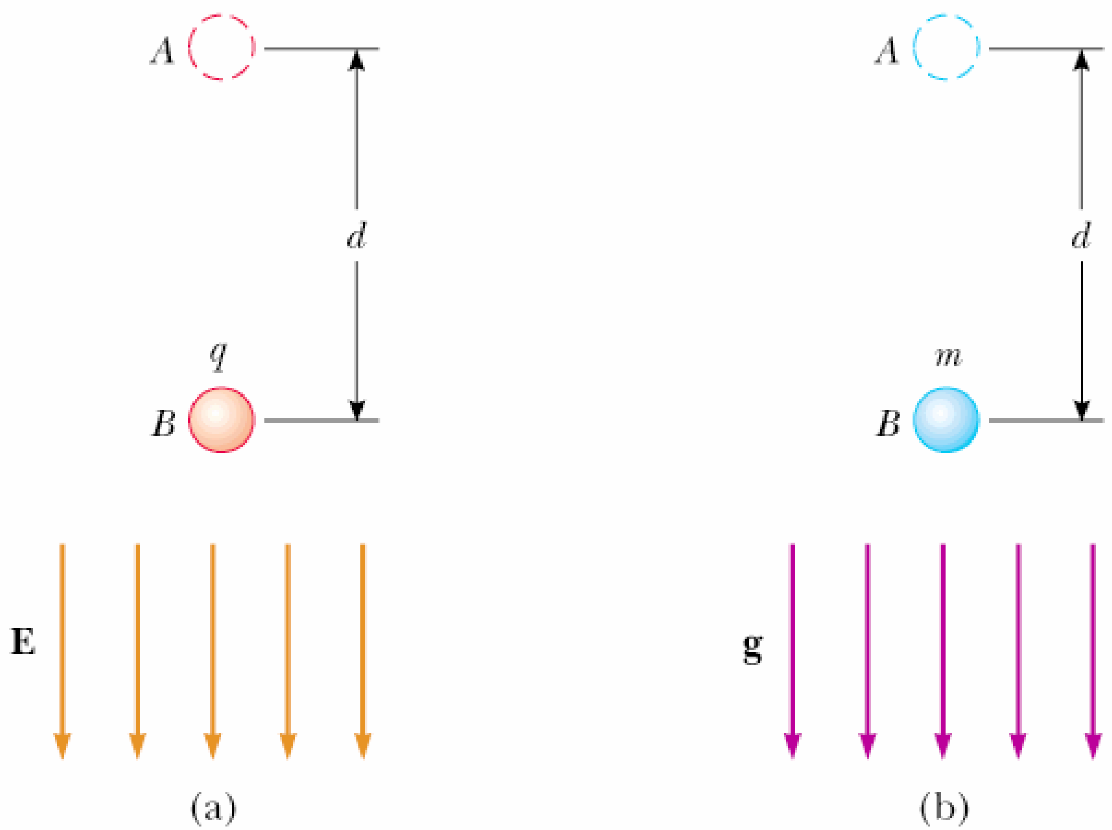
Figure 25.1



Imagine a unit positive charge moving **along** a uniform electric field from point A to B

- **E** and ds are parallel: $\theta = 0$ ($\cos\theta$=1):

$$\Delta V = -\int_A^B \mathbf{E} \cdot d\mathbf{s} = -\int_A^B E ds \tag{15}$$

$$= -E \int_A^B ds = -Ed \tag{16}$$

- **Electric field lines point in the direction of decreasing electric potential**

- If a charge q_0 is moved from A to B, the change in its potential energy:

$$\Delta u = q_0 \Delta V = -q_0 E d \tag{17}$$

- A positive charge loses potential energy when it moves in the direction of the electric field.

- The electric potential energy a particle loses is converted into kinetic energy (assuming no loss)

- A negative charge gains potential energy when it moves in the direction of the electric field.

Imagine a unit positive charge moving **perpendicular** to a uniform electric field from point A to B

- **E** and d**s** are perpendicular: $\theta = 90$ ($\cos\theta$=0):

$$\Delta V = -\int_A^B \mathbf{E} \cdot d\mathbf{s} = 0 \quad (18)$$

(19)

- **An equipotential surface is a surface consisting of a continuous distribution of points having the same potential**
- **The electric field at a point is perpendicular to the equipotential surface through that point**

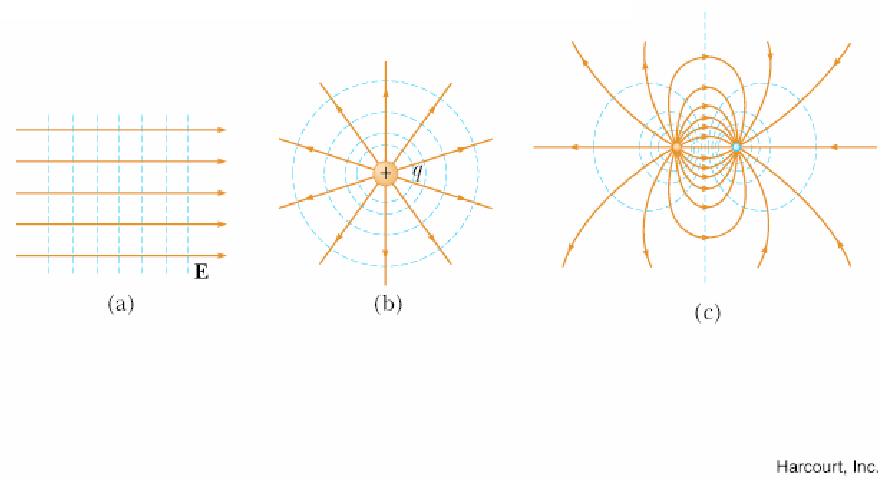


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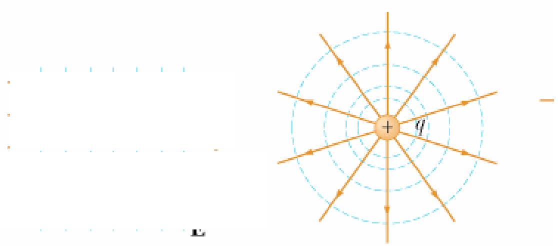
Figure 25.12



(a) (b)

- Potential difference in the field of a point charge:

$$\Delta V = -\int_A^B \mathbf{E} \cdot d\mathbf{s}$$

$$-\int_A^B k_e \frac{q}{r^2} \hat{\mathbf{r}} \cdot d\mathbf{s}$$

- **Potential difference is independent of the path from A to B**: So let us first go along the radial direction to a point C on the equipotential surface (in this case the sphere) that contains point B:

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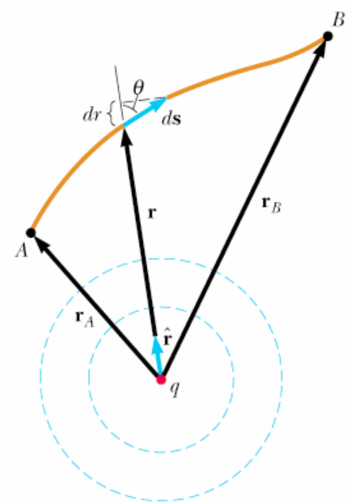
Figure 25.6



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$$V_C - V_A = -\int_A^C k_e \frac{q}{r^2}\hat{\mathbf{r}} \cdot d\mathbf{s} \tag{22}$$

$$= -\int_A^C k_e \frac{q}{r^2} dr \tag{23}$$

$$= -k_e q \int_{r_A}^{r_C} \frac{dr}{r^2} \tag{24}$$

$$= k_e q \left[\frac{1}{r}\right]_{r_A}^{r_C} \tag{25}$$

$$= k_e q \left[\frac{1}{r_C} - \frac{1}{r_A}\right] \tag{26}$$

- Now we will move on the equipotentail surface from C to B:

$$V_B - V_C = 0 \tag{27}$$

-

$$V_B - V_A = (V_B - V_C) + (V_C - V_A) \tag{28}$$

$$= 0 + k_e q \left[\frac{1}{r_C} - \frac{1}{r_A}\right] \tag{29}$$

$$r_C = r_B \Rightarrow \qquad V_B - V_A = k_e q \left[\frac{1}{r_B} - \frac{1}{r_A}\right] \tag{30}$$

- **Potential at a distance r from a point charge**

$$V_B - V_A = k_e q \left[\frac{1}{r_B} - \frac{1}{r_A} \right]$$

$$r_A = \infty, r_B = r \Rightarrow \qquad V(r) = k_e q \left[\frac{1}{r} - \frac{1}{\infty} \right]$$

$$V(r) = k_e \frac{q}{r}$$

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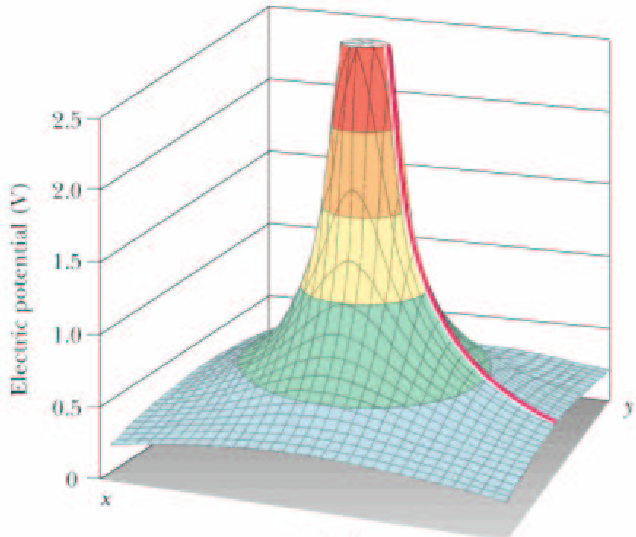
Figure 25.7a



(a)

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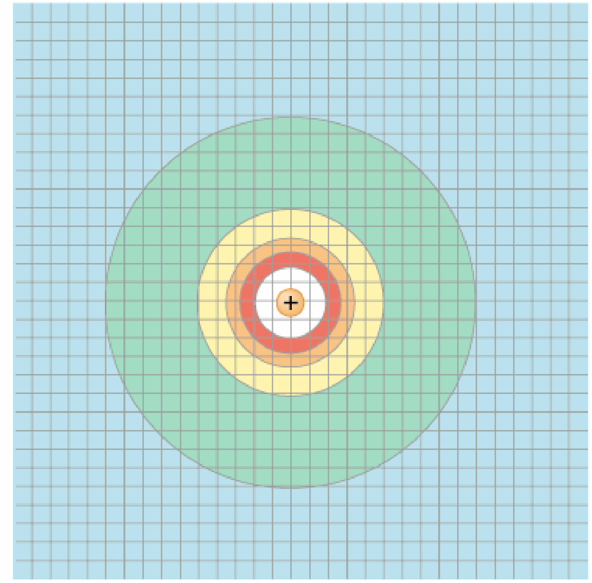
Figure 25.7b



(b)

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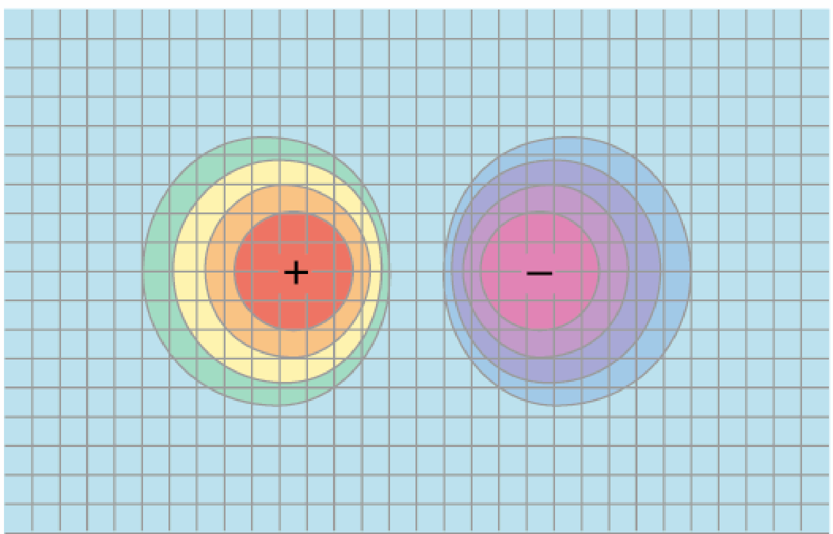
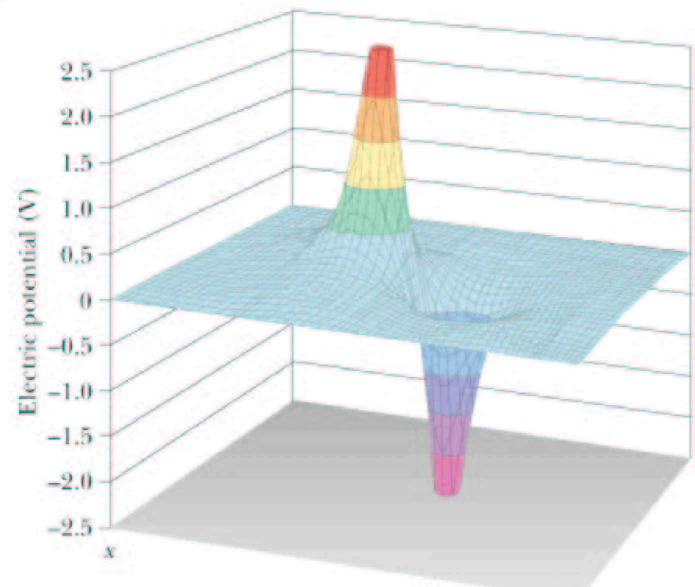
Figure 25.8b



(b)

25.8a



(a)