CAPACITORS AND CAPACITANCE

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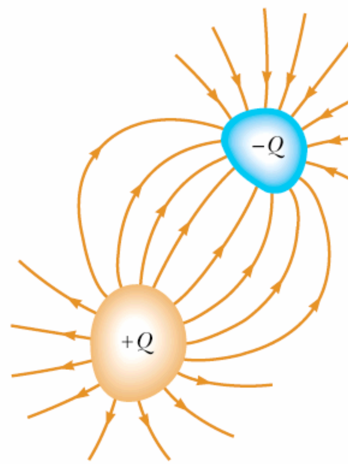
Figure 26.1



- An electric capacitor is a device to store electric charge.
- A capacitor can be constructed by placing two conductors close to each other.
- Consider a capacitor with charges $+Q$ and $-Q$;
- As Q increases, the electric field between the conductors increases.
- Hence the **potential difference** ΔV between the conductors increases.

$$Q \propto \Delta V \qquad (1)$$

- The capacitance of a capacitor can be thought of as its ability to store the highest amount of charge by generating the smallest ΔV.

$$C = \frac{Q}{\Delta V} \qquad (2)$$

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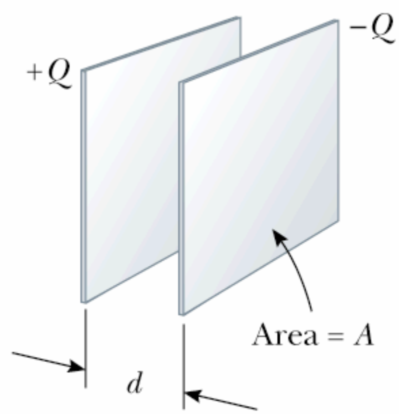
Figure 26.2



- Capacitance is defined to be a positive quantity: Q and ΔV are substituted without their signs. (i.e.. $\Delta V = |V_A - V_B|$

- Because of its unit ΔV is referred to as the **voltage**

- Unit of capacitance is **farad** (F)

$$1F = 1\frac{C}{V} \tag{3}$$

- 1F is HUGE. so the widely used unit of capacitance is the **picofarad** "pF" $(10^{-12}F)$

$$1pF = 1\frac{F}{1000000000000} \tag{4}$$

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Figure 26.2

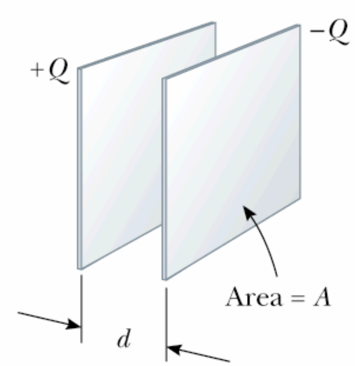

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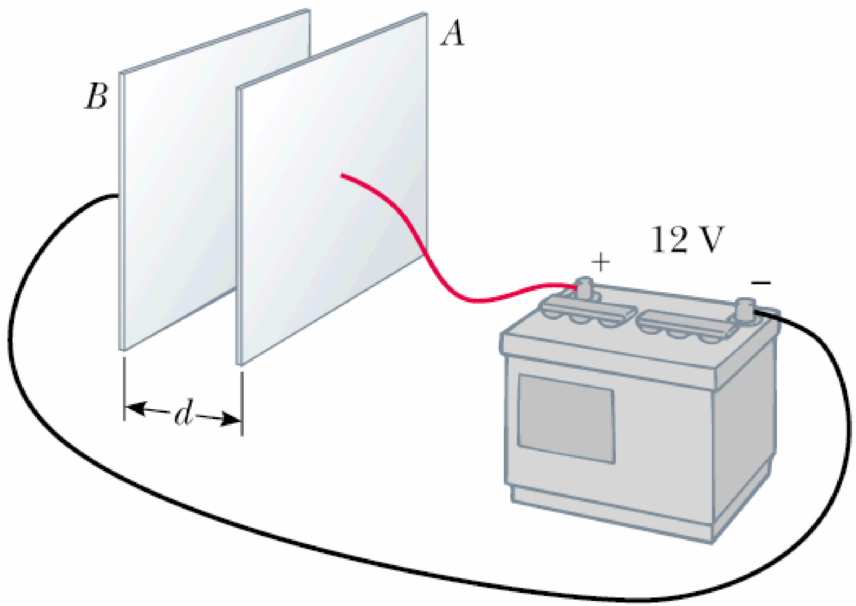
Figure 25.4



- Consider a capacitor with each plate connected to one terminal of a battery using conducting wires.

- Because a wire, and the terminal and the plate connected to it, must be at the same potential; **potential difference (voltage) across the capacitor plates must be equal to the voltage across the battery terminals**

- $\Rightarrow$ Electrons flow from the negative terminal of the battery to its plate (and from the other plate to the positive terminal) until the two voltages are equal.

Capacitance of a conducting sphere

- Consider a charge Q placed on a conducting sphere of radius R. The *other* conductor in this case can be considered as a conducting shell at infinity.

$$C = \frac{Q}{\Delta V} \tag{5}$$

$$\Delta V = V(R) - V(\infty) = \frac{k_e Q}{R} \Rightarrow \quad C = \frac{Q}{k_e Q/R} \tag{6}$$

$$= \frac{R}{k_e} \tag{7}$$

$$k_e = \frac{1}{4\pi\epsilon_0} \Rightarrow \quad C = 4\pi\epsilon_0 R \tag{8}$$

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Figure 25.18

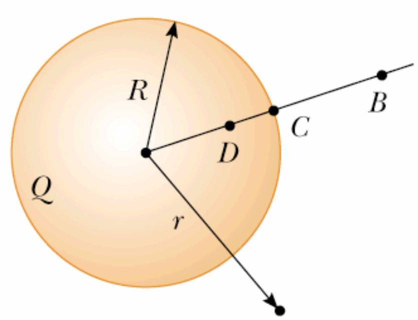

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Figure 26.2

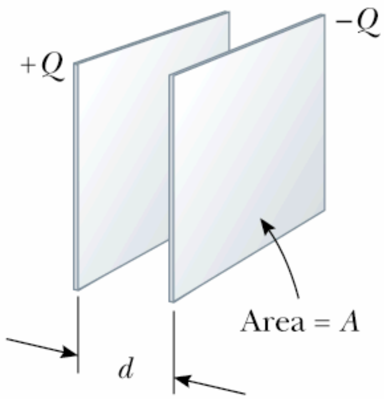

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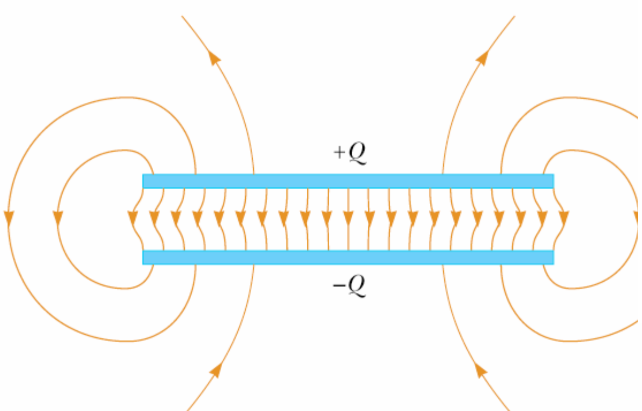
Figure 26.3a



(a)

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Parallel-Plate Capacitor

- If the distance between the plates is small compared to the area, the electric field between the plates is uniform.

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A} \quad (9)$$

- Potential difference between the plates:

$$V_A - V_B = -\int_A^B \mathbf{E} \cdot d\mathbf{s} = -Ed = -\frac{Qd}{\epsilon_0 A} \quad (10)$$

-

$$C = \frac{Q}{\Delta V} \quad (11)$$

$$= \frac{Q}{Qd/\epsilon_0 A} \quad (12)$$

$$= \frac{\epsilon_0 A}{d} \quad (13)$$

- **The capacitance of a parallel plate capacitor is proportional to its area and inversely proportional to the plate separation.**

Cylindrical Capacitor: Example 26.2

A solid cylindrical conductor of radius a and charge Q is coaxial with a cylindrical shell of negligible thickness, radius $b > a$, and charge $-Q$. Find the capacitance of this cylindrical capacitor if its length is l

From Gauss' law we had, $E_r = 2k_e\lambda/r$, and from symmetry we saw that **E** is radial :

$$V_a - V_b = -\int_a^b \mathbf{E}\cdot d\mathbf{s} \quad (14)$$

$$= -\int_a^b E dr \quad (15)$$

$$= -2k_e\lambda \int_a^b \frac{dr}{r} \quad (16)$$

$$= -2k_e\lambda\,[ln(r)]_a^b \quad (17)$$

$$= -2k_e\lambda\, ln\left(\frac{b}{a}\right) \quad (18)$$

$\lambda = Q/l$:

$$C = \frac{Q}{\Delta V} \quad (19)$$

$$= \frac{Q}{\frac{2k_e Q}{l}\, ln\left(\frac{b}{a}\right)} \quad (20)$$

$$= \frac{l}{2k_e\, ln\left(\frac{b}{a}\right)} \quad (21)$$

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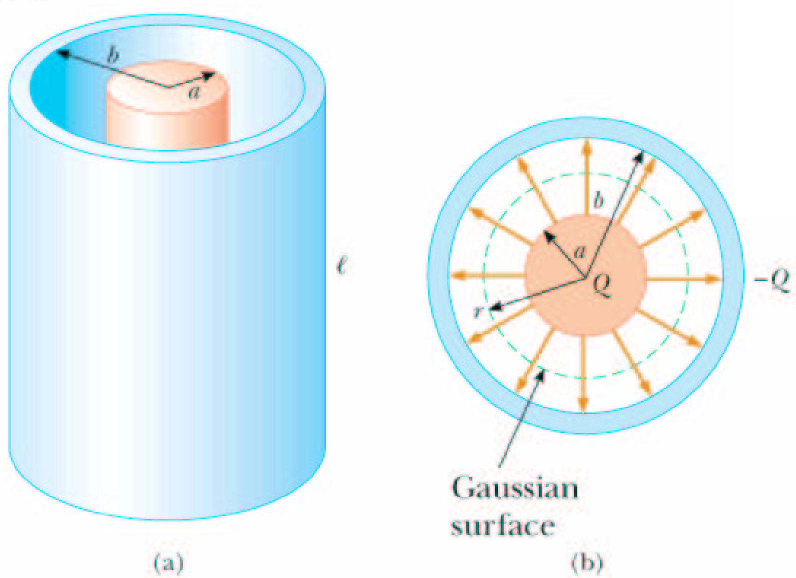
Figure 26.5



Spherical Capacitor: Example 26.3:

A spherical capacitor consists of a spherical conducting shell of radius b and charge $-Q$ concentric with a smaller conducting sphere of radius a and charge Q. Find the capacitance.

From Gauss' law we had, $E_r = k_e Q/r^2$ for $a < r < b$, and from symmetry we saw that **E** is radial :

$$V_a - V_b = -\int_a^b \mathbf{E} \cdot d\mathbf{s} \quad (22)$$

$$= -\int_a^b E\,dr \quad (23)$$

$$= -k_e Q \int_a^b \frac{dr}{r^2} \quad (24)$$

$$= -k_e Q \left[-\frac{1}{r}\right]_a^b \quad (25)$$

$$= -k_e Q \left(\frac{b-a}{ab}\right) \quad (26)$$

$$C = \frac{Q}{\Delta V} \quad (27)$$

$$= \frac{Q}{k_e Q \left(\frac{b-a}{ab}\right)} \quad (28)$$

$$= \frac{ab}{k_e (b-a)} \quad (29)$$

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Figure 26.6

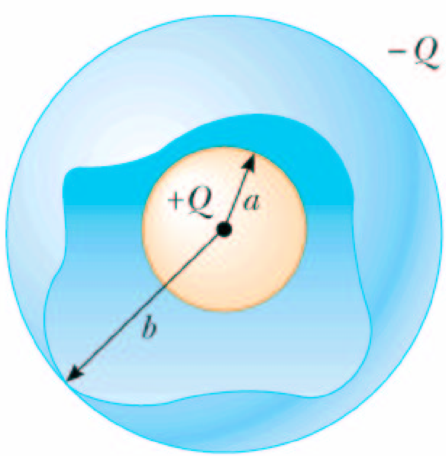

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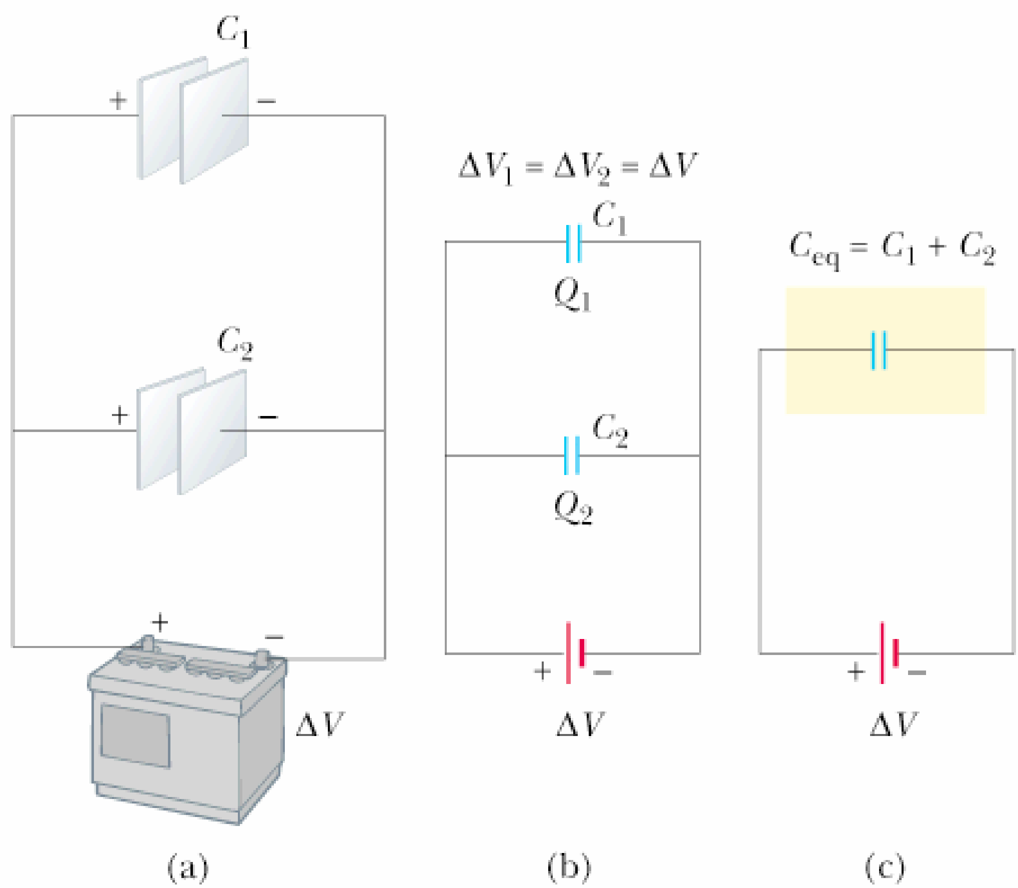
Figure 26.8



Combination of Capacitors

Parallel combination

- The individual potential differences across capacitors connected in parallel are all the same and are equal to the potential difference applied across the combination.

- For a parallel combination total charge Q,

$$Q = Q_1 + Q_2 \tag{30}$$

- Because the voltages are the same:

$$Q_1 = C_1 \Delta V \quad Q_2 = C_2 \Delta V \tag{31}$$

- For a capacitor equivalent to the combination;

$$Q = C_{eq} \Delta V \tag{32}$$

- Substituting:

$$C_{eq} \Delta V = C_1 \Delta V + C_2 \Delta V \tag{33}$$

$$C_{eq} = C_1 + C_2 \tag{34}$$

For a parallel combination of many capacitors:

$$C_{eq} = C_1 + C_2 + C_3 + C_4 .... \tag{35}$$

The equivalent capacitance of a parallel combination of capacitors is greater than any of the individual capacitances.

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Figure 26.8

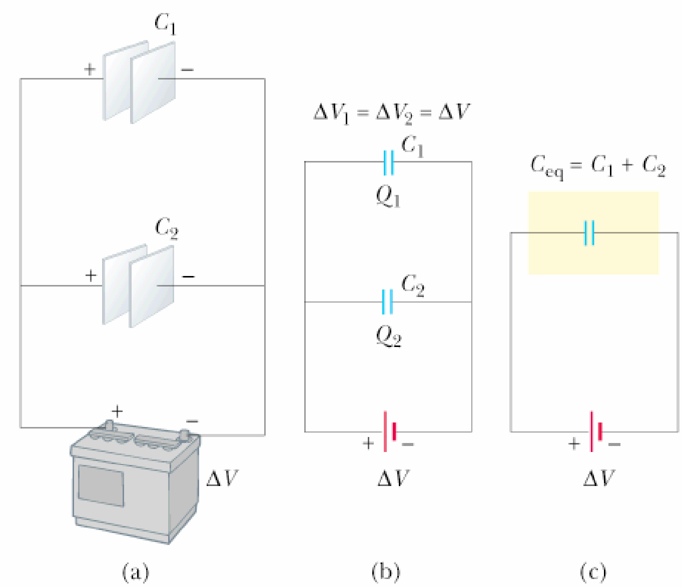

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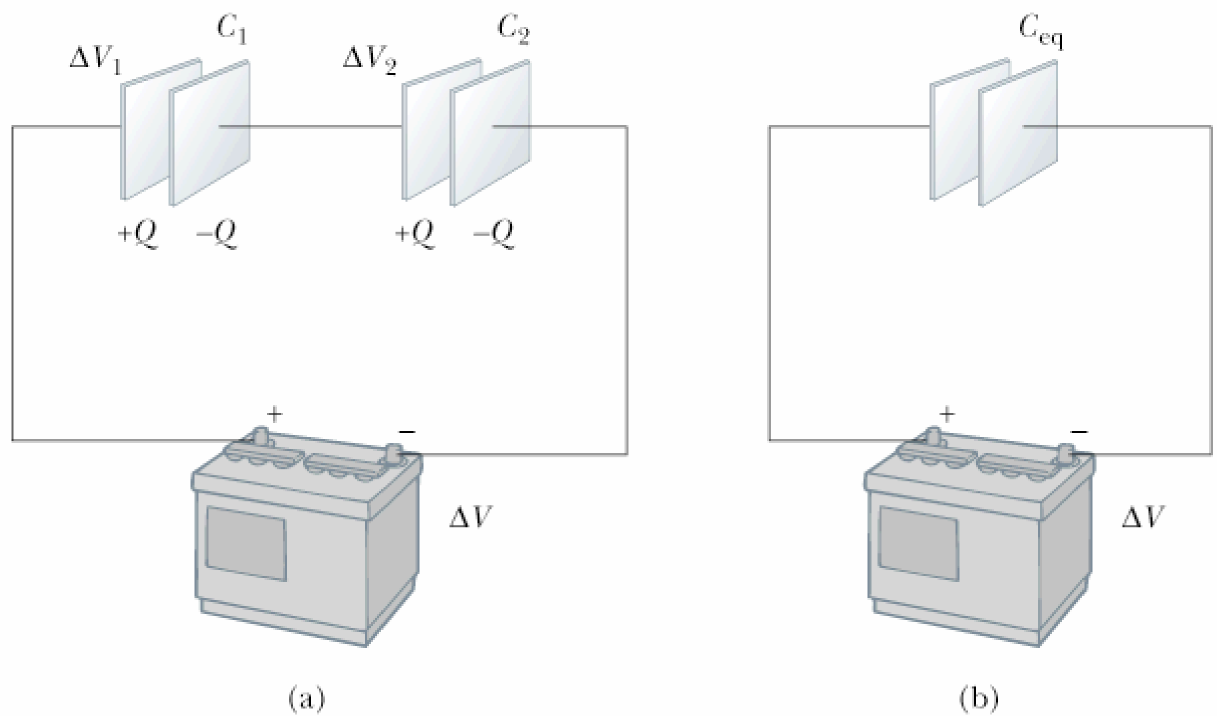
Figure 26.9



Series combination

- **The charges on capacitors connected in series are the same**
- For a series combination total voltage ΔV,

$$\Delta V = \Delta V_1 + \Delta V_2 \tag{36}$$

- Because the charges are the same:

$$\Delta V_1 = \frac{Q}{C_1} \quad \Delta V_2 = \frac{Q}{C_2} \tag{37}$$

- For a capacitor equivalent to the combination;

$$\Delta V = \frac{Q}{C_{eq}} \tag{38}$$

- Substituting:

$$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2} \tag{39}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \tag{40}$$

- For a series combination of many capacitors:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4} \ldots\ldots\ldots\ldots \tag{41}$$

- **The equivalent capacitance of a series combination of capacitors is always less than any individual capacitance in the combination.**

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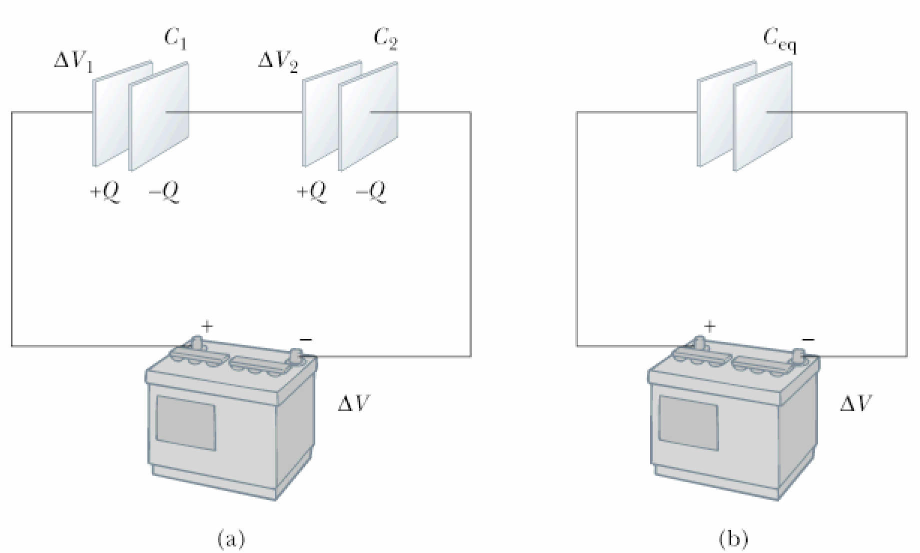
Figure 26.9



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Example 26.4

Find the equivalent capacitance between a and b for the combination shown.

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Figure 26.9