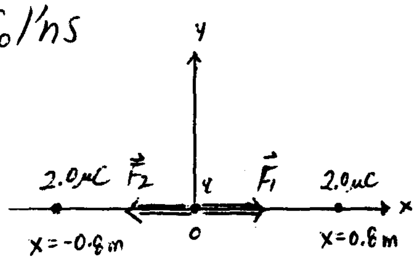
# PS #3 Sol'ns

25-16) a, b)




$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2$

By symmetry, $\vec{F}_1 = -\vec{F}_2 \Rightarrow \boxed{\vec{F}_{net} = 0}$

$\Rightarrow \boxed{\vec{E}_{net} = 0}$

25-16) c) $V_{net} = V_1 + V_2 = \frac{2}{4\pi\varepsilon_0}\left(\frac{2.0\mu C}{0.8m}\right) = \boxed{44.95 \times 10^3 \, V}$

__Remember__: Electric potential is explicitly defined as: $V = k_e \sum_i \frac{q_i}{\sqrt{\vec{r}_i \cdot \vec{r}_i}} = k_e \sum_i \frac{q_i}{\sqrt{r_i^2}} = k_e \sum_i \frac{q_i}{r_i}$

from the above def'n we see that the potential is pos. for pos. charges and neg. for neg. charges.

$q_3 = -q_1$, $r_{12} = r_{23}$

25-28) a) $U_3 = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \sum_{j<i} \frac{q_i q_j}{r_{ij}} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$

$\Rightarrow U_3 = -\left(8.99 \times 10^9 \frac{N \cdot m^2}{C^2}\right) \frac{(20.0\,nC)^2}{(0.08m)} \approx \boxed{-4.495 \times 10^{-5} \, J}$

25-28) b) from conservation of energy, $E_i = E_f$ & $\Rightarrow U_i = K_f + U_f$

$q_3 = -q_1$, $r_{14} = r_{34}$

Now, $U_i = U_3 + \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_4}{r_{14}} + \frac{q_2 q_4}{r_{24}} + \frac{q_3 q_4}{r_{34}}\right)$

$\Rightarrow U_i = U_3 + \frac{1}{4\pi\varepsilon_0} \frac{(10.0\,nC)(40.0\,nC)}{(0.03\,m)}$

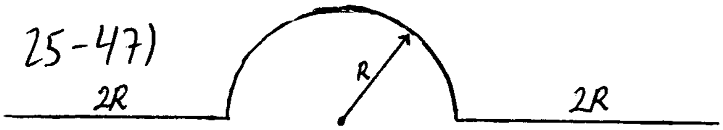
In calculating $U_f$, we must remember that the original configuration of three pt. charges remains fixed, so:

$U_f = U_3 + \lim_{r_{14}, r_{24}, r_{34} \to \infty} \left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{q_1 q_4}{r_{14}} + \frac{q_2 q_4}{r_{24}} + \frac{q_3 q_4}{r_{34}}\right) = U_3$

& $U_i - U_f = K_f \Rightarrow \frac{1}{4\pi\varepsilon_0} \frac{(10.0\,nC)(40.0\,nC)}{(0.03\,m)} = \frac{1}{2}mv^2$

$\Rightarrow \boxed{v = 3.46 \times 10^4 \, m/s}$

25-47)




The best way of handling this problem is by using the principle of superposition

$$V_{total} = V_{semi} + V_{straight} = \frac{\lambda}{4\pi\varepsilon_0}\left[\int_0^{\pi} \frac{r\,d\theta}{r} + 2\int_0^{2R} \frac{dx}{(x+R)}\right]$$

use polar co-ordinates to integrate over semi-circle.

Use the observation that each straight segment contributes equally

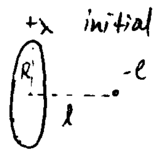
$$\Rightarrow V_{total} = \frac{\lambda}{4\pi\varepsilon_0}\left[\pi + 2\ln(x+R)\Big|_0^{2R}\right] = \boxed{\frac{\lambda}{4\pi\varepsilon_0}\left[\pi + 2\ln 3\right]}$$

25-50) a) Following Ex. 25.9:

$$\frac{q_1}{q_2} = \frac{r_1}{r_2} = 3 \Rightarrow \begin{matrix} q_1 = 0.9\,\mu C \\ q_2 = 0.3\,\mu C \end{matrix} \Rightarrow V = \frac{q_1}{4\pi\varepsilon_0 r_1} = \frac{q_2}{4\pi\varepsilon_0 r_2} \approx \boxed{134.85\ kV}$$

25-50) b) $E_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1^2} \approx \boxed{2.25 \times 10^6\ \frac{N}{C}}$ ; $E_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2^2} \approx \boxed{6.75 \times 10^6\ \frac{N}{C}}$

25-56)

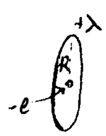



$l = 0.1\,m$ , $R = 0.2\,m$ , $\lambda = +0.1\,\mu C/m$

$-e = -1.6 \times 10^{-19} C$

$$K_i = 0 \quad ; \quad U_i = \frac{-eQk_e}{(l^2+R^2)^{1/2}} = \frac{-e(2\pi R\lambda)k_e}{(l^2+R^2)^{1/2}}$$

$Q = 2\pi R\lambda$



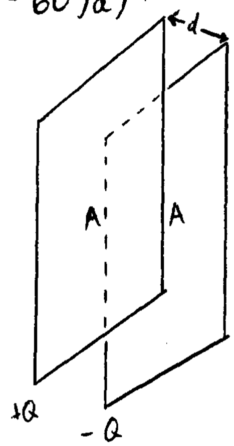

$$U_f = \frac{-e(2\pi R\lambda)k_e}{(R^2)^{1/2}} = -2\pi e\lambda k_e \quad ; K_f = \frac{1}{2}mv^2$$

$$\Rightarrow U_i + \cancel{K_i}^{\,0} = U_f + K_f$$

$$\Rightarrow \frac{1}{2}mv^2 = 2\pi e\lambda k_e\left(1 - \frac{R}{(l^2+R^2)^{1/2}}\right) \Rightarrow v = \sqrt{\frac{4\pi e\lambda k_e}{m}\left(1 - \frac{R}{(l^2+R^2)^{1/2}}\right)}$$

plugging in the given values $\Rightarrow$ $\boxed{v = 1.45 \times 10^7\ m/s}$

25-60)a)




In section, we found the electric field between the plates to be: $E = \frac{\sigma}{\varepsilon_0}$

$$\Delta V = V_+ - V_- = -\int_d^0 \frac{\sigma}{\varepsilon_0}\,dx = \frac{\sigma d}{\varepsilon_0} \Rightarrow \Delta V = 4\pi\left(8.99\times10^9 \frac{N\cdot m^2}{C^2}\right)\left(36.0\times10^{-9}\frac{C}{m^2}\right)d$$

$$d = 12.0\text{ cm} \Rightarrow \boxed{\Delta V \simeq 488.04\text{ V}}$$

25-60)b) Using Energy Conservation : $U = K_{final} = q\Delta V = 488.04\,eV \simeq \boxed{7.81\times10^{-17}\text{ J}}$ $\quad e = 1.6\times10^{-19}$ C

25-60)c) Why use Joules, when eV is so much easier!?!

$m_p \simeq 938.27$ MeV/$c^2$, where $c$ = speed of light = $3\times10^8$ m/s

$$q\Delta V = \frac{1}{2}m_p v^2 \Rightarrow v = \sqrt{\frac{2q\Delta V}{m_p}} \simeq c\sqrt{\frac{2(488.04\,eV)}{(938.27\,MeV)}} \simeq \boxed{3.06\times10^5\text{ m/s}}$$

25-60)d) This can be found using Newton's 2nd Law: $eE = ma$

$$\Rightarrow a = \frac{eE}{m} = \frac{4\pi e\left(8.99\times10^9\frac{N\cdot m^2}{C^2}\right)\left(36.0\times10^{-9}\text{ C/m}^2\right)}{(1.672623\times10^{-27}\text{ kg})} \simeq \boxed{3.89\times10^{11}\text{ m/s}^2}$$

25-60)e) $F = ma = (1.67\times10^{-27}\text{ kg})\,a \simeq \boxed{6.51\times10^{-16}\text{ N}}$

25-60)f) $F = qE \Rightarrow E = \frac{\sigma}{\varepsilon_0} = \frac{F}{q} \simeq \frac{6.51\times10^{-16}\text{ N}}{1.6\times10^{-19}\text{ C}} \simeq \boxed{4067\text{ N/C}}$ $\quad q = e$

25-70) $\vec{E} = -\vec{\nabla}V$ ; $V_{in} = V_0 \Rightarrow \boxed{\vec{E}_{in} = 0}$

$$V_{out} = V_0 - E_0 z + \frac{E_0 a^3 z}{(x^2+y^2+z^2)^{3/2}} \;;\; \vec{\nabla} = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$$

$\hat{z} = \hat{k}$

$$\Rightarrow \vec{E}_{out} = -E_0\hat{k} + \frac{E_0 a^3\hat{z}}{(x^2+y^2+z^2)^{3/2}} - 3E_0 a^3 z\left(\frac{x\hat{x}+y\hat{y}+z\hat{z}}{(x^2+y^2+z^2)^{5/2}}\right)$$

Using $r\hat{r} = \vec{r} = x\hat{x}+y\hat{y}+z\hat{z}$ & $r^2 = x^2+y^2+z^2$ yields : $\vec{E}_{out} = -E_0\hat{k} + \frac{E_0 a^3\hat{z}}{r^3} - \frac{3E_0 a^3 z r\hat{r}}{r^5}$

$$z = r\cos\theta \Rightarrow \vec{E}_{out} = -E_0\hat{k} + \frac{E_0 a^3}{r^3}\left(\hat{k} - 3\cos\theta\,\hat{r}\right)$$

$\hat{z} = \hat{k}$