PHYS 232 PS 5 Solutions

P28.26 Name the currents as shown in the figure to the right. Then $w + x + z = y$. Loop equations are

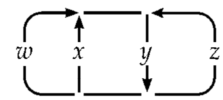

FIG. P28.26

$$-200w - 40.0 + 80.0x = 0$$
$$-80.0x + 40.0 + 360 - 20.0y = 0$$
$$+360 - 20.0y - 70.0z + 80.0 = 0$$

Eliminate y by substitution. $\begin{cases} x = 2.50w + 0.500 \\ 400 - 100x - 20.0w - 20.0z = 0 \\ 440 - 20.0w - 20.0x - 90.0z = 0 \end{cases}$

Eliminate x. $\begin{cases} 350 - 270w - 20.0z = 0 \\ 430 - 70.0w - 90.0z = 0 \end{cases}$

Eliminate $z = 17.5 - 13.5w$ to obtain $430 - 70.0w - 1\,575 + 1\,215w = 0$

$$w = \frac{70.0}{70.0} = \boxed{1.00\text{ A upward in }200\ \Omega}.$$

Now $z = \boxed{4.00\text{ A upward in }70.0\ \Omega}$

$x = \boxed{3.00\text{ A upward in }80.0\ \Omega}$

$y = \boxed{8.00\text{ A downward in }20.0\ \Omega}$

and for the 200 Ω, $\Delta V = IR = (1.00\text{ A})(200\ \Omega) = \boxed{200\text{ V}}$.

P28.30 We apply Kirchhoff's rules to the second diagram.

$50.0 - 2.00I_1 - 2.00I_2 = 0$ (1)

$20.0 - 2.00I_3 + 2.00I_2 = 0$ (2)

$I_1 = I_2 + I_3$ (3)

Substitute (3) into (1), and solve for I_1, I_2, and I_3

$I_1 = 20.0\text{ A}$; $I_2 = 5.00\text{ A}$; $I_3 = 15.0\text{ A}$.

Then apply $P = I^2R$ to each resistor:

$(2.00\ \Omega)_1$: $P = I_1^2(2.00\ \Omega) = (20.0\text{ A})^2(2.00\ \Omega) = \boxed{800\text{ W}}$

$(4.00\ \Omega)$: $P = \left(\frac{5.00}{2}\text{ A}\right)^2(4.00\ \Omega) = \boxed{25.0\text{ W}}$

(Half of I_2 goes through each)

$(2.00\ \Omega)_3$: $P = I_3^2(2.00\ \Omega) = (15.0\text{ A})^2(2.00\ \Omega) = \boxed{450\text{ W}}$.

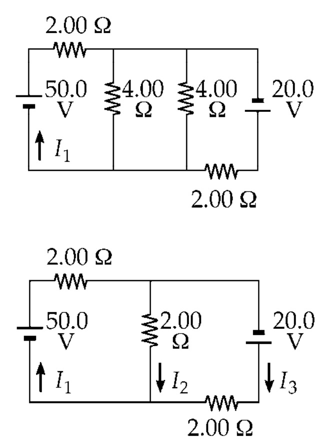

FIG. P28.30

P28.32 (a) $I(t) = -I_0 e^{-t/RC}$

$$I_0 = \frac{Q}{RC} = \frac{5.10\times10^{-6}\text{ C}}{(1\,300\ \Omega)(2.00\times10^{-9}\text{ F})} = 1.96\text{ A}$$

$$I(t) = -(1.96\text{ A})\exp\left[\frac{-9.00\times10^{-6}\text{ s}}{(1\,300\ \Omega)(2.00\times10^{-9}\text{ F})}\right] = \boxed{-61.6\text{ mA}}$$

(b) $$q(t) = Qe^{-t/RC} = (5.10\ \mu\text{C})\exp\left[\frac{-8.00\times10^{-6}\text{ s}}{(1\,300\ \Omega)(2.00\times10^{-9}\text{ F})}\right] = \boxed{0.235\ \mu\text{C}}$$

(c) The magnitude of the maximum current is $I_0 = \boxed{1.96\text{ A}}$.

P28.36 (a) $\tau = RC = (1.50\times10^{5}\ \Omega)(10.0\times10^{-6}\text{ F}) = \boxed{1.50\text{ s}}$

(b) $\tau = (1.00\times10^{5}\ \Omega)(10.0\times10^{-6}\text{ F}) = \boxed{1.00\text{ s}}$

(c) The battery carries current $\dfrac{10.0\text{ V}}{50.0\times10^{3}\ \Omega} = 200\ \mu\text{A}$.

The $100\text{ k}\Omega$ carries current of magnitude $I = I_0 e^{-t/RC} = \left(\dfrac{10.0\text{ V}}{100\times10^{3}\ \Omega}\right)e^{-t/1.00\text{ s}}$.

So the switch carries downward current $\boxed{200\ \mu\text{A} + (100\ \mu\text{A})e^{-t/1.00\text{ s}}}$.

P28.64 The battery supplies energy at a changing rate $\dfrac{dE}{dt} = \mathcal{P} = \varepsilon I = \varepsilon\left(\dfrac{\varepsilon}{R}e^{-1/RC}\right)$.

Then the total energy put out by the battery is $\displaystyle\int dE = \int_{t=0}^{\infty}\frac{\varepsilon^2}{R}\exp\left(-\frac{t}{RC}\right)dt$

$$\int dE = \frac{\varepsilon^2}{R}(-RC)\int_0^{\infty}\exp\left(-\frac{t}{RC}\right)\left(-\frac{dt}{RC}\right) = -\varepsilon^2 C\exp\left(-\frac{t}{RC}\right)\Big|_0^{\infty} = -\varepsilon^2 C[0-1] = \varepsilon^2 C.$$

The power delivered to the resistor is $\dfrac{dE}{dt} = \mathcal{P} = \Delta V_R I = I^2 R = R\dfrac{\varepsilon^2}{R^2}\exp\left(-\dfrac{2t}{RC}\right)$.

So the total internal energy appearing in the resistor is $\displaystyle\int dE = \int_0^{\infty}\frac{\varepsilon^2}{R}\exp\left(-\frac{2t}{RC}\right)dt$

$$\int dE = \frac{\varepsilon^2}{R}\left(-\frac{RC}{2}\right)\int_0^{\infty}\exp\left(-\frac{2t}{RC}\right)\left(-\frac{2dt}{RC}\right) = -\frac{\varepsilon^2 C}{2}\exp\left(-\frac{2t}{RC}\right)\Big|_0^{\infty} = -\frac{\varepsilon^2 C}{2}[0-1] = \frac{\varepsilon^2 C}{2}.$$

The energy finally stored in the capacitor is $U = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}C\varepsilon^2$. Thus, energy of the circuit is conserved $\varepsilon^2 C = \frac{1}{2}\varepsilon^2 C + \frac{1}{2}\varepsilon^2 C$ and resistor and capacitor share equally in the energy from the battery.

***P28.68** (a) With the switch closed, current exists in a simple series circuit as shown. The capacitors carry no current. For R_2 we have

$$P = I^2 R_2 \qquad I = \sqrt{\frac{P}{R_2}} = \sqrt{\frac{2.40 \text{ V}\cdot\text{A}}{7\,000 \text{ V/A}}} = 18.5 \text{ mA}.$$

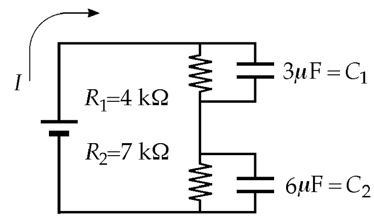

FIG. P28.68(a)

The potential difference across R_1 and C_1 is

$$\Delta V = IR_1 = (1.85\times10^{-2} \text{ A})(4\,000 \text{ V/A}) = 74.1 \text{ V}.$$

The charge on C_1

$$Q = C_1\Delta V = (3.00\times10^{-6} \text{ C/V})(74.1 \text{ V}) = \boxed{222\ \mu\text{C}}.$$

The potential difference across R_2 and C_2 is

$$\Delta V = IR_2 = (1.85\times10^{-2} \text{ A})(7\,000\ \Omega) = 130 \text{ V}.$$

The charge on C_2

$$Q = C_2\Delta V = (6.00\times10^{-6} \text{ C/V})(130 \text{ V}) = 778\ \mu\text{C}.$$

The battery emf is

$$IR_{eq} = I(R_1 + R_2) = 1.85\times10^{-2} \text{ A}\,(4\,000 + 7\,000) \text{ V/A} = 204 \text{ V}.$$

(b) In equilibrium after the switch has been opened, no current exists. The potential difference across each resistor is zero. The full 204 V appears across both capacitors. The new charge C_2

$$Q = C_2\Delta V = (6.00\times10^{-6} \text{ C/V})(204 \text{ V}) = 1\,222\ \mu\text{C}$$

for a change of $1\,222\ \mu\text{C} - 778\ \mu\text{C} = \boxed{444\ \mu\text{C}}$.

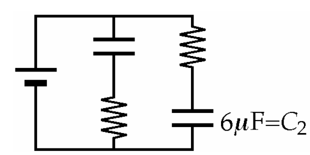

FIG. P28.68(b)

P28.75 The total resistance between points *b* and *c* is:

$$R = \frac{(2.00 \text{ k}\Omega)(3.00 \text{ k}\Omega)}{2.00 \text{ k}\Omega + 3.00 \text{ k}\Omega} = 1.20 \text{ k}\Omega.$$

The total capacitance between points *d* and *e* is:

$$C = 2.00\ \mu\text{F} + 3.00\ \mu\text{F} = 5.00\ \mu\text{F}.$$

The potential difference between point *d* and *e* in this series *RC* circuit at any time is:

$$\Delta V = \varepsilon\left[1 - e^{-t/RC}\right] = (120.0 \text{ V})\left[1 - e^{-1\,000t/6}\right].$$

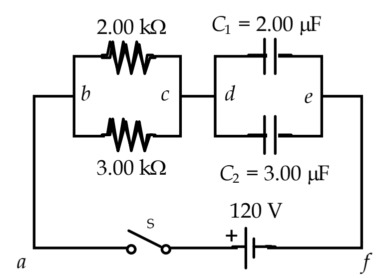

FIG. P28.75

Therefore, the charge on each capacitor between points *d* and *e* is:

$$q_1 = C_1\Delta V = (2.00\ \mu\text{F})(120.0 \text{ V})\left[1 - e^{-1\,000t/6}\right] = \boxed{(240\ \mu\text{C})\left[1 - e^{-1\,000t/6}\right]}$$

and $q_2 = C_2(\Delta V) = (3.00\ \mu\text{F})(120.0 \text{ V})\left[1 - e^{-1\,000t/6}\right] = \boxed{(360\ \mu\text{C})\left[1 - e^{-1\,000t/6}\right]}$.