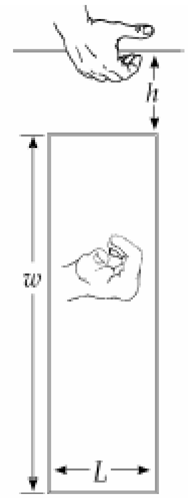
PHYS 232: Problem Set #8 Solutions.

P31.9 (a) $d\Phi_B = \mathbf{B}\cdot d\mathbf{A} = \frac{\mu_0 I}{2\pi x} L\,dx:\ \Phi_B = \int_h^{h+w} \frac{\mu_0 I L}{2\pi}\frac{dx}{x} = \boxed{\frac{\mu_0 I L}{2\pi}\ln\left(\frac{h+w}{h}\right)}$



(b) $\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left[\frac{\mu_0 I L}{2\pi}\ln\left(\frac{h+w}{h}\right)\right] = -\left[\frac{\mu_0 L}{2\pi}\ln\left(\frac{h+w}{h}\right)\right]\frac{dI}{dt}$

$$\varepsilon = -\frac{(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A})(1.00\ \text{m})}{2\pi}\ln\left(\frac{1.00+10.0}{1.00}\right)(10.0\ \text{A/s}) = \boxed{-4.80\ \mu\text{V}}$$

The long wire produces magnetic flux into the page through the rectangle, shown by the first hand in the figure to the right.

As the magnetic flux increases, the rectangle produces its own magnetic field out of the page, which it does by carrying $\boxed{\text{counterclockwise}}$ current (second hand in the figure).

P31.10 $\Phi_B = (\mu_0 n I)A_{\text{solenoid}}$

$$\varepsilon = -N\frac{d\Phi_B}{dt} = -N\mu_0 n\left(\pi r_{\text{solenoid}}^2\right)\frac{dI}{dt}$$

$$\varepsilon = -15.0\left(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(1.00\times 10^3\ \text{m}^{-1}\right)\pi(0.020\,0\ \text{m})^2(600\ \text{A/s})\cos(120t)$$

$$\boxed{\varepsilon = -14.2\cos(120t)\ \text{mV}}$$

P31.22 $F_B = I\ell B$ and $\varepsilon = B\ell v$

$I = \frac{\varepsilon}{R} = \frac{B\ell v}{R}$ so $B = \frac{IR}{\ell v}$

(a) $F_B = \frac{I^2\ell R}{\ell v}$ and $I = \sqrt{\frac{F_B v}{R}} = \boxed{0.500\ \text{A}}$

(b) $I^2 R = \boxed{2.00\ \text{W}}$

(c) For constant force, $\mathcal{P} = \mathbf{F}\cdot\mathbf{v} = (1.00\ \text{N})(2.00\ \text{m/s}) = \boxed{2.00\ \text{W}}$.

P31.28

(a) $\mathbf{B}_{ext} = B_{ext}\hat{\mathbf{i}}$ and B_{ext} decreases; therefore, the induced field is $\mathbf{B}_0 = B_0\hat{\mathbf{i}}$ (to the right) and the current in the resistor is directed $\boxed{\text{to the right}}$.

(b) $\mathbf{B}_{ext} = B_{ext}\left(-\hat{\mathbf{i}}\right)$ increases; therefore, the induced field $\mathbf{B}_0 = B_0\left(+\hat{\mathbf{i}}\right)$ is to the right, and the current in the resistor is directed $\boxed{\text{to the right}}$.

(c) $\mathbf{B}_{ext} = B_{ext}\left(-\hat{\mathbf{k}}\right)$ into the paper and B_{ext} decreases; therefore, the induced field is $\mathbf{B}_0 = B_0\left(-\hat{\mathbf{k}}\right)$ into the paper, and the current in the resistor is directed $\boxed{\text{to the right}}$.

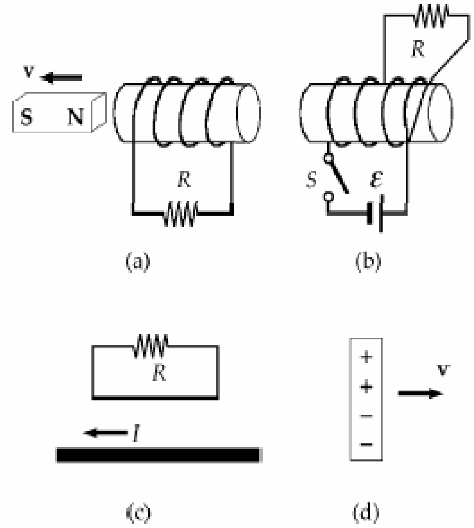


FIG. P31.28

(d) By the magnetic force law, $F_B = q(\mathbf{v} \times \mathbf{B})$. Therefore, a positive charge will move to the top of the bar if **B** is $\boxed{\text{into the paper}}$.

P31.31 Name the currents as shown in the diagram:

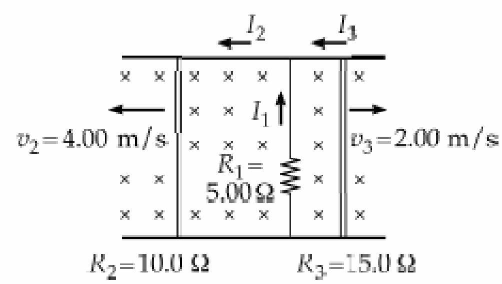


FIG. P31.31

Left loop: $+Bdv_2 - I_2R_2 - I_1R_1 = 0$

Right loop: $+Bdv_3 - I_3R_3 + I_1R_1 = 0$

At the junction: $I_2 = I_1 + I_3$

Then, $Bdv_2 - I_1R_2 - I_3R_2 - I_1R_1 = 0$

$$I_3 = \frac{Bdv_3}{R_3} + \frac{I_1R_1}{R_3}.$$

So, $$Bdv_2 - I_1(R_1 + R_2) - \frac{Bdv_3R_2}{R_3} - \frac{I_1R_1R_2}{R_3} = 0$$

$$I_1 = Bd\left(\frac{v_2R_3 - v_3R_2}{R_1R_2 + R_1R_3 + R_2R_3}\right) \text{ upward}$$

$$I_1 = (0.010\,0\text{ T})(0.100\text{ m})\left[\frac{(4.00\text{ m/s})(15.0\ \Omega) - (2.00\text{ m/s})(10.0\ \Omega)}{(5.00\ \Omega)(10.0\ \Omega) + (5.00\ \Omega)(15.0\ \Omega) + (10.0\ \Omega)(15.0\ \Omega)}\right] = \boxed{145\ \mu\text{A}} \text{ upward.}$$

P31.34 (a) $\oint \mathbf{E}\cdot d\boldsymbol{\ell} = \left|\frac{d\Phi_B}{dt}\right|$

$$2\pi r E = \left(\pi r^2\right)\frac{dB}{dt} \qquad \text{so} \qquad E = \boxed{(9.87\ \text{mV/m})\cos(100\pi t)}$$

(b) The E field is always opposite to increasing B. $\therefore$ $\boxed{\text{clockwise}}$.

P31.40 (a) $\varepsilon_{\text{max}} = BA\omega = B\left(\frac{1}{2}\pi R^2\right)\omega$

$$\varepsilon_{\text{max}} = (1.30\ \text{T})\frac{\pi}{2}(0.250\ \text{m})^2(4.00\pi\ \text{rad/s})$$

$$\varepsilon_{\text{max}} = \boxed{1.60\ \text{V}}$$

(b) $\overline{\varepsilon} = \int_0^{2\pi} \frac{\varepsilon}{2\pi}\,d\theta = \frac{BA\omega}{2\pi}\int_0^{2\pi} \sin\theta\, d\theta = \boxed{0}$

(c) The maximum and average ε would remain unchanged.

(d) See Figure 1 at the right.

(e) See Figure 2 at the right.

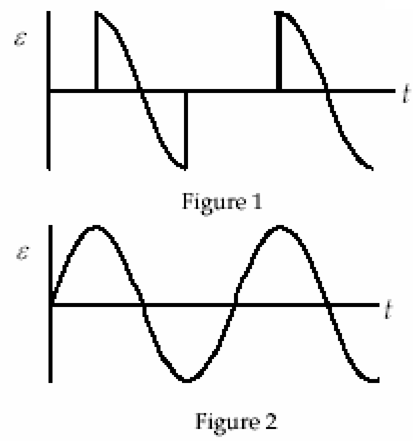


FIG. P31.40