PHYS 232: Problem Set #9 Solutions.

P32.12 $L = \frac{N\Phi_B}{I} = \frac{NBA}{I} \approx \frac{NA}{I} \cdot \frac{\mu_0 N I}{2\pi R} = \boxed{\frac{\mu_0 N^2 A}{2\pi R}}$

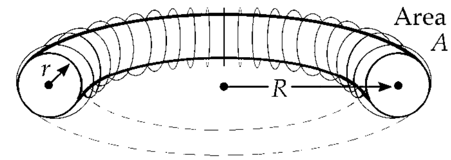

FIG. P32.12

P32.18 $I = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau}\right) = \frac{120}{9.00}\left(1 - e^{-1.80/7.00}\right) = 3.02\text{ A}$

$$\Delta V_R = IR = (3.02)(9.00) = 27.2\text{ V}$$

$$\Delta V_L = \varepsilon - \Delta V_R = 120 - 27.2 = \boxed{92.8\text{ V}}$$

P32.21

$$I = I_{\max}\left(1 - e^{-t/\tau}\right): \quad \frac{dI}{dt} = -I_{\max}\left(e^{-t/\tau}\right)\left(-\frac{1}{\tau}\right)$$

$$\tau = \frac{L}{R} = \frac{15.0\text{ H}}{30.0\ \Omega} = 0.500\text{ s}: \quad \frac{dI}{dt} = \frac{R}{L} I_{\max} e^{-t/\tau} \text{ and } I_{\max} = \frac{\varepsilon}{R}$$

(a) $t = 0: \frac{dI}{dt} = \frac{R}{L} I_{\max} e^0 = \frac{\varepsilon}{L} = \frac{100\text{ V}}{15.0\text{ H}} = \boxed{6.67\text{ A/s}}$

(b) $t = 1.50\text{ s}: \frac{dI}{dt} = \frac{\varepsilon}{L} e^{-t/\tau} = (6.67\text{ A/s})e^{-1.50/(0.500)} = (6.67\text{ A/s})e^{-3.00} = \boxed{0.332\text{ A/s}}$

P32.52 (a) $f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(0.100\text{ H})(1.00 \times 10^{-6}\text{ F})}} = \boxed{503\text{ Hz}}$

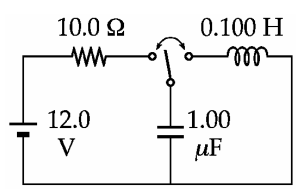

FIG. P32.52

(b) $Q = C\varepsilon = (1.00 \times 10^{-6}\text{ F})(12.0\text{ V}) = \boxed{12.0\ \mu\text{C}}$

(c) $\frac{1}{2}C\varepsilon^2 = \frac{1}{2}LI_{\max}^2$

$$I_{\max} = \varepsilon\sqrt{\frac{C}{L}} = 12\text{ V}\sqrt{\frac{1.00 \times 10^{-6}\text{ F}}{0.100\text{ H}}} = \boxed{37.9\text{ mA}}$$

(d) At all times $U = \frac{1}{2}C\varepsilon^2 = \frac{1}{2}(1.00 \times 10^{-6}\text{ F})(12.0\text{ V})^2 = \boxed{72.0\ \mu\text{J}}$.

P32.70 When switch is closed, steady current $I_0 = 1.20\text{ A}$. When the switch is opened after being closed a long time, the current in the right loop is

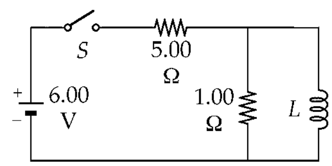

FIG. P32.70

$$I = I_0 e^{-R_2 t/L}$$

so $\quad e^{Rt/L} = \frac{I_0}{I} \quad$ and $\quad \frac{Rt}{L} = \ln\left(\frac{I_0}{I}\right)$.

Therefore,

$$L = \frac{R_2 t}{\ln(I_0/I)} = \frac{(1.00\ \Omega)(0.150\text{ s})}{\ln(1.20\text{ A}/0.250\text{ A})} = 0.095\,6\text{ H} = \boxed{95.6\text{ mH}}.$$

P32.72 (a) The instant after the switch is closed, the situation is as shown in the circuit diagram of Figure (a). The requested quantities are:

$$\boxed{I_L = 0,\ I_C = \frac{\varepsilon_0}{R},\ I_R = \frac{\varepsilon_0}{R}}$$

$$\boxed{\Delta V_L = \varepsilon_0,\ \Delta V_C = 0,\ \Delta V_R = \varepsilon_0}$$

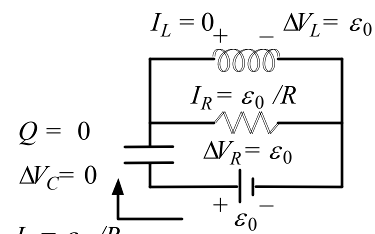

Figure (a)

(b) After the switch has been closed a long time, the steady-state conditions shown in Figure (b) will exist. The currents and voltages are:

$$\boxed{I_L = 0,\ I_C = 0,\ I_R = 0}$$

$$\boxed{\Delta V_L = 0,\ \Delta V_C = \varepsilon_0,\ \Delta V_R = 0}$$

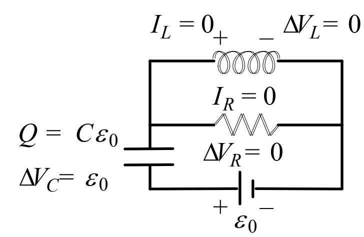

Figure (b)

FIG. P32.72

P32.79 (a)

$$B = \frac{\mu_0 N I}{\ell} = \frac{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(1\,400)(2.00\text{ A})}{1.20\text{ m}} = \boxed{2.93\times10^{-3}\text{ T (upward)}}$$

(b)

$$u = \frac{B^2}{2\mu_0} = \frac{(2.93\times10^{-3}\text{ T})^2}{2(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})} = \left(3.42\text{ J/m}^3\right)\left(\frac{1\text{ N}\cdot\text{m}}{1\text{ J}}\right) = 3.42\text{ N/m}^2 = \boxed{3.42\text{ Pa}}$$

(c) To produce a downward magnetic field, the surface of the superconductor must carry a $\boxed{\text{clockwise}}$ current.

(d) The vertical component of the field of the solenoid exerts an inward force on the superconductor. The total horizontal force is zero. Over the top end of the solenoid, its field diverges and has a radially outward horizontal component. This component exerts upward force on the clockwise superconductor current. The total force on the core is $\boxed{\text{upward}}$. You can think of it as a force of repulsion between the solenoid with its north end pointing up, and the core, with its north end pointing down.

P33.39 (a) $f = \frac{1}{2\pi\sqrt{LC}}$

$$C = \frac{1}{4\pi^2 f^2 L} = \frac{1}{4\pi^2\left(10^{10}/\text{s}\right)^2 400\times10^{-12}\ \text{V s}}\frac{\text{A}}{}\left(\frac{\text{C}}{\text{A s}}\right) = \boxed{6.33\times10^{-13}\ \text{F}}$$

(b) $C = \frac{\kappa \in_0 A}{d} = \frac{\kappa \in_0 \ell^2}{d}$

$$\ell = \left(\frac{Cd}{\kappa \in_0}\right)^{1/2} = \left(\frac{6.33\times10^{-13}\ \text{F}\times10^{-3}\ \text{mm}}{1\times8.85\times10^{-12}\ \text{F}}\right)^{1/2} = \boxed{8.46\times10^{-3}\ \text{m}}$$

(c) $X_L = 2\pi fL = 2\pi\times10^{10}/\text{s}\times400\times10^{-12}\ \text{V s/A} = \boxed{25.1\ \Omega}$

(e) $F = PA = (3.42\ \text{Pa})\left[\pi\left(1.10\times10^{-2}\ \text{m}\right)^2\right] = \boxed{1.30\times10^{-3}\ \text{N}}$

Note that we have not proven that energy density is pressure. In fact, it is not in some cases; Equation 21.2 shows that the pressure is two-thirds of the translational energy density in an ideal gas.

P33.44 (a) $\Delta V_{2,\text{rms}} = \frac{1}{13}(120\ \text{V}) = \boxed{9.23\ \text{V}}$

(b) $\Delta V_{1,\text{rms}} I_{1,\text{rms}} = \Delta V_{2,\text{rms}} I_{2,\text{rms}}$

$(120\ \text{V})(0.350\ \text{A}) = (9.23\ \text{V})\, I_{2,\text{rms}}$

$I_{2,\text{rms}} = \frac{42.0\ \text{W}}{9.23\ \text{V}} = \boxed{4.55\ \text{A}}$ for a transformer with no energy loss.

(c) $\mathcal{P} = \boxed{42.0\ \text{W}}$ from part (b).

P33.46 (a) $\left(\Delta V_{2,\text{rms}}\right) = \frac{N_2}{N_1}\left(\Delta V_{1,\text{rms}}\right)$

$$N_2 = \frac{(2\,200)(80)}{110} = \boxed{1\,600\ \text{windings}}$$

(b) $I_{1,\text{rms}}\left(\Delta V_{1,\text{rms}}\right) = I_{2,\text{rms}}\left(\Delta V_{2,\text{rms}}\right)$ $\quad I_{1,\text{rms}} = \frac{(1.50)(2\,200)}{110} = \boxed{30.0\ \text{A}}$

(c) $0.950 I_{1,\text{rms}}\left(\Delta V_{1,\text{rms}}\right) = I_{2,\text{rms}}\left(\Delta V_{2,\text{rms}}\right)$ $\quad I_{1,\text{rms}} = \frac{(1.20)(2\,200)}{110(0.950)} = \boxed{25.3\ \text{A}}$

P33.49 (a) $R = \left(4.50 \times 10^{-4}\ \Omega/\text{M}\right)\left(6.44 \times 10^{5}\ \text{m}\right) = 290\ \Omega$ and

$$I_{\text{rms}} = \frac{\mathcal{P}}{\Delta V_{\text{rms}}} = \frac{5.00 \times 10^{6}\ \text{W}}{5.00 \times 10^{5}\ \text{V}} = 10.0\ \text{A}$$

$$\mathcal{P}_{\text{loss}} = I_{\text{rms}}^{2} R = (10.0\ \text{A})^{2}(290\ \Omega) = \boxed{29.0\ \text{kW}}$$

(b) $$\frac{\mathcal{P}_{\text{loss}}}{\mathcal{P}} = \frac{2.90 \times 10^{4}}{5.00 \times 10^{6}} = \boxed{5.80 \times 10^{-3}}$$

(c) It is impossible to transmit so much power at such low voltage. Maximum power transfer occurs when load resistance equals the line resistance of $290\ \Omega$, and is

$$\frac{\left(4.50 \times 10^{3}\ \text{V}\right)^{2}}{2 \cdot 2(290\ \Omega)} = 17.5\ \text{kW}$$ far below the required 5 000 kW.

P33.58 The angular frequency is $\omega = 2\pi 60/\text{s} = 377/\text{s}$. When S is open, R, L, and C are in series with the source

$$R^{2} + \left(X_L - X_C\right)^{2} = \left(\frac{\Delta V_s}{I}\right)^{2} = \left(\frac{20\ \text{V}}{0.183\ \text{A}}\right)^{2} = 1.194 \times 10^{4}\ \Omega^{2}. \quad (1)$$

When S is in position 1, a parallel combination of two R's presents equivalent resistance $\frac{R}{2}$, in series with L and C:

$$\left(\frac{R}{2}\right)^{2} + \left(X_L - X_C\right)^{2} = \left(\frac{20\ \text{V}}{0.298\ \text{A}}\right)^{2} = 4.504 \times 10^{3}\ \Omega^{2}. \quad (2)$$

When S is in position 2, the current by passes the inductor. R and C are in series with the source:

$$R^{2} + X_C^{2} = \left(\frac{20\ \text{V}}{0.137\ \text{A}}\right)^{2} = 2.131 \times 10^{4}\ \Omega^{2}. \quad (3)$$

Take equation (1) minus equation (2):

$$\frac{3}{4}R^{2} = 7.440 \times 10^{3}\ \Omega^{2} \qquad \boxed{R = 99.6\ \Omega}$$

continued on next page

(only the positive root is physical.) Now equation (3) gives

$$X_C = \left[2.131\times10^4 - (99.6)^2\right]^{1/2}\ \Omega = 106.7\ \Omega = \frac{1}{\omega C}$$ (only the positive root is physical.)

$$C = (\omega X_C)^{-1} = \left[(377/\text{s})106.7\ \Omega\right]^{-1} = \boxed{2.49\times10^{-5}\ \text{F} = C}.$$

Now equation (1) gives

$$X_L - X_C = \pm\left[1.194\times10^4 - (99.6)^2\right]^{1/2}\ \Omega = \pm 44.99\ \Omega$$

$$X_L = 106.7\ \Omega + 44.99\ \Omega = 61.74\ \Omega \text{ or } 151.7\ \Omega = \omega L$$

$$L = \frac{X_L}{\omega} = \boxed{0.164\ \text{H or } 0.402\ \text{H} = L}$$

P33.61 (a) When ωL is very large, the bottom branch carries negligible current. Also, $\frac{1}{\omega C}$ will be negligible compared to $200\ \Omega$ and $\frac{45.0\ \text{V}}{200\ \Omega} = \boxed{225\ \text{mA}}$ flows in the power supply and the top branch.

(b) Now $\frac{1}{\omega C} \to \infty$ and $\omega L \to 0$ so the generator and bottom branch carry $\boxed{450\ \text{mA}}$.

P33.62 (a) With both switches closed, the current goes only through generator and resistor.

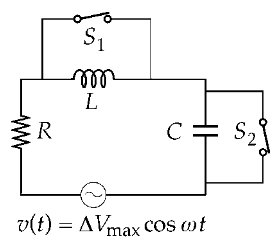

FIG. P33.62

$$\boxed{i(t) = \frac{\Delta V_{\max}}{R}\cos\omega t}$$

(b) $$\boxed{\mathcal{P} = \frac{1}{2}\frac{(\Delta V_{\max})^2}{R}}$$

(c) $$\boxed{i(t) = \frac{\Delta V_{\max}}{\sqrt{R^2+\omega^2L^2}}\cos\left[\omega t + \arctan\left(\frac{\omega L}{R}\right)\right]}$$

(d) For $$0 = \phi = \arctan\left(\frac{\omega_0 L - (1/\omega_0 C)}{R}\right).$$

We require $\omega_0 L = \frac{1}{\omega_0 C}$, so $\boxed{C = \frac{1}{\omega_0^2 L}}$.

(e) At this resonance frequency, $Z = \boxed{R}$.

(f) $$U = \frac{1}{2}C(\Delta V_C)^2 = \frac{1}{2}CI^2X_C^2$$

$$U_{\max} = \frac{1}{2}CI_{\max}^2X_C^2 = \frac{1}{2}C\frac{(\Delta V_{\max})^2}{R^2}\frac{1}{\omega_0^2C^2} = \boxed{\frac{(\Delta V_{\max})^2 L}{2R^2}}$$

(g) $U_{max} = \frac{1}{2} L I_{max}^2 = \boxed{\frac{1}{2} L \frac{(\Delta V_{max})^2}{R^2}}$

(h) Now $\omega = 2\omega_0 = \frac{2}{\sqrt{LC}}$.

So

$$\phi = \arctan\left(\frac{\omega L - (1/\omega C)}{R}\right) = \arctan\left(\frac{2\sqrt{L/C} - (1/2)\sqrt{L/C}}{R}\right) = \boxed{\arctan\left(\frac{3}{2R}\sqrt{\frac{L}{C}}\right)}.$$

(i) Now $\omega L = \frac{1}{2}\frac{1}{\omega C}$ $\qquad \omega = \boxed{\frac{1}{\sqrt{2LC}}} = \frac{\omega_0}{\sqrt{2}}$.