PHYS 232: Problem Set #13 Solutions.

P40.15 (a) $\lambda_c = \dfrac{hc}{\phi}$

Li: $$\lambda_c = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(2.30\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})} = 540\ \text{nm}$$

Be: $$\lambda_c = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(3.90\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})} = 318\ \text{nm}$$

Hg: $$\lambda_c = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(4.50\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})} = 276\ \text{nm}$$

$\lambda < \lambda_c$ for photo current. Thus, only lithium will exhibit the photoelectric effect.

(b) For lithium, $\dfrac{hc}{\lambda} = \phi + K_{\max}$

$$\frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{400\times10^{-9}\ \text{m}} = (2.30\ \text{eV})(1.60\times10^{-19}) + K_{\max}$$

$$K_{\max} = 1.29\times10^{-19}\ \text{J} = \boxed{0.808\ \text{eV}}$$

P40.42 (a) The wavelength of the student is $\lambda = \dfrac{h}{p} = \dfrac{h}{mv}$. If w is the width of the diffracting aperture,

then we need $$w \le 10.0\lambda = 10.0\left(\frac{h}{mv}\right)$$

so that $$v \le 10.0\frac{h}{mw} = 10.0\left(\frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{(80.0\ \text{kg})(0.750\ \text{m})}\right) = \boxed{1.10\times10^{-34}\ \text{m/s}}.$$

(b) Using $\Delta t = \dfrac{d}{v}$ we get: $$\Delta t \ge \frac{0.150\ \text{m}}{1.10\times10^{-34}\ \text{m/s}} = \boxed{1.36\times10^{33}\ \text{s}}.$$

(c) No. The minimum time to pass through the door is over 10^{15} times the age of the Universe.

P40.26 The energy of the incident photon is $E_0 = p_\gamma c = \frac{hc}{\lambda_0}$.

(a) Conserving momentum in the x direction gives

$$p_\lambda = p_e \cos\phi + p'_\gamma \cos\theta \text{, or since } \phi = \theta,\ \frac{E_0}{c} = \left(p_e + p'_\gamma\right)\cos\theta. \quad [1]$$

Conserving momentum in the y direction (with $\phi = 0$) yields

$$0 = p'_\gamma \sin\theta - p_e \sin\theta \text{, or } p_e = p'_\gamma = \frac{h}{\lambda'}. \quad [2]$$

Substituting Equation [2] into Equation [1] gives

$$\frac{E_0}{c} = \left(\frac{h}{\lambda'} + \frac{h}{\lambda'}\right)\cos\theta \text{, or } \lambda' = \frac{2hc}{E_0}\cos\theta. \quad [3]$$

By the Compton equation, $\lambda' - \lambda_0 = \frac{h}{m_e c}(1-\cos\theta)$, $\quad \frac{2hc}{E_0}\cos\theta - \frac{2hc}{E_0} = \frac{h}{m_e c}(1-\cos\theta)$

which reduces to $\quad \left(2m_e c^2 + E_0\right)\cos\theta = m_e c^2 + E_0$.

Thus, $\quad \boxed{\phi = \theta = \cos^{-1}\left(\frac{m_e c^2 + E_0}{2m_e c^2 + E_0}\right)}$.

(b) From Equation [3], $\quad \lambda' = \frac{2hc}{E_0}\cos\theta = \frac{2hc}{E_0}\left(\frac{m_e c^2 + E_0}{2m_e c^2 + E_0}\right)$.

Therefore, $\quad E'_\gamma = \frac{hc}{\lambda'} = \frac{hc}{(2hc/E_0)\left(m_e c^2 + E_0\right)/\left(2m_e c^2 + E_0\right)} = \boxed{\frac{E_0}{2}\left(\frac{2m_e c^2 + E_0}{m_e c^2 + E_0}\right)}$,

and $\quad p'_\gamma = \frac{E'_\gamma}{c} = \boxed{\frac{E_0}{2c}\left(\frac{2m_e c^2 + E_0}{m_e c^2 + E_0}\right)}$.

(c) From conservation of energy, $K_e = E_0 - E'_\gamma = E_0 - \frac{E_0}{2}\left(\frac{2m_e c^2 + E_0}{m_e c^2 + E_0}\right)$

or $\quad K_e = \frac{E_0}{2}\left(\frac{2m_e c^2 + 2E_0 - 2m_e c^2 - E_0}{m_e c^2 + E_0}\right) = \boxed{\frac{E_0^2}{2\left(m_e c^2 + E_0\right)}}$.

Finally, from Equation (2), $p_e = p'_\gamma = \boxed{\frac{E_0}{2c}\left(\frac{2m_e c^2 + E_0}{m_e c^2 + E_0}\right)}$.

P40.51 With $\Delta x = 2\times10^{-15}$ m m, the uncertainty principle requires $\Delta p_x \ge \frac{\hbar}{2\Delta x} = 2.6\times10^{-20}$ kg·m/s.

The average momentum of the particle bound in a stationary nucleus is zero. The uncertainty in momentum measures the root-mean-square momentum, so we take $p_{rms} \approx 3\times10^{-20}$ kg·m/s. For an electron, the non-relativistic approximation $p = m_e v$ would predict $v \approx 3\times10^{10}$ m/s, while v cannot be greater than c.

Thus, a better solution would be $$E = \left[\left(m_e c^2\right)^2 + (pc)^2\right]^{1/2} \approx 56 \text{ MeV} = \gamma m_e c^2$$

$$\gamma \approx 110 = \frac{1}{\sqrt{1 - v^2/c^2}} \quad \text{so} \quad v \approx 0.999\,96c.$$

For a proton, $v = \frac{p}{m}$ gives $v = 1.8\times10^7$ m/s, less than one-tenth the speed of light.

P40.54 $\Delta V_S = \left(\frac{h}{e}\right)f - \frac{\phi}{e}$

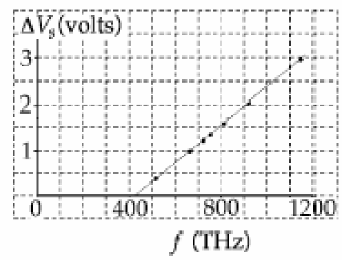

FIG. P40.54

From two points on the graph $$0 = \left(\frac{h}{e}\right)\left(4.1\times10^{14}\text{ Hz}\right) - \frac{\phi}{e}$$

and $$3.3\text{ V} = \left(\frac{h}{e}\right)\left(12\times10^{14}\text{ Hz}\right) - \frac{\phi}{e}.$$

Combining these two expressions we find:

(a) $\phi = \boxed{1.7\text{ eV}}$

(b) $\frac{h}{e} = \boxed{4.2\times10^{-15}\text{ V}\cdot\text{s}}$

(c) At the cutoff wavelength $\frac{hc}{\lambda_c} = \phi = \left(\frac{h}{e}\right)\frac{ec}{\lambda_c}$

$$\lambda_c = \left(4.2\times10^{-15}\text{ V}\cdot\text{s}\right)\left(1.6\times10^{-19}\text{ C}\right)\frac{\left(3\times10^8\text{ m/s}\right)}{(1.7\text{ eV})\left(1.6\times10^{-19}\text{ J/eV}\right)} = \boxed{730\text{ nm}}$$

P40.57 $\Delta\lambda = \frac{h}{m_p c}(1-\cos\theta) = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})}{(1.67\times10^{-27}\ \text{kg})(3.00\times10^{8}\ \text{m/s})}(0.234) = 3.09\times10^{-16}\ \text{m}$

$$\lambda_0 = \frac{hc}{E_0} = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(200\ \text{MeV})(1.60\times10^{-13}\ \text{J/MeV})} = 6.20\times10^{-15}\ \text{m}$$

$$\lambda' = \lambda_0 + \Delta\lambda = 6.51\times10^{-15}\ \text{m}$$

(a) $E_\gamma = \frac{hc}{\lambda'} = \boxed{191\ \text{MeV}}$

(b) $K_p = \boxed{9.20\ \text{MeV}}$

P42.19 (a) $\int |\psi|^2 dV = 4\pi \int_0^\infty |\psi|^2 r^2 dr = 4\pi\left(\frac{1}{\pi a_0^3}\right)\int_0^\infty r^2 e^{-2r/a_0} dr$

Using integral tables, $\int |\psi|^2 dV = -\frac{2}{a_0^2}\left[e^{-2r/a_0}\left(r^2 + a_0 r + \frac{a_0^2}{2}\right)\right]_0^\infty = \left(-\frac{2}{a_0^2}\right)\left(-\frac{a_0^2}{2}\right) = \boxed{1}$

so the wave function as given is normalized.

(b) $P_{a_0 2 \to 3a_0 2} = 4\pi \int_{a_0/2}^{3a_0/2} |\psi|^2 r^2 dr = 4\pi\left(\frac{1}{\pi a_0^3}\right)\int_{a_0/2}^{3a_0/2} r^2 e^{-2r/a_0} dr$

Again, using integral tables,

$$P_{a_0 2 \to 3a_0 2} = -\frac{2}{a_0^2}\left[e^{-2r/a_0}\left(r^2 + a_0 r + \frac{a_0^2}{2}\right)\right]_{a_0/2}^{3a_0/2} = -\frac{2}{a_0^2}\left[e^{-3}\left(\frac{17a_0^2}{4}\right) - e^{-1}\left(\frac{5a_0^2}{4}\right)\right] = \boxed{0.497}.$$

***P42.53** (a) $\Delta E = \frac{ehB}{m_e} = \frac{1.60\times10^{-19}\ \text{C}(6.63\times10^{-34}\ \text{J}\cdot\text{s})(5.26\ \text{T})}{2\pi(9.11\times10^{-31}\ \text{kg})}\left(\frac{\text{N}\cdot\text{s}}{\text{T}\cdot\text{C}\cdot\text{m}}\right)\left(\frac{\text{kg}\cdot\text{m}}{\text{N}\cdot\text{s}^2}\right) = 9.75\times10^{-23}\ \text{J}$

$= \boxed{609\ \mu\text{eV}}$

(b) $k_B T = (1.38\times10^{-23}\ \text{J/K})(80\times10^{-3}\ \text{K}) = 1.10\times10^{-24}\ \text{J} = \boxed{6.90\ \mu\text{eV}}$

(c) $f = \frac{\Delta E}{h} = \frac{9.75\times10^{-23}\ \text{J}}{6.63\times10^{-34}\ \text{J}\cdot\text{s}} = \boxed{1.47\times10^{11}\ \text{Hz}}$

$$\lambda = \frac{c}{f} = \frac{3\times10^{8}\ \text{m/s}}{1.47\times10^{11}\ \text{Hz}} = \boxed{2.04\times10^{-3}\ \text{m}}$$