PS #2 Sol'ns

23-26) We can obtain our answer directly from Ex. 23.7:

The upper limit in the integral changes from $l+a$ to ∞:

$$|\vec{E}| = k_e \lambda_0 \int_{x_0}^{\infty} \frac{dx}{x^2} = k_e \lambda_0 \left[-\frac{1}{x} \right]_{x_0}^{\infty} = \frac{k_e \lambda_0}{x_0} \Rightarrow \boxed{\vec{E} = -\frac{k_e \lambda_0}{x_0} \hat{x}}$$

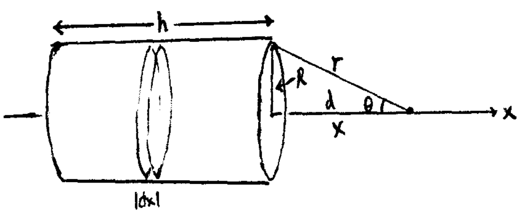
Likewise, we could've also considered the limiting case: $l \gg x_0 = a$, which gives:

$$\lim_{l\to\infty} |\vec{E}| = \lim_{l\to\infty} \frac{k_e Q}{a(l+a)} = \frac{k_e Q}{la} = \frac{k_e \lambda_0}{a} = \frac{k_e \lambda_0}{x_0}$$

from Ex. 23.7

For a distribution of positive charges, the electric field is to the left of the origin along the x-axis.

23-34) a)



Start off in the same fashion as Ex. 23.8:

$$dE_x = dE\cos\theta = \frac{k_e\, dq\, x}{(x^2+R^2)^{3/2}}$$

$$\cos\theta = \frac{x}{r} \; ; \; dE = \frac{k_e\, dq}{r^2}$$

$r = \sqrt{x^2+R^2}$; $A_{ring} = 2\pi R\, dx$

We define dq as $dq = 2\pi R\sigma\, dx$, where σ is the area charge density of the hollow cylinder. This yields:

$$dE_x = k_e \frac{2\pi R\sigma\, x\, dx}{(x^2+R^2)^{3/2}},$$

Now integrate both sides to obtain the electric field:

$$\vec{E} = k_e \cdot 2\pi R\sigma\hat{x} \int_d^{h+d} \frac{x\,dx}{(x^2+R^2)^{3/2}} = k_e \cdot \pi R\sigma\hat{x} \int_{d=x}^{h+d=x} \frac{du}{u^{3/2}} = -k_e \frac{2\pi R\sigma\hat{x}}{(x^2+R^2)^{1/2}}\Bigg|_d^{h+d}$$

Substitute: $u = x^2+R^2$

$du = 2x\,dx$

$$\sigma = \frac{Q}{2\pi R h} \Rightarrow \boxed{\vec{E} = \frac{k_e Q}{h}\left[(d^2+R^2)^{-1/2} - ((h+d)^2+R^2)^{-1/2}\right]\hat{x}}$$

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23-34)b) If we construct our cylinder of many infinitely thin disks, then we can use the result of Ex. 23.9 and integrate over the axis to obtain the electric field:

volume charge density

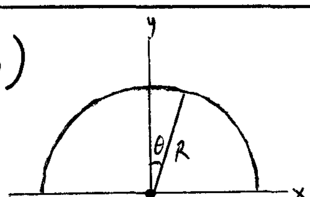
Given: $E_{x,disk} = 2\pi k_e \sigma\left[1 - \dfrac{x}{(x^2+R^2)^{1/2}}\right]$; $\sigma = \dfrac{Q}{\pi R^2} = \rho\, dx$

$\Rightarrow dE_x = 2\pi k_e \rho\, dx\left[1 - \dfrac{x}{(x^2+R^2)^{1/2}}\right] \Rightarrow \vec{E} = 2\pi k_e \rho\, \hat{x}\displaystyle\int_d^{h+d} dx\left[1 - \dfrac{x}{(x^2+R^2)^{1/2}}\right]$; 2nd term: $u = x^2+R^2$, $du = 2x\,dx$

$\Rightarrow \vec{E} = 2\pi k_e \rho\,\hat{x}\left(h - \dfrac{1}{2}\displaystyle\int_{d=x}^{h+d=x} \dfrac{du}{u^{1/2}}\right) = 2\pi k_e \rho\,\hat{x}\left(h - u^{1/2}\Big|_{d=x}^{h+d=x}\right)$

$$\Rightarrow \boxed{\vec{E} = \frac{2k_eQ}{R^2h}\left[h + \sqrt{d^2+R^2} - \sqrt{(h+d)^2+R^2}\right]\hat{x}}$$

23-63)



$\vec{E} = \dfrac{1}{4\pi\varepsilon_0}\displaystyle\int \dfrac{\lambda\, d\vec{l}}{r^2}$; $r = R = \text{const.}$; $d\vec{l} = r\,d\theta\,\hat{\theta}$

$\lambda = \lambda_0 \cos\theta \Rightarrow \vec{E} = \dfrac{\lambda_0}{4\pi\varepsilon_0 R}\displaystyle\int_{+\pi/2}^{-\pi/2} \cos\theta\, d\theta\,\hat{\theta}$; $\hat{\theta} = -\sin\theta\,\hat{x} + \cos\theta\,\hat{y}$

$\Rightarrow \vec{E} = \dfrac{\lambda_0}{4\pi\varepsilon_0 R}\displaystyle\int_{+\pi/2}^{-\pi/2}\left(\overset{0,\text{ odd fn.}}{-\cos\theta\sin\theta\,\hat{x}} + \cos^2\theta\,\hat{y}\right) = \dfrac{\lambda_0\hat{y}}{4\pi\varepsilon_0 R}\displaystyle\int_{+\pi/2}^{-\pi/2}\dfrac{1}{2}(\cos 2\theta + 1)\,d\theta = \left(\dfrac{-1}{4\pi\varepsilon_0}\right)\dfrac{\pi\lambda_0}{2R}\hat{y}$

since we know $\vec{E} = -E\hat{y}$, by symmetry! $\cos^2\theta = \frac{1}{2}(\cos 2\theta + 1)$

Now, $Q = \int \lambda\, dl = \lambda_0 R\displaystyle\int_{-\pi/2}^{\pi/2}\cos\theta\, d\theta = 2\lambda_0 R \Rightarrow \lambda_0 = \dfrac{Q}{2R}$

$\Rightarrow \vec{E} = -\left(\dfrac{1}{4\pi\varepsilon_0}\right)\dfrac{\pi Q}{(2R)^2}\hat{y} = \boxed{-0.707\,N\,\hat{y}}$

24-13) $\Phi = \displaystyle\oint_S \vec{E}\cdot\hat{n}\,dA = \oint_S \vec{E}\cdot d\vec{A} = \oint_{\text{DISK}} \vec{E}\cdot d\vec{A} + \oint_{\text{PARABOLOID}} \overset{0}{(\vec{E}_{in} - \vec{E}_{out})}\cdot d\vec{A} = \oint_{\text{DISK}} \vec{E}\cdot d\vec{A}$

$\vec{E}_{in} = \vec{E}_{out}$

$$\boxed{\Phi = E_0\int_{\text{DISK}} dA = E_0\pi R^2}$$

$R = r$

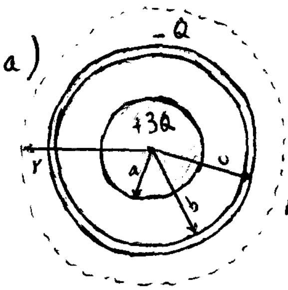
24-24)a) The electric fields inside & outside a solid charged sphere are derived in Ex. 24.5:

$$\boxed{E_{r<a} = k_e \frac{Qr}{a^3} = 0\ \frac{N}{C}\ ,\ \text{at } r=0}$$

24-24)b) $E_{r<a}(r=0.1m) = \left(8.99\times10^9\ \frac{N\cdot m^2}{C^2}\right)\left(2.6\times10^{-5}C\right)\cdot\frac{0.1m}{(0.4m)^3} \simeq \boxed{3.65\times10^5\ \frac{N}{C}}$

24-24)c) $E_{r=a} = k_e\frac{Q}{r^2} = \frac{\left(8.99\times10^9\ \frac{N\cdot m^2}{C^2}\right)\left(2.6\times10^{-5}C\right)}{(0.4\,m)^2} \simeq \boxed{1.46\times10^6\ \frac{N}{C}}$

24-24)d) $E_{r>a} = \frac{k_e Q}{r^2} = \frac{k_e Q}{(0.6m)^2} \simeq \boxed{6.49\times10^5\ \frac{N}{C}}$

24-55)a) 

← spherical gaussian surface

$q_{enc} = -Q + 3Q \Rightarrow \boxed{q_{enc} = +2Q}$

24-55)b) Since the charge enclosed is positive, $\boxed{\vec{E}_{r>c} = +|\vec{E}_{r>c}|\,\hat{r}}$, or radially outwards!

24-55)c) $\oint_{r>c} \vec{E}\cdot d\vec{A} = |\vec{E}_{r>c}|\,4\pi r^2 = \frac{q_{enc}}{\varepsilon_0} \Rightarrow \boxed{\vec{E}_{r>c} = \frac{Q\hat{r}}{2\pi\varepsilon_0 r^2}}$

24-55)d) $\boxed{\vec{E}_{c>r>b} = 0}$, the electric field is zero inside a conductor

24-55)e) Looking at the situation in more detail, one sees that a charge of +3Q is induced on the inner surface of the conductor. This means that the outer surface of the conductor must have a charge +2Q (as expected), since the net charge on the conductor is -Q.

$$\Rightarrow \boxed{q_{enc,\,c>r>b} = -3Q + 3Q = 0}$$

24-55)f) $\boxed{q_{enc,\,b>r>a} = +3Q}$

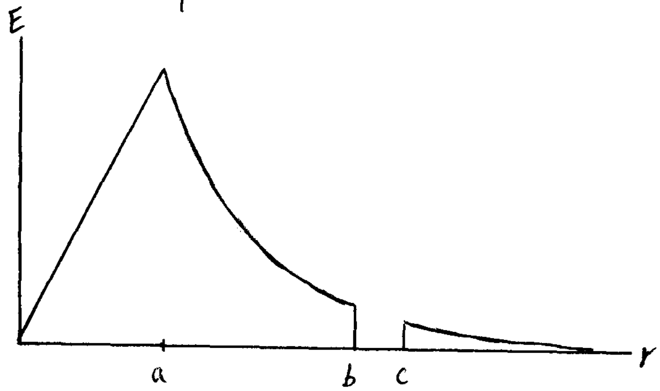
24-55)g) $\oint_{b>r>a} \vec{E}\cdot d\vec{A} = \left|\vec{E}_{b>r>a}\right| 4\pi r^2 = \frac{+q_{enc}}{\varepsilon_0} \Rightarrow \boxed{\vec{E}_{b>r>a} = \frac{+3Q\hat{r}}{4\pi\varepsilon_0 r^2}}$

24-55)h) $\boxed{q_{enc} = 3Q\left(\frac{r}{a}\right)^3}$, by inspection

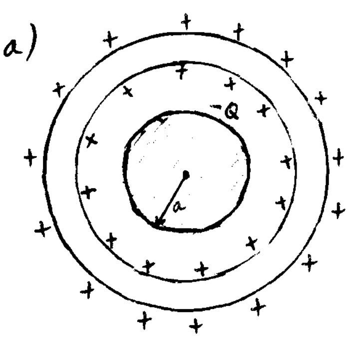
24-55)i) $\oint_{a>r} \vec{E}\cdot d\vec{A} = \left|\vec{E}_{a>r}\right| 4\pi r^2 = \frac{q_{enc}}{\varepsilon_0} \Rightarrow \boxed{\frac{3Qr\hat{r}}{4\pi\varepsilon_0 a^3} = \vec{E}_{a>r}}$

24-55)j)k) These questions were answered in part e).

24-55)l)



24-58)a)



Ben Franklin said, "Charge resides on the outside of the metal cup!"

$\vec{E}_{a<r<b} = \frac{q\hat{r}}{4\pi\varepsilon_0 r^2} \Rightarrow q = \frac{\left(-3.60\times10^{3}\,\frac{N}{C}\right)}{\left(8.99\times10^{9}\,\frac{N\cdot m^2}{C^2}\right)}\cdot(0.1\,m)^2$

$\Rightarrow \boxed{q_{\substack{insulating\\ sphere}} = q = -4.00\times10^{-9}\,C}$

24-58)b)c) $\vec{E}_{r>c} = \frac{q\hat{r}}{4\pi\varepsilon_0 r^2} \Rightarrow q_{out} = \frac{\left(2.00\times10^{2}\,\frac{N}{C}\right)(0.5\,m)^2}{\left(8.99\times10^{9}\,\frac{N\cdot m^2}{C^2}\right)} \simeq 5.56\times10^{-9}\,C$

$\Rightarrow \boxed{q_{net} = q_{in} + q_{out} = 4.00\times10^{-9}\,C + 5.56\times10^{-9}\,C = 9.56\times10^{-9}\,C}$