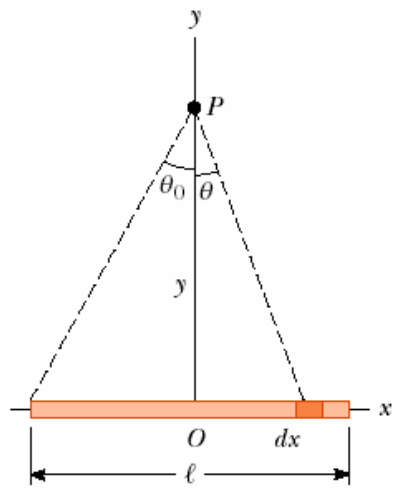
232 Lecture supplement 3

35.a A thin rod of length ℓ and uniform charge per unit length λ lies along the x axis, as shown in Figure P23.35. Show that the electric field at P, a distance y from the rod along its perpendicular bisector, has no x component and is given by $E = 2k_e \lambda \sin \theta_0 / y$.



P23.35 (a) The electric field at point P due to each element of length dx, is $dE = \dfrac{k_e dq}{x^2 + y^2}$ and is directed along the line joining the element to point P. By symmetry,

$E_x = \int dE_x = 0$ and since $dq = \lambda\, dx$,

$E = E_y = \int dE_y = \int dE \cos\theta$ where $\cos\theta = \dfrac{y}{\sqrt{x^2 + y^2}}$.

Therefore, $E = 2k_e \lambda y \displaystyle\int_0^{\ell/2} \frac{dx}{\left(x^2 + y^2\right)^{3/2}} = \boxed{\frac{2k_e \lambda \sin\theta_0}{y}}$.

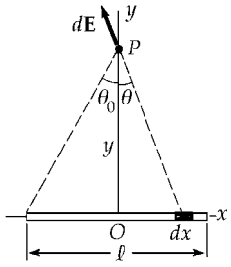


FIG. P23.35