Special Relativity

- **Postulates of special relativity:**
  - The laws of physics must be the same in all inertial reference frames
  - The speed of light has the same value in all inertial frames.

- **Proper Time Interval** : (Δt_p) is the time interval between two events measured by an observer who sees the events occur at the same point in space. The time interval Δt measured by an observer moving with respect to the first one is longer than Δt_p:

$$\Delta t = \frac{\Delta t_p}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma \Delta t_p$$

- **Proper Length** : (L_p) of an object is the length measured by an observer at rest relative to the object. The length L measured by an observer moving with respect to the first one is less than L_p.

$$L = \frac{L_p}{\gamma}$$

- length contraction takes place only along the direction of motion.

- **Lorentz transformation equations**

$$x' = \gamma(x - vt)$$
$$y' = y$$
$$z' = z$$
$$t' = \gamma(t - \frac{v}{c^2}x)$$

- When $v \ll c$ Lorentz transformation equations reduce to Galilean equations.

- **Lorentz Velocity transformation equations**

$$u'_x = \frac{u_x - v}{1 - \frac{u_x v}{c^2}}$$

$$u'_y = \frac{u_y}{\gamma\left(1 - \frac{u_x v}{c^2}\right)}$$

$$u'_z = \frac{u_z}{\gamma\left(1 - \frac{u_x v}{c^2}\right)}$$

- When $u, v \ll c$ Lorentz velocity transformation equations reduce to Galilean equation.
$u'_x = u_x - v$
- When $u_x = c$, $u'_x = c$: speed of light is c in any frame.

8. An astronomer on Earth observes a meteoroid in the southern sky approaching the Earth at a speed of 0.800*c*. At the time of its discovery the meteoroid is 20.0 ly from the Earth. Calculate

(a) the time interval required for the meteoroid to reach the Earth as measured by the Earthbound astronomer,

(b) this time interval as measured by a tourist on the meteoroid, and

(c) the distance to the Earth as measured by the tourist.

(a) The $0.8c$ and the 20 ly are measured in the Earth frame,

so in this frame, $\Delta t = \frac{x}{v} = \frac{20\text{ ly}}{0.8c} = \frac{20\text{ ly}}{0.8c}\frac{1c}{1\text{ ly/yr}} = \boxed{25.0\text{ yr}}$.

(b) We see a clock on the meteoroid moving, so we do not measure proper time; that clock measures proper time.

$\Delta t = \gamma \Delta t_p$: $\Delta t_p = \frac{\Delta t}{\gamma} = \frac{25.0\text{ yr}}{1/\sqrt{1 - v^2/c^2}} = 25.0\text{ yr}\sqrt{1 - 0.8^2} = 25.0\text{ yr}(0.6) = \boxed{15.0\text{ yr}}$

(c) Method one: We measure the 20 ly on a stick stationary in our frame, so it is proper length. The tourist measures it to be contracted to

$L = \frac{L_p}{\gamma} = \frac{20\text{ ly}}{1/\sqrt{1 - 0.8^2}} = \frac{20\text{ ly}}{1.667} = \boxed{12.0\text{ ly}}$.

Method two: The tourist sees the Earth approaching at $0.8c$

$(0.8\text{ ly/yr})(15\text{ yr}) = \boxed{12.0\text{ ly}}$.

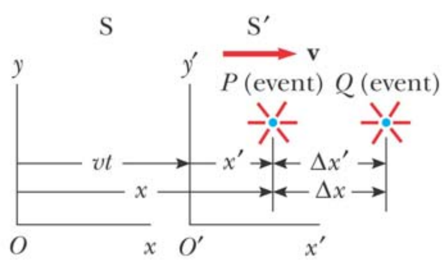
Relativistic Doppler Effect

When a light source and an observer approach each other:

$$f_{obs} = \frac{\sqrt{1+v/c}}{\sqrt{1-v/c}} f_{source}$$

Edwin Hubble discovered that the spectral lines in the light from distant galaxies are shifted towards lower frequencies (Red shifted)

- These galaxies are receding from us
- The Universe is expanding



- **Lorentz transformation equations**

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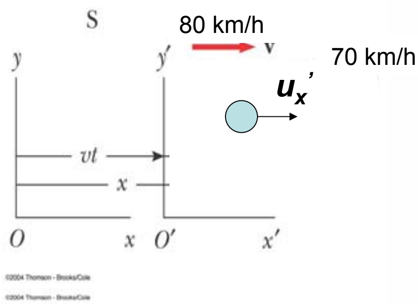


$$u'_x = \frac{u_x - v}{1 - \frac{u_x v}{c^2}}$$

$$u'_y = \frac{u_y}{\gamma\left(1 - \frac{u_x v}{c^2}\right)}$$

$$u'_z = \frac{u_z}{\gamma\left(1 - \frac{u_x v}{c^2}\right)}$$

- When $u, v \ll c$ Lorentz velocity transformation equations reduce to Galilean equation.
  $u'_x = u_x - v$
- When $u_x = c$, $u'_x = c$: speed of light is c in any frame.

39.63. An alien spaceship traveling 0.600*c* toward the Earth launches a landing craft that travels in the same direction with a speed of 0.800*c* relative to the mother ship. As observed on the Earth, the spaceship is 0.200 ly from the Earth when the lander is launched.

(a) What speed do the Earth observers measure for the approaching lander?

(b) What is the distance to the Earth at the time of lander launch, as observed by the aliens on the mother ship?

(c) How long does it take the lander to reach the Earth as observed by the aliens on the mother ship?

(a) Take the spaceship as the primed frame, moving toward the right at $v = +0.600c$.

Then $u'_x = +0.800c$, and $$u_x = \frac{u'_x + v}{1 + (u'_x v)/c^2} = \frac{0.800c + 0.600c}{1 + (0.800)(0.600)} = \boxed{0.946c}.$$

(b) $L = \frac{L_p}{\gamma}$: $$L = (0.200 \text{ ly})\sqrt{1 - (0.600)^2} = \boxed{0.160 \text{ ly}}$$

(c) The aliens observe the 0.160-ly distance closing because the probe nibbles into it from one end at 0.800*c* and the Earth reduces it at the other end at 0.600*c*.

Thus, $$\text{time} = \frac{0.160 \text{ ly}}{0.800c + 0.600c} = \boxed{0.114 \text{ yr}}.$$

- **Relativistic Momentum** of a particle moving with velocity $\mathbf{u}$

$$\mathbf{p} = \frac{m\mathbf{u}}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma m\mathbf{u}$$

$$\mathbf{F} = \frac{d\mathbf{p}}{dt}$$

$$a \propto (1-\frac{u^2}{c^2})^{3/2}$$

- When $u \ll c$, $\mathbf{p}$ approaches the classical value $m\mathbf{u}$.

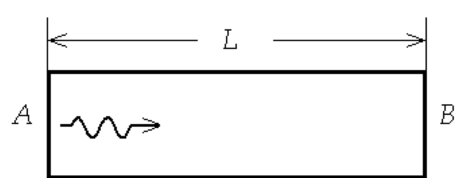
- As $u \to c$, $a \to 0$: it is impossible to accelerate a particle to a velocity greater than c: Speed of light is the ultimate speed.

Photons: Particle of Light

$$E = hf$$

$$E = pc$$

Einstein's Box



$$\boxed{E = mc^2}$$

- **Relativistic Energy** of a particle moving with speed u

$$E = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma mc^2$$
$$= K + mc^2$$
$$E^2 = p^2c^2 + (mc^2)^2$$

- **Mass is a form of energy**: a particle at rest carries an energy $E_R = mc^2$.

- The laws of energy conservation and mass conservation are one and the same:

  The energy of a system of particles is conserved; where the energy of the i^{th} particle is given by:

$$E_i = \frac{m_i c^2}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma m_i c^2$$