Light: particles or waves?

The particle (photon) model of light and the wave model of light complement each other; some experiments can only be described by the wave model while some others can only be described by the particle model.

The Wave Properties of Particles

Louis de Broglie Postulated: All forms of matter have both wave and particle properties, just like photons do.

We know that for a photon: \( p = \frac{h}{\lambda} \)

So for any particle: \( \lambda = \frac{h}{p} = \frac{h}{\gamma mv} \) \text{ de Broglie wave length }

And the frequency of a particle: \( f = \frac{E}{h} \)

Wave-Particle duality

Soon after de Broglie’s proposal, Davidson and Germer showed that electrons show diffraction effects and measured the wave length of electrons!
34. Calculate the de Broglie wavelength for an electron that has kinetic energy (a) 50.0 eV and (b) 50.0 keV. Are these electrons moving at relativistic velocities?

The rest energy of an electron = $m_e c^2 = 8.2 \times 10^{-14}$ J

$$= 8.2 \times 10^{-14} / 1.6 \times 10^{-19} \text{ eV}$$

$$= 0.511 \text{ MeV}$$

So the total energy of the electron in the two given cases:

(a) $E = 511000 + 50 \text{ eV} = 0.51105 \text{ MeV}$

(b) $E = 511000 + 50000 \text{ eV} = 0.551 \text{ MeV}$

And $\gamma$ for the two cases are:

(a) $\gamma = \frac{E}{E_R} = 1.0001$  \hspace{1cm} (b) $\gamma = \frac{E}{E_R} = 1.08$
34. Calculate the de Broglie wavelength for an electron that has kinetic energy (a) 50.0 eV and (b) 50.0 keV.

(a) \[ \frac{p^2}{2m} = (50.0)(1.60 \times 10^{-19} \text{ J}) \]

\[ p = 3.81 \times 10^{-24} \text{ kg \cdot m/s} \]

\[ \lambda = \frac{h}{p} = 0.174 \text{ nm} \]

About 200 times smaller than the wavelength of light

(b) \[ \frac{p^2}{2m} = (50.0 \times 10^3)(1.60 \times 10^{-19} \text{ J}) \]

\[ p = 1.20 \times 10^{-22} \text{ kg \cdot m/s} \]

\[ \lambda = \frac{h}{p} = 5.49 \times 10^{-14} \text{ m} \]

The relativistic answer is slightly more precise:

\[ \lambda = \frac{h}{p} = \frac{hc}{\left( m c^2 + K \right)^{1/2} - m c^4} = 5.37 \times 10^{-12} \text{ m} \]

**Electron Microscope**

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\[ p_c = 6000 \text{ MeV} \]
\[ \lambda = \frac{hc}{p_c} \approx 7 \times 10^{-17} \text{ m} \]
An ideal particle would have zero size and would be localized in space.
An ideal wave has a single frequency and is infinitely long.
Consider combining two waves with slightly different frequencies:

When a large number of waves with varying frequencies are combined, there is destructive interference everywhere except near $x=0$.

The result is a wave packet that is localized in space.
The quantum particle is a wave packet.
Werner Heisenberg used quantum theoretical arguments to show that it is impossible to make simultaneous measurements of a particle’s position and momentum with infinite accuracy. This is known as the Heisenberg Uncertainty Principal.

If a measurement of the position of a particle is made with uncertainty \( \Delta x \) and a simultaneous measurement of its \( x \) component of measurement made with uncertainty \( \Delta p_x \), the product of the two uncertainties can never be smaller than \( \hbar / 2 \).

\[
\Delta x \Delta p_x \geq \frac{\hbar}{2}
\]

\( (\hbar = \hbar / 2\pi) \)