PHYS 241E

Homework Solutions for 10^{th} assignment, Due the week of Nov. 8, 1999.

Chapter 32 : 19

Chapter 33 : 9, 13, 30 37, 52, 61

19. Inside of the ferromagnetic material the atoms partially align <u>adding</u> to the field inside, $\vec{B}$.

$\vec{B} = \vec{B}_o + \mu_o \vec{M}$ — the magnetic field inside the metal is due to the field caused by the coil plus that caused by the aligned dipoles.

$= \vec{B}_o + \mu_o \chi_m \vec{H}$ — the dipole alignment is proportional to the magnetic intensity.

$= B_o + \mu_o \chi_m \frac{\vec{B}_o}{\mu_o}$ - this intensity is also related to the field caused by the coils

$= B_o(1+\chi_m)$ → The effects of the original field plus that due to alignment caused by the original field can be combined in one factor

$= B_o \cdot \frac{\mu}{\mu_o}$

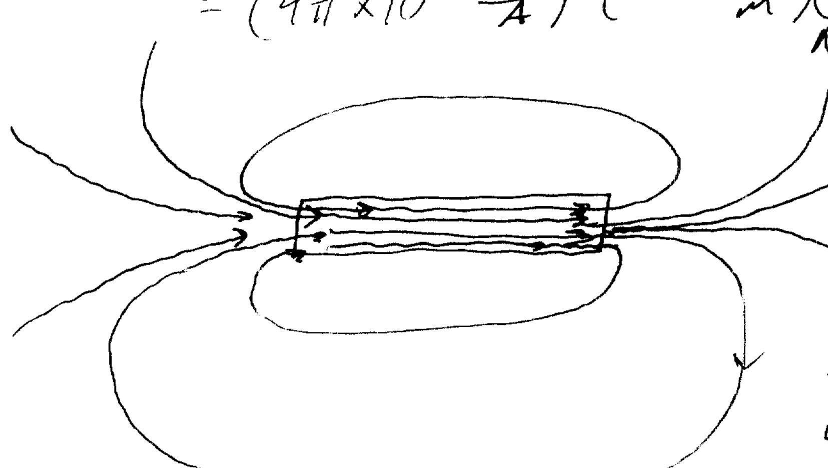
$= \mu_o n I \cdot \frac{\mu}{\mu_o}$ The initial field is entirely due to the solenoid.

$\vec{B}_{iron} = \left(4\pi \times 10^{-7} \frac{T\,m}{A}\right) \cdot \left(400 \frac{turns}{m}\right)(.5A) \cdot (640) = \boxed{0.16\,T}$

In the air next to the iron there is no net alignment of dipoles. The magnetic field near the <u>ends</u> of the ferromagnet (but outside of it) is greatly increased by the presence of the iron... but alongside of the iron there is <u>very</u> nearly no effect.

$\vec{B}_{air} = \mu_o n I$

$= \left(4\pi \times 10^{-7} \frac{T\,m}{A}\right) \cdot \left(400 \frac{turns}{m}\right)(.5A) = \boxed{2.5 \times 10^{-4}\,T}$



Note that the field lines leaving the iron are close together, adding strongly to the local field... but along side the metal they extend over all space, having very little effect at any given place.

Ch 33-19.

If you have a coil in which you vary the amount of current passing through, the flux inside of this coil will change, and by Lenz's Law, A current will be induced in the same coil to counter the change. By Faraday's Law, the induced emf (related to the current by $V = IR$) is:

$$\mathcal{E} = -\frac{d\Phi}{dt} = -\frac{d}{dt}(\Phi) = -\frac{d}{dt}(\vec{B}\cdot\vec{A}) = -\frac{d}{dt}(BA\cos\theta)$$

The magnetic field is affected by the geometry of the source, + the area and angle are related to the relative geometry of the 'receiver' loop. In the case of one loop affecting itself, the source + receiver are obviously the same.

$$\mathcal{E} = -\frac{d}{dt}(\mu_0 nINA\cos\theta) = -\mu_0 nNA\cos\theta \cdot \frac{d}{dt}I$$

Since the geometry of the source + receiver doesn't change, these terms may come out of the derivative, + can be combined in 1 variable:

$$\mathcal{E} = -L\frac{dI}{dt}$$

$$\therefore L_{sol} = \mu_0 nNA = \mu_0 n^2 lA = \mu_0 N^2 A/l$$

if only 1 coil is connected to a circuit, we can calculate the self inductance of this coil:

a) $\boxed{L_1 = \mu_0 N_1^2 A/l \quad L_2 = \mu_0 N_2^2 A/l}$

b) if the windings are in the same direction, the fluxes add:

$$\boxed{L_{1+2} = \mu_0 (N_1+N_2)^2 A/l}$$

c) wired oppositely, they create fluxes in opposite directions:

$$L'_{1+2} = \mu_0 (N_1 - N_2)^2 A/l$$

d) If one coil creates + another receives the flux, then $\Phi = (\mu_0 n_1 I)\cdot(n_2 A_2)$ and the mutual inductance is $\boxed{M_{21} = \frac{\Phi_{21}}{I} = \mu_0 N_1 N_2 A/l}$

33-13. From problem 9, where we have a similar geometry

$$M_{21} = \frac{\Phi_{21}}{I} = \frac{\mu_0 N_1 N_2 A_2}{l}$$ w/ $N_2 = 1$ because there is only 1 loop.

$$= \frac{\left(4\pi \times 10^{-7} \frac{T\cdot m}{A}\right)(120\,\text{turns})(1\,\text{turn})\cdot(\pi)(0.75\times 10^{-2}\,m)^2}{(0.2\,m)}$$

$$\boxed{M_{21} = 1.3\times 10^{-7} = .13\,\mu H}$$

A changing current in one loop causes a changing flux, and therefore an ~~other~~ emf in the second loop.

$$\mathcal{E}_2 = -M_{21}\frac{dI}{dt}$$

If the current rises linearly by 30A in 0.3s, & falls to 0 in the same time, then:

$$I_1 = \begin{cases} \left(\frac{30\,A}{0.3\,s}\right)t & 0 \le t \le .3\,s \\ -\left(\frac{30\,A}{0.3\,s}\right)t & .3\,s \le t \le .6\,s \\ 0 & \forall \text{ other times.} \end{cases} \rightarrow \frac{dI_1}{dt} = \begin{cases} 30/.3 = 100\,A/s \\ -30/.3 = -100\,A/s \\ 0 \end{cases}$$

The induced current in the second loop is the induced emf divided by the resistance:

$$I_2(0 \le t \le .3s) = \frac{\mathcal{E}_{2,\le .3}}{R} = -\frac{M_{21}}{R}\cdot\frac{dI_1}{dt} = -\frac{(1.3\times 10^{-7}H)\cdot(100\,A/s)}{33\,\Omega}$$

$$= -3.9\overline{3}\times 10^{-7}A = \boxed{-4\times 10^{-7}A = I_{2,ind}(0 \le t \le .3s)}$$

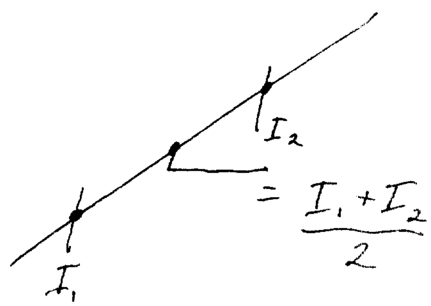
for the next ~~0.3~~ 0.3 s only the sign changes

$$\boxed{I_{2_{ind}}(.3 \le t \le .6s) = 4\times 10^{-7}A}$$

$I_{ind} = 0$ for all other times.

33-30. $V = L\, dI/dt$

$V + L$ are given as constants, therefore $\frac{dI}{dt}$ is a constant... this means that the current changes linearly, which means we can use endpoints to find an average for the current:



$P = IV$

$P_{ave} = I_{ave} V$

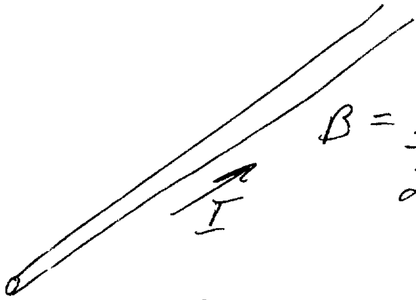
$= \left(\frac{I_1 + I_2}{2}\right) V$

a) $P_{ave} = \frac{1}{2}(0A + .1A) \cdot 12V = 0.60W$

b) $P_{ave} = \frac{1}{2}(.1A + .2A) \cdot 12V = 1.8\ W$

c) $P_{ave} = \frac{1}{2}(.2A + .3A) \cdot 12V = 3.0W$

32-37



$$B = \frac{\mu_0 I}{2\pi r}$$

The Energy density for a magnetic field is

$$u_B = \frac{B^2}{2\mu_0} = \frac{\mu_0^2 I^2}{2\mu_0 \cdot 4\pi^2 r^2} = \frac{\mu_0 I}{8\pi^2 r^2}$$

$$= \frac{\left(4\pi \times 10^{-7} \frac{T \cdot m}{A}\right)(20A)^2}{8 \cdot \pi^2 \cdot r^2}$$

$$= \left(6.4 \times 10^{-6}\, J/m\right) \cdot \frac{1}{r^2}$$

The energy density of an electric field is:

$$u_E = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0\left(\frac{V}{d}\right)^2 = \frac{1}{2}\varepsilon_0\left(\frac{Q}{Cd}\right)^2$$

... Using the fact that the electric field in a capacitor is constant ($E = V/d$) and that $Q = VC$ for any capacitor.

$$u_E = \frac{1}{2}\frac{\left(8.85 \times 10^{-12} F/m\right)\left(10^{-7} C\right)^2}{\left(6.3 \times 10^{-9} F\right)^2 \left(1.5 \times 10^{-3} m\right)^2}$$

$$= 5.0 \times 10^{-4}\, J/m$$

$$u_E = u_B \text{ when } r^2 = \frac{6.4 \times 10^{-6}\, J/m}{5 \times 10^{-4}\, \cancel{J/m}^3}$$

$$\boxed{r = .11 m = 11 cm}$$

33-52

From equation 33-25, the charge on the capacitor is:

$$Q = Q_0 \cos(\omega t + \phi)$$

And the current in the circuit is the derivative of Q:

$$I = \frac{dQ}{dt} = -Q_0 \omega \sin(\omega t + \phi)$$

The initial conditions (when $t = 0$) ~~state thate~~

$$Q(t=0) = q \rightarrow q = Q_0 \cos\phi$$

$$I(t=0) = 0 \qquad 0 = -Q_0 \omega \sin\phi$$

solving this tells us that

$$Q_0 = q$$

$$\phi = 0$$

The current as a function of time is then:

$$I(t) = -q\omega \sin \omega t$$

This is maximized when $\sin \omega t = 1$ which occurs when

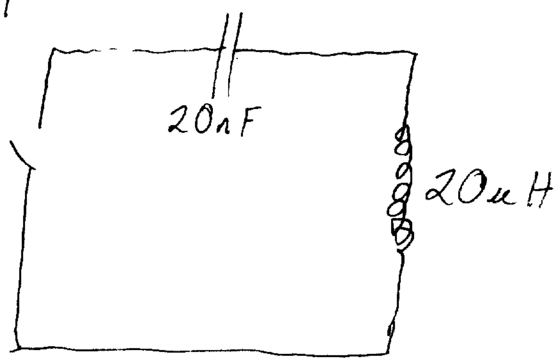
$$\omega t = \left(\frac{2n+1}{2}\right)\pi, \quad n = 0, 1, 2 \ldots$$

$$I_{max} = \boxed{q\omega = \frac{q}{(LC)^{1/2}}}$$

which occurs at times

$$\boxed{t = \left(\frac{2n+1}{2}\right)\frac{\pi}{\omega}}$$

33-61



from the previous problem, w/ $q = 30nC$

a) $I_{max} = \frac{q}{(LC)^{1/2}} = \frac{30\times10^{-9}C}{[(2\times10^{-5}H)\cdot(20\times10^{-9}F)]^{1/2}}$

$\boxed{= 4.7\times10^{-2}A = 47mA}$

b) $U_{L\,max} = \frac{1}{2}LI_{max}^2$

$= \frac{1}{2}(2\times10^{-5}H)(4.7\times10^{-2}A)^2$

$= \boxed{2.2\times10^{-8}J}$

c) $\frac{U_{L\,max}}{U_{C\,max}} = \frac{\frac{1}{2}LI^2}{\frac{1}{2}Q^2/C} = \frac{L(Q_0\omega)^2}{Q_0^2/C} = LC\omega^2 = 1$

As should be the case if energy is to be conserved