LECTURE # 10 & 11

Application of Gauss' Law

Example 1: A point charge q = situated on corner of a cube.
For three faces touching the corner the flux is parallel to face $\therefore \int E.dA = 0$ for these faces. The remaining three faces are all equivalent by symmetry. What is the flux through them? Construct 7 more cubes to fill space around the charge q.

According to Gauss' law total $f = \frac{q}{\in_0}$

Therefore $Total / cube = \frac{q}{8\in_0}$ since there are eight cubes

$\therefore Total / \ face = \left(\frac{q}{24\in_0} \right)$ since there are 3 faces with non-zero flux per cube.

Example 2: Electric Field due to a line charge: Enclose the line charge with a cylinder only on a portion (line charge density=λ)

By symmetry all the flux can only come out radially.

Charge enclosed inside cylinder is λh.

$$\oint E.ds = \frac{l\,h}{\in_0}$$

$$\int_{top\ surface} \vec{E}.ds + \int_{Bot.cap} \vec{E}.ds + \int_{Cylin} E.ds = \frac{l\,h}{\in_0}$$ Compare this method with E.g., 23-5 in text.

$\searrow 0$ $\searrow 0$ $\searrow$ E=2$p$rh*

$\Rightarrow$ E=$\frac{l}{2p\,\mathrm{r}\in_0}$

- Because we have chosen a cylinder as our Gaussian surface.

Example 3 (Sheet charge): Infinite extent; charge density σ
Gaussian surface = cylinder

$$\in_0 \oint \vec{E}.ds = q$$

$$\in_0 \left(E.A + E - A \right) = q$$

$$or \quad E = \frac{q}{2\in_0 A} = \frac{s}{2\in_0}$$

Example 4 (Spherical shell)

(a) Outside the shell:

$$\frac{Q}{\in_0} = \oint \vec{E}.d\vec{A} = E.4p\,r^2$$

$$E = \frac{Q}{4p \in_0 r^2}$$

(b) Inside the shell

$$: \frac{Q}{\in_0} = \oint_{\text{(inside)}} \vec{E}.dt = E\oint da = 0$$

$$or \quad E = 0$$

Example 5 (**Charged sphere)** (Special charge distribution.)

Charge density depends only on radial distance r.

(a) E outside the spherical distribution - applying Gauss' law just as for a point charge.

$E = \left(\frac{q}{4p \in_0 r^2}\right)$ where q is the total charge.

$$\left\{\oint \text{E.ds} = \frac{\text{q}}{\in_0} \Rightarrow E.4p\,r^2 = q \ \text{ or } E = \left[\frac{q}{4p \in_0 r^2}\right]\right\}$$

(b) Inside the sphere : Gauss' law

$$\in_0 \oint \vec{E}.d\vec{s} = \in_0 E4p\,r^2 = q^1$$

$$\text{or } E = \left(\frac{1}{4p \in_0}\right)\frac{q^1}{r^2}$$

Where q' is part of q contained within r.

(The rest of the q makes no contribution to E inside)

Example 7 (Uniform sphere) : In this case

$$q^1 = q\frac{4/3p\,r^3}{4/3p\,R^3} = q\left(\frac{r}{R}\right)^3$$

$$\therefore E = \left(\frac{q}{4p \in_0} \ \frac{r}{R^3}\right)$$

Conducting sphere: Suppose we make the sphere conduct "all if a sudden". In a conductor charges are free to move---the charges move to the surface and stay there with uniform density, We can also say that the electric field Has to be perpendicular to the surface everywhere with no tangential comp. Because tangential component will make charges move. This statement applies for any conducting surface. And as long as we are outside and near the surface of the sphere $E = \frac{1}{4\pi \in_0} \frac{q}{r^2} \sim \frac{1}{r^2}$

Cylinder $E \sim \frac{1}{r}$

Relevance to lightning conductor----Electrostatic precipitator.