ELECTRIC POTENTIAL ENERGY

In order to discuss charges and their motion we started first by considering the forces between charges. An alternate method was to introduce an electric field and then consider the motion of charges in such a field. We will now see that there is also a third method of considering charges and their interactions. This is the method of considering "electrical potential energy" between charges.

Recall your knowledge about gravitational potential energy. When we write the potential energy of an object of mass m as P.E. = m.g.h at a height h we are making two assumptions:

(a) that the 'zero' of the P.E. lies on the surface of the earth
(b) the radius of the earth is much larger than the height h.

A more correct way to define P.E. is to start from Newton's Law of gravitation and consider the work needed to overcome the force of attraction between the two objects. (work done is stored as P.E.). Of course it is always true that it is only the difference in the energy that matters.

We know that there is also a force involved between charges:

$$F = \frac{kqq_0}{r^2}\hat{r}$$

Consider a charge 'q' and a test charge 'q0' which is initially at a distance ra from q and is then moved to rb. We define the change in the potential energy as:

$$\Delta U = [U(\vec{r}_b) - U(\vec{r}_a)] = -\int_{\vec{r}_a}^{\vec{r}_b} \vec{F}\,'\,d\vec{s}$$

The negative sign is attached since when work is done (i.e.'expended') we gain P.E.

Substituting for F explicitly

$$\Delta U = -\int_a^{b_b} \vec{F}\,'\,d\vec{r} = -kqq_0\int_a^b \frac{dr}{r^2} = kqq_0\left(\frac{1}{r_b} - \frac{1}{r_a}\right)$$

From this define:

$U(r) = \dfrac{kqq_0}{r}$ is the potential energy (change) of charge q0 brought from infinity to r in the presence of charge "q".

Electric Potential:

The charge 'q' gives rise to an electric field E and any other charge 'q0' would interact with E as given by the equation: $\vec{F} = q_0\vec{E}$. Just as we obtained E from F and made it independent of q0 we can define a quantity which looks like the potential energy but is independent of the 'q0'. We can do this as:

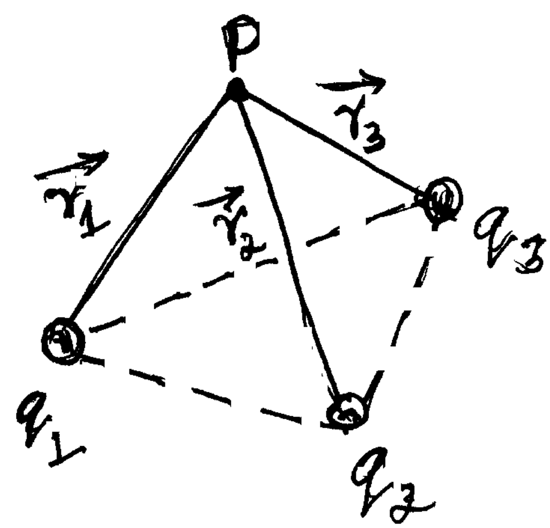
Since $\Delta U = [U(\vec{r}_b) - U(\vec{r}_a)] = -\int_{\vec{r}_a}^{\vec{r}_b} \vec{F}\,'d\vec{s}$ we can write $\frac{\Delta U}{q_0} = -\int_{\vec{r}_a}^{\vec{r}_b} \vec{E}.d\vec{s}$. For a point charge this becomes: $\frac{U(r)}{q_0} = \frac{kq}{r} = V(r)$ - is the potential due to q.

In practice we only deal with potential differences:

$\Delta V = V_b - V_a = \frac{kq}{r_b} - \frac{kq}{r_a}$. The physical interpretation of this equation is that this is the work done to move unit charge from ra to rb. Alternately we can also write:

$\Delta V = \left(\frac{U_b - U_a}{q_0}\right) = -\int_{r_a}^{r_b} \vec{E}.d\vec{s}$ an equation which applies to any charge distribution which gives rise to a field distribution E(r).

Superposition Principle: We have seen that the electric field obeys the superposition principle. Therefore the electric potential of a system of charges can also be determined by superposition (i.e. simply by adding). Thus referring to the figure below the potential at the point P when charges q1, q2 and q3 are present as shown is given by:



$$V_p = \frac{k.q}{r_1} + \frac{k.q}{r_2} + \frac{k.q}{r_3}$$

This can be extended to continuous charge distributions as:

$$V = \int dV = k\int \frac{dq}{r}$$

See worked examples for applications.

UNITS: Electric potential is measured in Volts where 1 Volt = 1 Joule/Coulomb. This definition of the volt also leads to a unit of energy called the electron volt (1 eV). 1 eV is the energy gained by an electron accelerated through a potential difference of 1 Volt. This energy is 1.6 x 10^{-19} Joule.