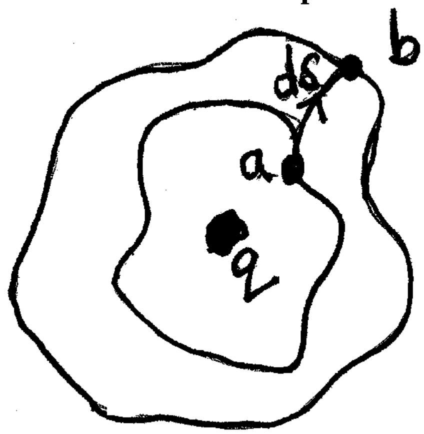
Electron Contour Maps



Contour maps of mountains -- give "paths" of constant gravitational potential energy -- if one managed to walk on these lines then no energy expended ! Similarly one can produce electron potential contour maps. This means that when a test charge moves along this line no work is done !
If the points 'a' and 'b' lie on different surfaces then work is done -- and this work is equal to

$$q_0 \int_a^b \vec{E} d\vec{s}$$

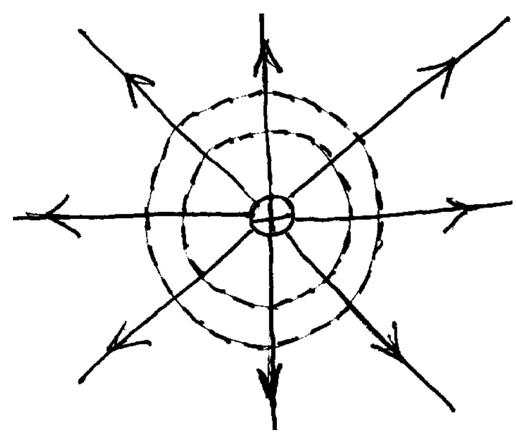
Since no work is done on a constant potential curve -- it does not mean there is no electric field. It ⇒E field is always ⊥ to the constant potential line.

EXAMPLES :

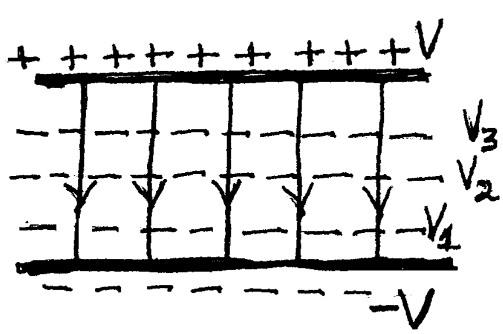
(a) Point charge: The potential due to a point charge is given by

U(r)~1/r ⇒ constant potential lines are circles.

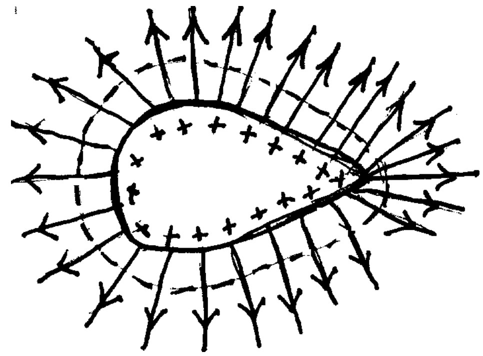
We already know that the E field lines are "radial".



(b) Parallel plates:



(c) Complicated metallic object: The surface of the metal is a constant potential and
therefore electron field lines are always ⊥ to the surface.



Fields from potentials: In all the above examples we drew field lines first and then potentials. Can we do the opposite?

$$V_b - V_a = \int_{r_a}^{r_b} dV = -\int_{r_a}^{r_b} \vec{E}.d\vec{s}$$

$dV = -\vec{E}.d\vec{s}$ Independent of the path

$\therefore\ \mathrm{E} = -\frac{\mathrm{dV}}{\mathrm{ds}}$ OR $dV = -Eds$

If ds is chosen along perpendicular to the equipotential surface. Therefore
And perpendicular to the surface.

How do we do the differentials in 3-Dimensions?

EXAMPLE:

(1) Charged disk and its potential. And how two get field.

(2) Electric dipole & its potential. And how to get field.

(3) Charged spherical shell - Electric field of shell

$$E = \frac{Q}{4p\ell_0 r^z} \qquad \mathrm{r} > R$$

$$= 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots \mathrm{r} < \mathrm{R}$$

Therefore $\Delta V = V(r) - V(\infty) = -\int_{\infty}^{r} E.dr = -kQ\int_{\infty}^{r} \frac{dr}{r^2} = kQ\left(\frac{1}{r} - \frac{1}{\infty}\right)$

If $\mathrm{V}(\infty) = 0 \Rightarrow \mathrm{V(r)} = \frac{\mathrm{Q}}{4pe\mathrm{r}}$ for r > R and therefore

$$\Delta\mathrm{V} = -\left(\int_{\infty}^{\mathrm{r}} \mathrm{E_{out}}\,dr + \int_{R}^{r} E_{in}\,dr\right) = \frac{Q}{4peR}$$

POTENTIALS & FIELDS NEAR CONDUCTORS : The role of sharp points on conducting surfaces.

Start with two spheres

$$V_1 = \frac{Q}{4pe\, r_1} \quad \neq \quad V_2 = \frac{q'}{4pe\, r_2}.$$

When connected $\mathrm{V} = \frac{\mathrm{q}_1}{4pe_0 \mathrm{r}_1} = \frac{q_2}{4pe_0 r_2} \Rightarrow \frac{q_1}{r_1} = \frac{q_2}{r_2}$

But $s = \frac{q_1}{4p\, r_1^2}$ & $s_2 = \frac{q_2}{4p\, r_2 z} \Rightarrow s_1 r_1 = s_2 r_2$

$\Rightarrow$ smaller the radius ---larger the surface charge density.