<u>Faraday's Law of Induction</u>: States that there is an emf induced around a closed path enclosing a magnetic flux Φ_M whenever this flux changes with time. This emf is given by:

$$E = -\frac{d\phi_m}{dt}.$$

Another way to state the same is: The time rate of change of flux through a surface equals the negative EMF induced on a <u>path</u> enclosing that surface.
This 'path' can be physical or imaginary and the changing flux can be due to either a changing magnetic field or a changing area.

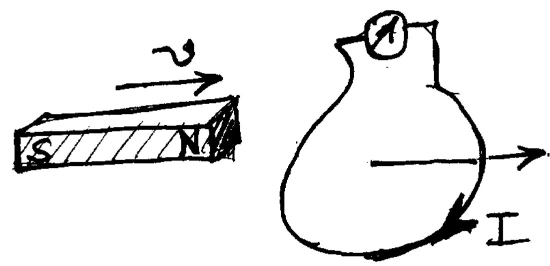
What is the meaning of the negative sign? This will become clear shortly!
Electromotive force = work done on unit charge.
This work done = Force x distance = $(qE).d\ell = E.d\ell$ for unit charge.

Since we have a closed loop this "voltage" $= \oint \vec{E} \cdot \overrightarrow{d\ell}$

Hence Faraday's law can be expressed as $\therefore \quad \oint \vec{E} \cdot \overrightarrow{d\ell} = -\frac{d\phi_m}{dt}$

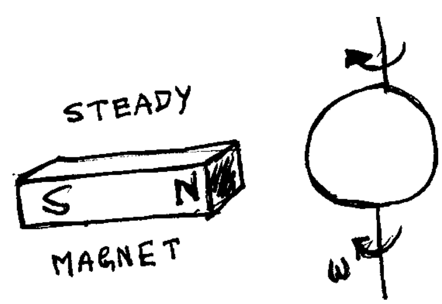
Meaning of –ve sign starts to become clear now. While evaluating flux we need to consider an area. Then we need to fix the direction in which we are going to go around the loop that encloses this area. If the fingers of the right hand are in the same direction as the sense in which we go around the loop then the thumb (stuck out) gives the normal to the area. Thus we can arrive at a 'sign' for the flux and hence a sign for the rate of change of flux. The reversal of this sign specifies the polarity of the induced emf with respect to the direction of the path around the area chosen.

<u>Example 1</u>: The figure on the right shows shows a bar magnet being moved closer to a closed loop of wire at a certain velocity. The arrow through the loop gives the direction of the magnetic field. The direction of the induced current (due to the emf induced) is also shown. Further explanation of this phenomenon comes from Lenz' law.

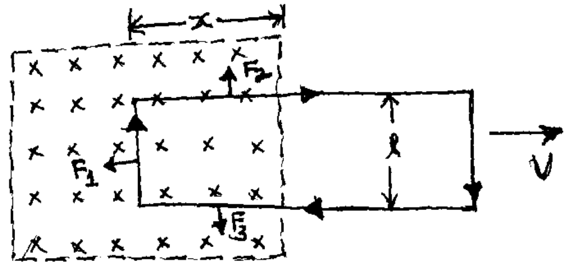


<u>Lenz's Law</u>: The induced EMF causes a current such that the induced current produces an effect which opposes the change that produced it.

<u>Example 2</u>: Changing "A" by rotating the loop. Or you can "blow down" the area as in flux compression experiments!
[Flux same ⇒ enormous fields as Area shrinks!]



Quantitative Example I: Consider a rectangular loop of wire being drawn through a region of uniform magnetic field with a velocity v as shown in the figure.



The flux through the rectangle is $\phi_m = B \cdot \ell \cdot R$

$$\therefore E = \frac{-d\phi_m}{dt} = -B \cdot \ell \cdot \frac{dR}{dt}$$

$$= B\ell \cdot v \ .$$

This EMF sets up a current in the loop.

$$i = \frac{E}{R} = \frac{B\ell v}{R};$$

This current must be clockwise according to Lenz' law and causes forces F_1, F_2, F_3 as per the equation $F = i\ell \times B$. The forces F_2 & F_3 cancel each other and the force F1 is given by:

$$F_1 = i\ell B \sin 90^\circ = \left(\frac{B\ell v}{R}\right)\ell \cdot B$$

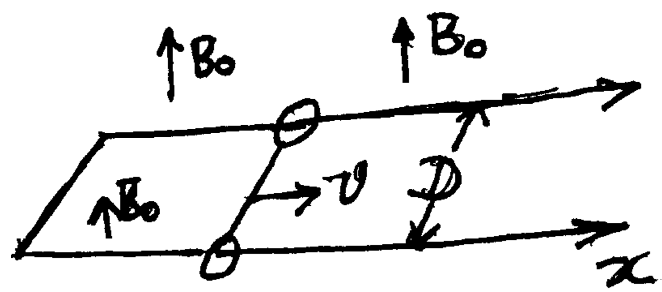
$$= B^2 \ell^2 \frac{v}{R}$$

Since this force to the left the agent that pulls the loop to the right must do work at the steady rate.

$$P = F_1 . v = B^2 \ell^2 \frac{v^2}{R}$$

<u>Note</u>: Joule heating $= i^2 R = \frac{B^2 \ell^2 v^2}{R^2} \cdot R = \frac{B^2 \ell^2 v^2}{R}$ same as P $\rightarrow$ as it should be!

<u>Quantitative Example II (variation on I)</u>: Resistance of loop varies as R = αL where L is the total length of the loop.



$$\Phi_m = B \cdot A = B \cdot D \cdot v \cdot t$$

$$\frac{d\phi_m}{dt} = B_0 \cdot D \cdot v$$

$$E = -B_0 D v = I_{ind} \cdot R$$

or

$$I_{ind} = \frac{-BDv}{2\alpha(D + vt)}$$

Example III (Also a variation on I)
Loop is imaginary [but has to be within the region of field and fixed].

E = - BLv here also.

This EMF makes charges move until there is no more motion - accumulation of charges which is positive at bottom $\rightarrow$ this is the same as what Lorentz forces would say!

Example IV: Generator:

$$\Phi_m = \vec{A} \cdot \vec{B} = AB\cos\theta$$

$$\therefore \frac{d\phi_m}{dt} = AB\frac{d}{dt}\cos(\omega t) = -AB\omega\sin(\omega t)$$

$$E = -N\frac{d\Phi_B}{dt} \text{ for N turns}$$

$$= -NAB\omega\sin(\omega t)$$

Current induced $\quad I = \frac{E}{R} = \frac{NAB\omega}{R}\sin\omega t$

Power $P = E \cdot I = I \cdot N \cdot A \cdot B\omega\sin\omega t = \frac{(NAB\omega)^2}{R}\sin^2\omega t > 0$

$$\vec{\tau} = \left|\vec{\mu} \times \vec{B}\right| = \mu B\sin\theta$$

Power = $\tau\omega = \mu B\sin\theta \cdot \omega = (I \cdot N \cdot A)B\sin\theta\omega$ $\rightarrow$ same as electrical power.

AC motors $\rightarrow$ no commutator necessary.