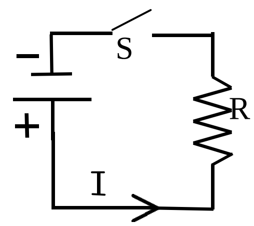
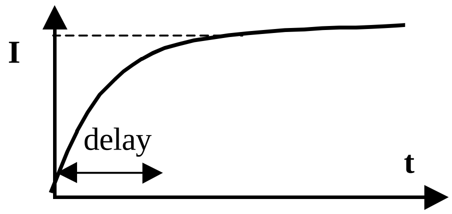
Inductance:

Consider the circuit shown below:
When the switch S is closed at t=0 we can expect the current to rise to behave according to $E = I \cdot R$ .or $I = \varepsilon / R$.



But what really happens is shown in the figure below:



This is because as soon as S is closed a closed loop is created and as the current builds up there is a flux though the loop which is increasing and therefore according to Lenz' law $\Rightarrow$ induced current to oppose this flux.

This induced current is given by e $I_{ind} = \frac{E_{ind}}{R} = \frac{\frac{-d\phi_m}{dt}}{R} = \frac{-1}{R} \frac{d\phi_m}{dt}$.

Therefore the total EMF is $\therefore \overbrace{\left(E - \frac{d\phi_m}{dt} \right)}^{\text{Total EMF}} = \frac{I}{R}$. This quantity is time dependent.

In order to say more – we do not know what is $\left(\frac{d\phi_m}{dt} \right)$ – we need more info.

Of course the definition of flux is always $\phi_m = B.A.$ but note that the B field produced by circuit – depends on geometry of circuit.

For a circular loop we know that $B = \frac{\mu_o I}{2R}$.

And for a solenoid $B = \mu_o nI$ etc.

Hence for a loop $\therefore \Phi_m = \frac{\mu_o I}{2R} \cdot A = \frac{\mu_o I}{2} \pi R = \left(\frac{\mu_o \pi R}{2} \right) \cdot I$

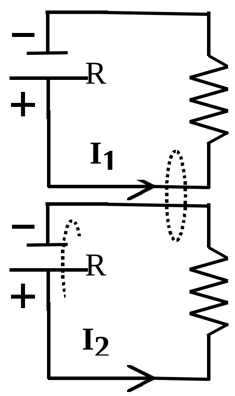
And for a solenoid $\phi_m = \mu_o nI(\pi R^2 \cdot N) = (\mu_o n^2 \pi R^2 \ell) \cdot I$

In all cases: $\phi_m = L \cdot I$ where L is a geometry dependent factor which is a constant for given geometry.

Therefore the induced EMF becomes $\therefore \frac{d\phi_m}{dt} = L \frac{dI}{dt} = -E_{ind}$

We call "L" the 'inductance' $\rightarrow$ it is a "temporary" resistance in DC circuits. In this e.g. "L" is the "self-inductance". For a solenoid it is given by $L = m_0 n^2 p R^2 \ell$

Suppose we put two circuits together as shown in the figure below then the flux created by current in one circuit will couple to the other in addition to the flux created by the current in itself. In this case the total flux linking circuit 1 (top circuit) for example is given by:



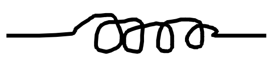
$$\Phi_m(1) = L_1 I_1 + M_{12} I_2$$

where we introduce a new term M12 called the mutual Inductance which gives the flux due to current in coil 2 on coil 1. Similarly the total flux on circuit 2 is :

$$\phi_m(2) = L_2 I_2 + M_{21} I_1 \qquad \& \qquad M_{12} = M_{21}$$

<u>Notes</u>:

(a) <u>Unit of inductance</u>: is a Henry defined as $1 \text{ Henry} = \dfrac{1 \text{ Weber}}{1 \cdot \text{Amp}} = \dfrac{1\text{T} \cdot \text{m}^2}{1 \cdot \text{Amp}}$

(b) <u>Symbol</u>: 

(c) <u>Modification of Kirchoff's loop rule</u>: The EMF created due to induction has to be counted while applying Kirchoff's rule. While moving through the loop in the presumed direction of current I, the potential drop is $V = -L\, dI/dt$. The actual EMF can be either +ve or -ve depending on whether dI/dt is -ve or +ve.

(d) <u>Effect of magnetic materials</u>: Inductance for a solenoid is $L = m_0 n^2 p R^2 \ell$. When a magnetic material with permeability μ is inserted into the solenoid $\mu_o \to \mu_o(1 + X_m)$ where Xm is the magnetic susceptibility of the material. Thus iron core inductors have a much larger L than air-core inductors.

(e) Example – numerical: – superconducting magnets ~ 10H $\qquad I_{max} \sim 100\text{A}$

$$E = -L\frac{dI}{dt} \quad \Rightarrow \quad 10 \times 100\, A/S \quad \Rightarrow \quad 10\text{v}$$

100

$$10 \times \frac{100}{1\text{S}} \quad \Rightarrow \quad 1000 \text{ v} \quad \Rightarrow \quad \text{large}$$

(f) Inductors in series and parallel work just like resistors if they are not coupled to each other. If they are coupled in any way then the answer is different. This is illustrated in the following example. Consider two coils wound on top of each other.

(a) Let us first calculate the self inductance when the coils are connected in series with the current in the same direction:

We know that the individual inductances are $L_1 = \mu_0 A \ell n_1^2 = \mu_0 A \ell\, N_1^2/\ell$ and $L_2 = \mu_0 A \ell n_2^2$

Also $B_{total} = \mu_0 (n_1 + n_2) I$. This means that the total flux is:

$I_B = \left[\mu_0 (n_1 + n_2) I\right] \cdot A \cdot \left[N_1 + N_2\right]$. This implies that $\therefore\ L = \mu_0 (n_1 + n_2) \cdot A$.

$L = \mu_0 \left[N_1 + N_2\right]^2 \cdot A/\ell$.

(b) For opposite direction

$L = \mu_0 \left[N_2 - N_1\right]^2 A/\ell$.

(c) The mutual inductance is obtained by first finding $B_1 = \mu_0 N_1 I = \mu_0 N_1 I/\ell$. Therefore the flux

$$\phi_{321} = B_1 \cdot A \cdot N_Z = \mu_0 A N_1 N_2 I/\ell \ .$$

$$L = \mu_0 A N_1 N_2/\ell \ .$$

Energy Considerations: Go back to e.g. $\rightarrow$
If induced current is opposite to current from battery, the battery has to do additional work to set up the current. In the steady state energy dissipated per second is: $E_0^2/R \rightarrow$ Joule.

Additional work is $\rightarrow \left(L^{dI}/dt\right)^2 \rightarrow$ is not constant. R. $-$ need to integrate t = 0 $\rightarrow$ t = ∞ to get energy.

Let us take a simpler approach: Consider only an inductor being driver by E_o
Power delivered by battery: Power = $I \cdot E_o$
But $E_o + E_{ind} = 0$ [loop rule]

$$E_o = -E_{ind} = +L\frac{dI}{dt}$$

$$\therefore\ P = LI\frac{dI}{dt} = \frac{dw}{dt}$$

Change in energy: as current goes from I_1 to I_2 is:

$$dv = L\int_{I_1}^{I_2} IdI = \frac{1}{2}L\left(I_2^2 - I_1^2\right)$$

$$v_L = \frac{1}{2}LI^2 \qquad\qquad [\text{compare with } v_c = \frac{1}{2}c \cdot v^2]$$

Energy in Magnetic Fields: The above energy is stored in the inductor

$$v_L = \frac{1}{2}LI^2 = \frac{1}{2}\mu_o n^2 \left(\pi R^2\right)\ell \cdot I^2 = \frac{1}{2}\frac{B^2}{\mu_o} \cdot A \cdot \ell$$

$$\frac{v_L}{A \cdot \ell} = \mu_B = \frac{1}{2}\frac{B^2}{\mu_o} \text{ Energy density.} \quad [\text{compare w/ } \mu_E = \frac{1}{2}\ e_0\ E^2\ ]$$

The energy density is $\mu = \mu_B + \mu_E$ when both fields present.