Transformers

Motivation: It is more efficient to transmit power at high voltages. Consider power being transmitted through power lines which have a resistance R_w. Not all power P_t arrives at the output. A part P_{diss} is lost in the wires and only a part P_{av} is available for use. Thus $P_t = P_{av} + P_{diss} = I.V$. Since we would like to transmit large amounts of power in principle this can be done in either of two ways - with large I or with large V. However, there is a difference. The power dissipated in the wire is

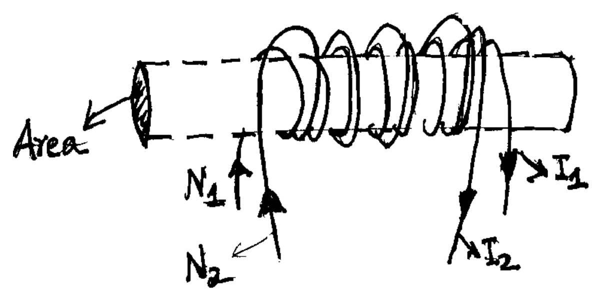
$$P_{diss} = I^2.R_w = \frac{P_t^2}{V^2} R_w$$

Hence the ratio $\frac{P_{diss}}{P_t} = \frac{P_t}{V^2} R_w$. Since P_t and R_w are fixed this ratio decreases as V increases. The decrease occurs quite rapidly and the gains realized can be quite substantial.

So how does one transform voltages UP to high values ? i.e. we need a device like the one in the figure below:

Design 1: Consider two solenoids 1 and 2 each with N_1 and N_2 turns respectively carrying currents I_1 and I_2 which are wound on the same cross-sectional area a shown in the figure.

If there is a changing current I1 in the first solenoid then there is an emf across this solenoid which is given by:



$$e_1 = -L \frac{dI_1}{dt}$$ and due to the mutual inductance between the two solenoids we also have:

$$e_2 = -M_{21} \frac{dI_1}{dt} = -M \frac{dI_1}{dt}$$

Therefore

$\frac{e_2}{e_1} = \frac{M}{L}$ =constant means that coil 2 has emf which scales with the emf in coil 1. For an ideal solenoid we know how to calculate L and M explicitly. We have:

$L = \frac{m_0 N_1^2 A}{\ell}$ and $M = m_0 \frac{N_1 N_2 A}{\ell}$. These equations imply that: $\frac{e_2}{e_1} = \frac{N_2}{N_1}$.

For $N_2 > N_1$ we call the transformer step-up and for $N_2 < N_1$ it is a step-down.

How are the currents related in the two coils ? Is it possible to have the same current in the secondary ? No ! remember that energy must be conserved. Since power is the same we have $I_1 e_1 = I_2 e_2$. And this means that:

$\frac{I_1}{I_2} = \frac{N_2}{N_1}$. A step-up transformer thus lowers I.