DC - Circuits

EMF: Consider the problem of a 9-V battery which is driving 50 mA through a bulb for 8 hours. How much energy does the battery provide ? This energy is stored in the battery as chemical energy. What happens inside a battery actually is a chemical reaction – and in the process chemical energy is converted to electrical energy. Similarly the little "generator" attached to your bicycle converts mechanical energy into electrical energy. This process also occurs for example when water comes down the Hoover Dam - the potential energy stored in the water is converted to electrical energy. In all these instances the electrical energy generated is given a special name "emf' or electromotive force. (Note: EMF is not a force – rather an energy). This source of energy is hooked up to a load say a lamp, R to form an arrangement which we call as circuit through which an electrical current flows as shown in figure A below.

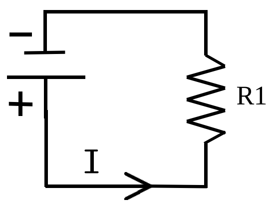


Figure (A)

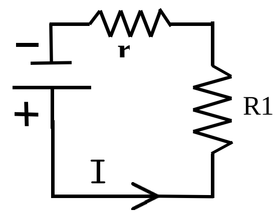


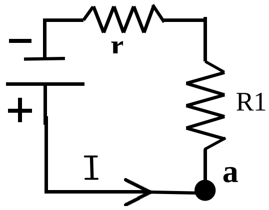
Figure (B)

For the time being the current flows only one way and such a circuit is called a D.C. circuit. In the above circuit the current "I" goes from the + terminal of the battery through R to the – terminal by convention. The electrons go the opposite way. Inside the battery through the electrons are going from + to -. This is the important role of a battery. Normally electrons get attracted to +ve poles. However batteries are able to do the opposite. This needs work. Let us say that dW is the work done to move dq of electrons. Then per unit charge the work done **E (emf) = dW/dq**. A dry cell has E = 1.5 V. The EMF across the battery terminals combined with the "size" of the battery is a measure of the "energy" stored in the battery. The current in this circuit is given according to Ohms law by **I = E/R** and the power through the "load" R is $\mathbf{P = E^2/R}$.

Internal Resistance of a Battery: Batteries are not "perfect" devices. Of the total energy stored in them not all of it can be extracted in a useful manner ! This is because they have an "internal resistance". We have to modify the circuit in figure (A) to include the internal resistance 'r'. The current in the circuit then gets modified to **I = E/(R + r)** and the power, $\mathbf{P = E^2/(R + r)}$. Thus the power P is non zero even if there is no load R connected outside. This is the reason batteries get hot immediately if they are 'shorted' because all the power is converted to heat inside the battery.

Kirchoff's I Law: Now let us look at the second circuit in terms of voltages. Let the current = I . Since there is only one path I is same everywhere. Thus the voltage drop across the resistor R, $\mathbf{V_R = I\,R}$ and $\mathbf{V_R + V_r = E}$. In this equation using Ohms law we get **IR + Ir = E** which can be rewritten as **IR + Ir - E = 0**. What is the physical meaning of this last equation?

Start from the point "a" in the figure below and move counter clockwise.



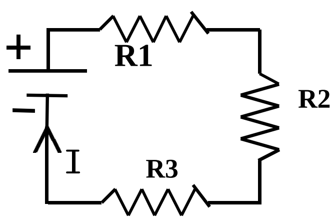
There is a I.R drop across R and similarly a drop I.r across r and then a '–E' across the battery and back to "a". This is nothing but the statement of Kirchoff`s I Law also called the loop rule.

$$\sum_{closedloop} \Delta V_i = 0$$

This rule applies to any closed path in any circuit. Examples: wiring in an - air plane, automobile, etc.

Applications of Kirchoffs loop rule:

(a) Resistors in series:



$$IR_1 + IR_2 + IR_3 - E = 0$$

$$\sum_{closedloop} \Delta V_i = IR_1 + IR_2 + IR_3 - E = 0 \text{ or } (R_1 + R_2 + R_3).I = E \text{ or } IR_{eq} = E \text{ where}$$

$$R_{eq} = (R_1 + R_2 + R_3)$$

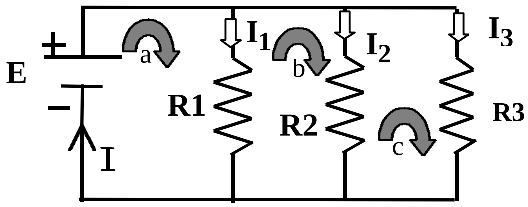
Progress to more complicated circuits now! What if we had two or more loops and they "intersected" ~ multi-loops. It is "handy" to establish a second rule – Kirchoff's second law or "junction rule".

Kirchoff's Second Law: states that the algebraic sum of the current entering a junction equals zero. In applying this law we should remember to assign "polarity" to current. Thus this law is merely is a statement of the conservation of current.

$$\sum_{junction} \Delta I_i = 0$$

Applications of the "junction" rule:

(a) Resistors in parallel:



For loop 'a': $E - I_1 R_1 = 0$

For loop 'b': $I_1 R_1 - I_2 R_2 = 0$

For loop 'c': $I_2 R_2 - I_3 R_3 = 0$ OR $E = I_1 R_1 = I_2 R_2 = I_3 R_3$ OR

$I_1 = \frac{E}{R_1} = \frac{E}{R_2} = \frac{E}{R_3}$. Since current is conserved $I = I1 + I2 + I3$
Therefore,

$$I = \frac{E}{R_{eq}} = \frac{E}{R_1} + \frac{E}{R_2} + \frac{E}{R_3} \qquad \text{OR} \qquad \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Resistors in parallel add inversely.