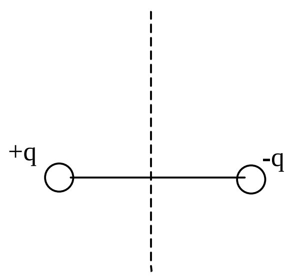
THE ELECTRIC DIPOLE

Electric Dipole: If two equal and opposite charges do not sit on top of each other - you get a dipole. Individually the field due to each pole falls off as 1/r2 but together how do they behave?

Define electric dipole moment as product of the charge and the separation between them.
Field along ⊥ line of intersection:



Net $\vec{E} = \vec{E}_1 + \vec{E}_2$; but in this vector addition y-components cancel & only x-components add.
Thus $\vec{E} = E_x\hat{i} = 2E_{1x}\hat{i}$ where

$$E_{1x} = \frac{q}{4\pi\epsilon_0 r^2} Cos\theta$$ but Cosθ =L/2r

Therefore $\vec{E} = \frac{2q}{4\pi\epsilon_0 r^2}\frac{L}{2r}\hat{i} = \frac{qL}{4\pi\epsilon_0 r^3}\hat{i}$ OR $\vec{E} = \frac{-\vec{p}}{4\pi\epsilon_0 r^3}$ since p is opposite to i.

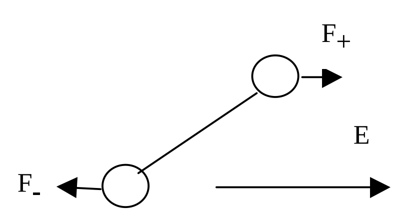
This is true for all "r".
Electric dipoles abound in Nature. Some examples are water molecule, ...

Energy of a dipole in an external field: The electric field does +ve work when aligning a dipole that is misaligned.

This work done is stored as potential energy of the dipole in the field. If $\Delta U = U - U_0$ then,

$$\Delta U = W = \int_{\theta i}^{\theta f} pESin\theta d\theta = pE(Cos\theta_i - Cos\theta_f)$$

Suppose $\theta_i = \pi/2$ & we choose $U_i = 0$ and $\theta_f = \theta$ then U = - PE Cos θ or
The electric dipole in an external field:



Force on +q: $\vec{F}_+ = q\vec{E}$

Force on -q: $\vec{F}_- = -q\vec{E}$

Forces cancel but there is a torque.

$\vec{t} = \vec{r} x \vec{F}$ where r is from '0' to '+q' or '-q'.

In terms of the dipole moment this torque is $\vec{t} = \vec{p} x \vec{E}$

Maximum torque occurs when p and E are ⊥.

Energy stored in a dipole = work done to rotate dipole. This is dW = τ d θ = q L E sin θ d θ

<u>Electric field of a continuous charge distribution</u>:

By definition: $\Delta E = \frac{\Delta q}{4pe_0 r^2} \hat{r}$

To get total electric field integrate over volume.

$$\vec{E} = \frac{1}{4pe_0} \int \frac{dq}{r^2} \hat{r}$$

<u>Motion of charge in a field:</u> Given an electric field, E_{ext} we can always find a force, on a charge say an electron. This principle is utilized in many practical devices such as electrostatic accelerator, electrostatic precipitator which removes dust particles. Dust particles almost always have some charge.

The equation of motion of such a charged particle is given by: $\vec{F} = q\vec{E}_{ext}$

Suppose σ is the surface charge density then $\vec{E} = E_x = \frac{s}{e_0}$ because there are two plates.

Therefore the acceleration $a_x = \frac{qs}{me_0}$

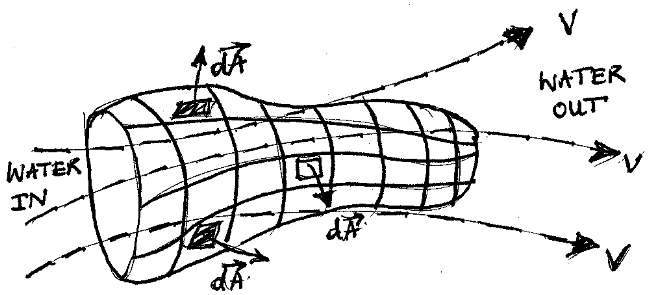
Suppose we inject a charged particle in a direction ⊥ to the field. Acceleration is along x axis:

We can write in general $a = a_x \hat{i} + a_y \hat{j}$ but $a_y = 0$

Time taken to travel distance "L_1" is T=L1/Vo

Deflection along x is: $x = \frac{1}{2} a_y t^2 = \frac{1}{2}\left(\frac{qE}{m}\right) T^2 = \frac{1}{2}\left(\frac{qE}{m}\right)\left(\frac{L_1}{V_0}\right)^2$

Electric Flux: Analogy to flow of water. Velocity profile.



Amount of water through the area element dA is:

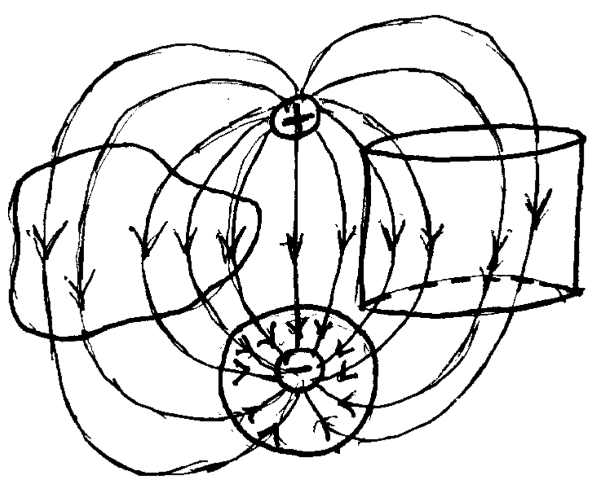
$$d\Phi_w = \vec{v}.d\vec{A}$$

Therefore the amount of water through the surface of the net is:

$$\Phi_w = \int_{surfaceofnet} d\Phi_w = \int_S \vec{v}.d\vec{A}$$

Instead of velocity lines - replace with electric field lines: - however nothing really flows in this case - then we get Electric flux. The surface "S" does not have to be "real" either. Any "useful" or "necessary" surface can be taken. consider the case of the dipole - can have many different fish nets - called Gaussian surfaces.

Example: Consider the case of an electric dipole. The field lines are shown below



Notice: Something interesting here: - the number of field lines entering the chosen surface is equal to that leaving it (i.e. the flux ϕ) is zero in two cases. In the limit of small cylinder all field lines are $\perp$ in and out. In both these cases the net charge enclosed inside is zero also whenever there is a net charge enclosed (as in the case of the sphere) there is an x-s of field of lines either entering or leaving. Based on this we can suggest that the flux is related to the "total charge" enclosed by the surface.