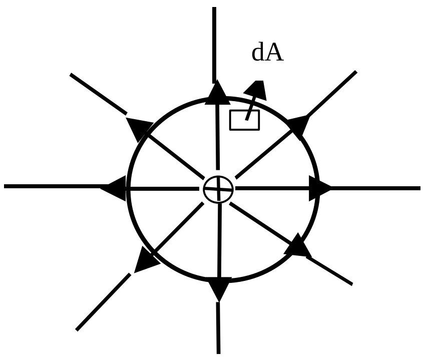
GAUSS' LAW

Consider a point charge +q. The electric field due to q is given by the equation

$E = \left(\dfrac{q}{4\pi\epsilon_0 r^2}\right)\hat{r}$ and is radially out. Consider an area element dA as shown in the figure



- the normal to this is also radially out. Therefore the flux out of the sphere is:

$$\Phi = \int_{Sphere} \vec{E}.d\vec{A} = \int_{sphere} \left(\frac{q}{4\pi\epsilon_0 R^2}\right) dA$$

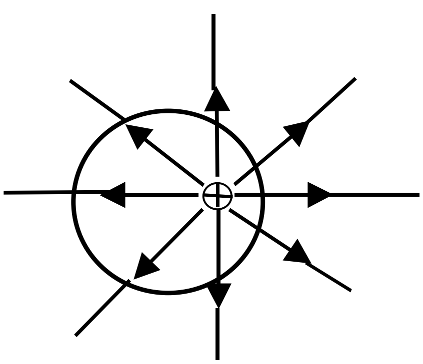
$$\Phi = \left(\frac{q}{4\pi\epsilon_0 R^2}\right) \int_{sphere} dA = \frac{q}{\epsilon_0}$$

i.e. the number of field lines is independent of the radius of the sphere we chose.

Now we could also choose the gaussian sphere not be centered on the charge but instead as shown in the figure below. Note however that the number of field lines crossing the sphere does not change. Thus the flux out of the sphere is same as before and we can conjecture that
Or for that matter we can replace the sphere with an arbitrary shape which encloses the charge +q and still have the same number of lines crossing the surface. Therefore:

$$\Phi = \int_{dispalcedsphere} \vec{E}.d\vec{A} = \frac{q}{\epsilon_0}$$

Now instead of a single charge we had a distribution of charges q1, q2 etc.then the total flux is:



$$\Phi = \int_{closedsurface} \vec{E}.d\vec{A} = \frac{q}{\epsilon_0}$$

$$\Phi = \int_{closedsurface} \vec{E}_1.d\vec{A} + \int_{closedsurface} \vec{E}_2.d\vec{A} + \int_{closedsurface} \vec{E}_3.d\vec{A} = \frac{q_1}{\epsilon_0} + \frac{q_2}{\epsilon_0} + \frac{q_3}{\epsilon_0} = \frac{Q}{\epsilon_0}$$

where Q=q1+q2+q3 etc.
This result is referred to as Gauss' Law.